CHAPTER 4 Rational Expressions, Equations, and Functions

SECTION 4.1 Integer Exponents and Scientific Notation

1. The graph of an equation is the set of solution points of the equation on a rectangular coordinate system.

2. The point-plotting method for graphing an equation begins by creating a table of solution points of the equation. Plot these points on a rectangular coordinate system, and connect the points with a smooth curve or line.

3. $g(x) = \sqrt{x-2}$

$g(2) = \sqrt{2-2} = \sqrt{0} = 0$ $\quad (2, 0)$

$g(6) = \sqrt{6-2} = \sqrt{4} = 2$ $\quad (6, 2)$

4. To find the x-intercept, let $y = 0$ and solve the equation for x. To find the y-intercept, let $x = 0$ and solve the equation for y.

5. $(7x^2)(2x^3) = 14x^{2+3} = 14x^5$

6. $(y^2z^3)(z^2)^4 = (y^2z^3)(z^8) = y^2z^{3+8} = y^2z^{11}$

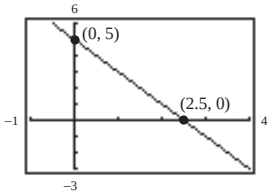
7. $\dfrac{a^4b^2}{ab^2} = a^{4-1}b^{2-2} = a^3b^0 = a^3$

8. $(x+2)^4 \div (x+2)^3 = (x+2)^{4-3} = (x+2)^1 = x+2$

9. $f(x) = 5 - 2x$

Keystrokes:

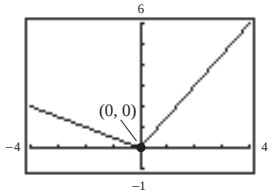
Y= 5 − 2 X,T,θ GRAPH



10. $h(x) = \frac{1}{2}x + |x|$

Keystrokes:

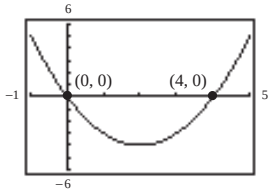
Y= (1 ÷ 2) X,T,θ + ABS X,T,θ GRAPH



11. $g(x) = x^2 - 4x$

Keystrokes:

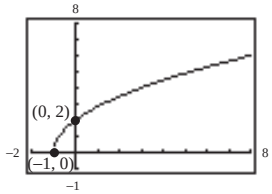
Y= X,T,θ x² − 4 X,T,θ GRAPH



12. $f(x) = 2\sqrt{x+1}$

Keystrokes:

Y= 2 √ (X,T,θ + 1) GRAPH



SECTION 4.2 Rational Expressions and Functions

1. Slope $= m = \dfrac{y_2 - y_1}{x_2 - x_1}$

2. (a) $m > 0$ (b) $m < 0$
(c) $m = 0$ (d) m is undefined.

3. $2(x + 5) - 3 - (2x - 3) = 2x + 10 - 2x$
$= 10$

4. $3(y + 4) + 5 - (3y + 5) = 3y + 12 + 5 - 3y - 5$
$= 12$

5. $4 - 2[3 + 4(x + 1)] = 4 - 2[3 + 4x + 4]$
$= 4 - 2[7 + 4x]$
$= 4 - 14 - 8x$
$= -10 - 8x$

6. $5x + x[3 - 2(x - 3)] = 5x + x[3 - 2x + 6]$
$= 5x + x[9 - 2x]$
$= 5x + 9x - 2x^2$
$= -2x^2 + 14x$

7. $\left(\dfrac{5}{x^2}\right)^2 = \dfrac{25}{x^4}$

8. $-\dfrac{(2u^2v)^2}{-3uv^2} = -\dfrac{4u^4v^2}{-3uv^2} = \dfrac{4u^3}{3}$

9. *Verbal Model:* $\boxed{\text{Gallons solution 1}} \cdot 30\% + \boxed{\text{Gallons solution 2}} \cdot 60\% = \boxed{\text{Total gallons}} \cdot 40\%$

Labels: Gallons solution 1 $= x$
Gallons solution 2 $= 20 - x$
Total gallons $= 20$

Equation: $0.30x + 0.60(20 - x) = 20(0.40)$
$0.30x + 12 - 0.60x = 8$
$-0.30x = -4$
$x = 13\frac{1}{3}$ gallons at 30%
$20 - x = 6\frac{2}{3}$ gallons at 60%

10. *Verbal Model:* $\boxed{\text{Original price}} \cdot 75\% = \boxed{\text{Sale price}}$

Labels: Original price $= x$
Sale price $= \$375$

Equation: $x \cdot 0.75 = 375$
$x = \$500$

SECTION 4.3 Multiplying and Dividing Rational Expressions

1. $9t^2 - 4 = (3t - 2)(3t + 2)$

2. $4x^2 - 12x + 9 = (2x - 3)^2$

3. $8x^3 + 64 = (2x + 4)(4x^2 - 8x + 16)$

4. $3x^2 + 13x - 10 = (3x - 2)(x + 5)$

5. $5x - 20x^2 = 5x(1 - 4x)$

6. $64 - (x - 6)^2 = [8 - (x - 6)][8 + (x - 6)]$
$= (8 - x + 6)(8 + x - 6)$
$= (14 - x)(2 + x)$

