

111. Verbal Model: $\boxed{\text{Rate of person 1}} + \boxed{\text{Rate of person 2}} = \boxed{\text{Rate together}}$

Labels: Supervisor's time = 12
Your time = 15
Time together = x

Equation: $\frac{1}{12} + \frac{1}{15} = \frac{1}{x}$

$$60x\left(\frac{1}{12} + \frac{1}{15}\right) = \left(\frac{1}{x}\right)60x$$

$$5x + 4x = 60$$

$$9x = 60$$

$$x = \frac{60}{9}$$

$$x = \frac{20}{3} = 6\frac{2}{3} \text{ min or 6 min 40 sec}$$

113. (a) $N = \frac{20[4 + 3(5)]}{1 + 0.05(5)} = 304,000$

$$N = \frac{20[4 + 3(10)]}{1 + 0.05(10)} \approx 453,333$$

$$N = \frac{20[4 + 3(25)]}{1 + 0.05(25)} \approx 702,222$$

(b) $752 = \frac{20(4 + 3t)}{1 + 0.05t}$

$$752(1 + 0.05t) = 20(4 + 3t)$$

$$752 + 37.6t = 80 + 60t$$

$$672 = 22.4t$$

$$29.8 \text{ years} \approx t$$

Chapter Test for Chapter 4

1. $2^{-2} + 2^{-3} = \frac{1}{2^2} + \frac{1}{2^3}$

$$= \frac{1}{4} + \frac{1}{8}$$

$$= \frac{2}{8} + \frac{1}{8}$$

$$= \frac{3}{8}$$

2. $\frac{6.3 \times 10^{-3}}{2.1 \times 10^2} = 3 \times 10^{-3-2}$

$$= 3 \times 10^{-5}$$

3. $(5a^{-3})(2a^2) = 10a^{-3+2}$

$$= 10a^{-1}$$

$$= \frac{10}{a}$$

4. $\frac{r^2s^{-3}}{r^5s^2} = r^{2-5}s^{-3-2}$

$$= r^{-3}s^{-5}$$

$$= \frac{1}{r^3s^5}$$

5. $(x^2y^{-3})^4 = x^{2 \cdot 4}y^{-3 \cdot 4}$

$$= x^8y^{-12}$$

$$= \frac{x^8}{y^{12}}$$

6. $(3x^2y^2)^3(2x^{-2}y)^2 = (3^3x^6y^6)(2^2x^{-4}y^2)$

$$= 27 \cdot 4x^{6+(-4)}y^{6+2}$$

$$= 108x^2y^8$$

7. $0.000032 = 3.2 \times 10^{-5}$

8. $3.04 \times 10^7 = 30,400,000$

9. $y^2 - 25 \neq 0$

$(y - 5)(y + 5) \neq 0$

$y \neq 5, -5$

$D = (-\infty, -5) \cup (-5, 5) \cup (5, \infty)$

10. Least common denominator: $x^3(x - 3)(x + 3)$

11. (a) $\frac{2-x}{3x-6} = \frac{2-x}{-3(-x+2)} = -\frac{1}{3}, x \neq 2$

12. $\frac{4z^3}{5} \cdot \frac{25}{12z^2} = \frac{4 \cdot z^2 \cdot z \cdot 5 \cdot 5}{5 \cdot 4 \cdot 3 \cdot z^2} = \frac{5z}{3}, z \neq 0$

(b) $\frac{2a^2 - 5a - 12}{5a - 20} = \frac{(2a+3)(a-4)}{5(a-4)}$
 $= \frac{2a+3}{5}, a \neq 4$

13. $\frac{y^2 + 8y + 16}{2(y-2)} \cdot \frac{8y-16}{(y+4)^3} = \frac{(y+4)^2 \cdot 8(y-2)}{2(y-2)(y+4)^2(y+4)}$
 $= \frac{4}{y+4}, y \neq 2$

14. $(4x^2 - 9) \cdot \frac{2x+3}{2x^2 - x - 3} = \frac{(2x-3)(2x+3)(2x+3)}{(2x-3)(x+1)}$
 $= \frac{(2x+3)^2}{x+1}, x \neq \frac{3}{2}$

15. $\frac{(2xy^2)^3}{15} \div \frac{12x^3}{21} = \frac{(2xy^2)^3}{15} \cdot \frac{21}{12x^3}$
 $= \frac{8x^3y^6 \cdot 7 \cdot 3}{5 \cdot 3 \cdot 4 \cdot 3x^3}$
 $= \frac{14y^6}{15}, x \neq 0$

16. $\frac{\left(\frac{3x}{x+2}\right)}{\left(\frac{12}{x^3+2x^2}\right)} = \frac{3x}{x+2} \div \frac{12}{x^3+2x^2}$
 $= \frac{3x}{x+2} \cdot \frac{x^2(x+2)}{12}$
 $= \frac{x^3}{4}, x \neq 0, -2$