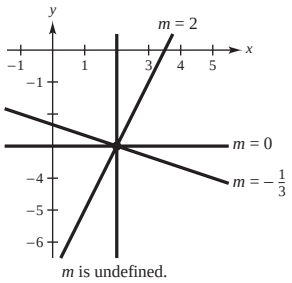
7. $15x^2 - 16x - 15 = (5x + 3)(3x - 5)$

8. $16t^2 + 8t + 1 = (4t + 1)^2$

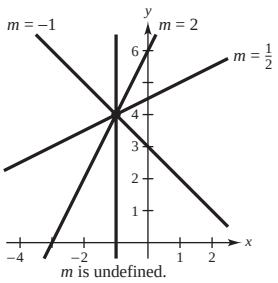
9. $y^3 - 64 = (y - 4)(y^2 + 4y + 16)$

10. $8x^3 + 1 = (2x + 1)(4x^2 - 2x + 1)$

11.



12.



SECTION 4.4 Adding and Subtracting Rational Expressions

1. (a) $5y - 3x - 4 = 0$

$5y = 3x + 4$

$y = \frac{3}{5}x + \frac{4}{5}$

(b) $5y - 3x - 4 = 0$

$5y = 3x + 4$

$y = \frac{3}{5}x + \frac{4}{5}$

$y - y_1 = \frac{3}{5}(x - x_1) + \frac{4}{5}$ Let $x_1 = 1$.

$y - y_1 = \frac{3}{5}(x - 1) + \frac{4}{5}$

$y - \frac{7}{5} = \frac{3}{5}(x - 1)$ $y_1 = \frac{7}{5}$

(Many answers)

2. If $m > 0$, the line rises from left to right.
If $m < 0$, the line falls from left to right.

3. $-6x(10 - 7x) = -60x + 42x^2$

$= 42x^2 - 60x$

4. $(2 - y)(3 + 2y) = 6 + 4y - 3y - 2y^2$

$= 6 + y - 2y^2$

5. $(11 - x)(11 + x) = 121 - x^2$

6. $(4 - 5z)(4 + 5z) = 16 + 20z - 20z - 25z^2$

$= 16 - 25z^2$

7. $(x + 1)^2 = (x + 1)(x + 1)$

$= x^2 + 2x + 1$

8. $t(t^2 + 1) - t(t^2 - 1) = t^3 + t - t^3 + t$

$= 2t$

9. $(x - 2)(x^2 + 2x + 4) = x^3 + 2x^2 + 4x - 2x^2 - 4x - 8$

$= x^3 - 8$

10. $t(t - 4)(2t + 3) = t(2t^2 + 3t - 8t - 12)$

$= t(2t^2 - 5t - 12)$

$= 2t^3 - 5t^2 - 12t$

11. Perimeter = Sum of all sides

$= 7(x) + (x + 3) + (2x) + (2x + 3)$

$= 12x + 6$

Area = Area rectangle 1 + Area rectangle 2 + Area rectangle 3

$= (x \cdot x) + (x + 3)(3x) + (x \cdot x)$

$= x^2 + 3x^2 + 9x + x^2$

$= 5x^2 + 9x$

12. Perimeter = Sum of all sides

$= 3x + 4x + 5x$

$= 12x$

Area $= \frac{1}{2} \cdot$ Base $\cdot$ Height

$= \frac{1}{2} \cdot 3x \cdot 4x$

$= 6x^2$

SECTION 4.5 Dividing Polynomials

1. $\frac{120y}{90} = \frac{30 \cdot 4y}{30 \cdot 3} = \frac{4y}{3}$ Divide the numerator and the denominator by 30.

2. $(2n + 1)(2n + 3) = 4n^2 + 6n + 2n + 3 = 4n^2 + 8n + 3$

3. $(2n + 1) + (2n + 3) = 4n + 4$

4. $2n(2n + 2) = 4n^2 + 4n$

5.
$$\begin{aligned} 3(2 - x) &= 5x \\ 6 - 3x &= 5x \\ 6 - 3x + 3x &= 5x + 3x \\ 6 &= 8x \\ \frac{6}{8} &= \frac{8x}{8} \\ \frac{3}{4} &= x \end{aligned}$$

6.
$$\begin{aligned} 125 - 50x &= 0 \\ 125 - 125 - 50x &= 0 - 125 \\ -50x &= -125 \\ \frac{-50x}{-50} &= \frac{-125}{-50} \\ x &= \frac{5}{2} \end{aligned}$$

7.
$$\begin{aligned} 8y^2 - 50 &= 0 \\ 2(4y^2 - 25) &= 0 \\ 2(2y - 5)(2y + 5) &= 0 \\ 2y - 5 = 0 \quad &\text{or} \quad 2y + 5 = 0 \\ y = \tfrac{5}{2} \quad &\text{or} \quad y = -\tfrac{5}{2} \end{aligned}$$

8.
$$\begin{aligned} t^2 - 8t &= 0 \\ t(t - 8) &= 0 \\ t = 0 \quad &\text{or} \quad t - 8 = 0 \\ &\text{or} \quad t = 8 \end{aligned}$$

9. $x^2 + x - 42 = 0$

$(x + 7)(x - 6) = 0$

$x + 7 = 0 \qquad x - 6 = 0$

$x = -7 \qquad x = 6$

10. $x(10 - x) = 25$

$10x - x^2 = 25$

$-x^2 + 10x - 25 = 0$

$x^2 - 10x + 25 = 0$

$(x - 5)^2 = 0$

$x - 5 = 0$

$x = 5$

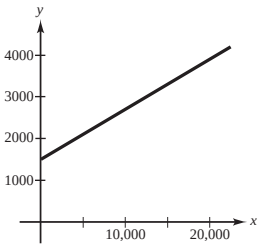
11. *Verbal model:* Salary = Salary + Commission

Labels: Monthly wage $= y$

Salary $= 1500$

Commission $= 0.12x$

Function: $y = 1500 + 0.12x$

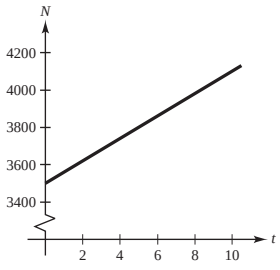


12. *Verbal model:* Total enrollment = Enrollment per year + 3500

Labels: Total enrollment $= n$

Enrollment per year $= 60t$

Function: $n = 60t + 3500$



SECTION 4.6 Solving Rational Equations

1. $(-2, y)$ can be located in quadrants II or III.

2. $(x, 3)$ can be located in quadrants I or II.

3. Points whose y-coordinates are 0 are located on the x-axis.

4. $(9, -6)$

5. $7 - 3x > 4 - x$

$-2x > -3$

$x < \frac{3}{2}$

6. $2(x + 6) - 20 < 2$

$2x + 12 - 20 < 2$

$2x - 8 < 2$

$2x < 10$

$x < 5$

7. $|x - 3| < 2$

$-2 < x - 3 < 2$

$1 < x < 5$

8. $|x - 5| > 3$

$x - 5 > 3 \quad \text{or} \quad x - 5 < -3$

$x > 8 \quad \text{or} \quad x < 2$

9. $\left|\frac{1}{4}x - 1\right| \geq 3$

$\frac{1}{4}x - 1 \geq 3$ or $\frac{1}{4}x - 1 \leq -3$

$\frac{1}{4}x \geq 4$ $\frac{1}{4}x \leq -2$

$x \geq 16$ or $x \leq -8$

10. $\left|2 - \frac{1}{3}x\right| \leq 10$

$-10 \leq 2 - \frac{1}{3}x \leq 10$

$-12 \leq -\frac{1}{3}x \leq 8$

$36 \geq x \geq -24$

$-24 \leq x \leq 36$

11.

Labels: Distance $= d$

1st jogger's rate $= 6$; 1st jogger's time $= x + \frac{5}{60}$

2nd jogger's rate $= 8$; 2nd jogger's time $= x$

Equation: $d = 6\left(x + \frac{1}{12}\right)$

$d = 8x$

$6\left(x + \frac{1}{12}\right) = 8x$

$6x + \frac{1}{2} = 8x$

$\frac{1}{2} = 2x$

$\frac{1}{4} = x$ hours, or 15 minutes

$d = 8\left(\frac{1}{4}\text{ hour}\right) = 2$ miles

12. *Verbal Model:* $\boxed{\text{Amount at 7.5\%}} + \boxed{\text{Amount at 9\%}} = 24{,}000$

$7.5\% \cdot \boxed{\text{Amount at 7.5\%}} + 9\% \cdot \boxed{\text{Amount at 9\%}} = 1935$

Labels: Amount at 7.5% $= x$

Amount at 9% $= y$

System: $x + y = 24{,}000$

$0.075x + 0.09y = 1935$

$y = 24{,}000 - x$

$0.075x + 0.09(24{,}000 - x) = 1935$

$0.075x + 2160 - 0.09x = 1935$

$-0.015x = -225$

$x = \$15{,}000$ at 7.5%

$y = \$9000$ at 9%

CHAPTER 5 Radicals and Complex Numbers

SECTION 5.1 Radicals and Rational Exponents

1. $a^m \cdot a^n = a^{m+n}$ **2.** $(ab)^m = a^m b^m$ **3.** $(a^m)^n = a^{mn}$ **4.** $\frac{a^m}{a^n} = a^{m-n}$, if $m > n$