17. $\frac{\left(\frac{9x-1}{x}\right)}{\left(\frac{1}{x}-3\right)} = \frac{\left(\frac{9x-1}{x}\right)}{\left(\frac{1}{x}-3\right)} \cdot \frac{x}{x}$
 $= \frac{9x(x) - \frac{1}{x}(x)}{\frac{1}{x}(x) - 3(x)}$
 $= \frac{9x^2 - 1}{1 - 3x}$
 $= \frac{(3x-1)(3x+1)}{-1(-1+3x)}$
 $= -(3x+1), x \neq 0, \frac{1}{3}$

18. $2x + \frac{1-4x^2}{x+1} = 2x\left(\frac{x+1}{x+1}\right) + \frac{1-4x^2}{x+1}$
 $= \frac{2x^2+2x}{x+1} + \frac{1-4x^2}{x+1}$
 $= \frac{-2x^2+2x+1}{x+1}$

$$\begin{aligned}
 19. \quad \frac{5x}{x+2} - \frac{2}{x^2-x-6} &= \frac{5x}{x+2} - \frac{2}{(x-3)(x+2)} \\
 &= \frac{5x}{x+2} \left(\frac{x-3}{x-3} \right) - \frac{2}{(x-3)(x+2)} \\
 &= \frac{5x^2 - 15x - 2}{(x+2)(x-3)}
 \end{aligned}$$

$$\begin{aligned}
 20. \quad \frac{3}{x} - \frac{5}{x^2} + \frac{2x}{x^2+2x+1} &= \frac{3}{x} - \frac{5}{x^2} + \frac{2x}{(x+1)^2} \\
 &= \frac{3 \left[\frac{x(x+1)^2}{x(x+1)^2} \right] - \frac{5 \left[\frac{x(x+1)^2}{x^2(x+1)^2} \right] + \frac{2x}{(x+1)^2} \left(\frac{x^2}{x^2} \right)}{x^2(x+1)^2} \\
 &= \frac{3x(x^2+2x+1) - 5(x^2+2x+1) + 2x^3}{x^2(x+1)^2} \\
 &= \frac{3x^3 + 6x^2 + 3x - 5x^2 - 10x - 5 + 2x^3}{x^2(x+1)^2} \\
 &= \frac{5x^3 + x^2 - 7x - 5}{x^2(x+1)^2}
 \end{aligned}$$

$$\begin{aligned}
 21. \quad \frac{4}{x+1} + \frac{4x}{x+1} &= \frac{4+4x}{x+1} \\
 &= \frac{4(1+x)}{x+1} \\
 &= 4, \quad x \neq -1
 \end{aligned}$$

$$\begin{aligned}
 22. \quad \frac{t^4 + t^2 - 6t}{t^2 - 2} &= t^2 - 2 \overline{) t^4 + 0t^3 + t^2 - 6t + 0} \\
 &\quad \underline{t^4 \quad \quad - 2t^2} \\
 &\quad \quad \quad 3t^2 - 6t \\
 &\quad \quad \quad \underline{3t^2 \quad \quad - 6} \\
 &\quad \quad \quad \quad \quad -6t + 6
 \end{aligned}$$

$$\begin{aligned}
 23. \quad \frac{2x^4 - 15x^2 - 7}{x-3} & \\
 3 \left| \begin{array}{cccccc} 2 & 0 & -15 & 0 & -7 \\ & 6 & 18 & 9 & 27 \\ \hline & 2 & 6 & 3 & 9 & 20 \end{array} \right. & \\
 \frac{2x^4 - 15x^2 - 7}{x-3} &= 2x^3 + 6x^2 + 3x + 9 + \frac{20}{x-3}
 \end{aligned}$$

$$\begin{aligned}
 24. \quad \frac{3}{h+2} &= \frac{1}{8} \\
 3(8) &= h+2 \\
 24 &= h+2 \\
 22 &= h
 \end{aligned}$$

Check:

$$\begin{aligned}
 \frac{3}{22+2} &= \frac{1}{8} \\
 \frac{3}{24} &= \frac{1}{8} \\
 \frac{1}{8} &= \frac{1}{8}
 \end{aligned}$$

$$25. \quad \frac{2}{x+5} - \frac{3}{x+3} = \frac{1}{x}$$

$$2x(x+3) - 3x(x+5) = (x+5)(x+3)$$

$$2x^2 + 6x - 3x^2 - 15x = x^2 + 3x + 5x + 15$$

$$-2x^2 - 17x - 15 = 0$$

$$(-2x - 15)(x + 1) = 0$$

$$-2x - 15 = 0$$

$$-2x = 15$$

$$x = -\frac{15}{2}$$

$$x + 1 = 0$$

$$x = -1$$

Check:

$$\frac{2}{-\frac{15}{2} + 5} - \frac{3}{-\frac{15}{2} + 3} \stackrel{?}{=} \frac{1}{-\frac{15}{2}}$$

$$-\frac{12}{15} + \frac{10}{15} \stackrel{?}{=} -\frac{2}{15}$$

$$-\frac{2}{15} = -\frac{2}{15}$$

Check:

$$\frac{2}{-1 + 5} - \frac{3}{-1 + 3} \stackrel{?}{=} \frac{1}{-1}$$

$$\frac{2}{4} - \frac{3}{2} \stackrel{?}{=} 1$$

$$\frac{1}{2} - \frac{3}{2} \stackrel{?}{=} -1$$

$$-1 = -1$$

$$26. \quad \frac{1}{x+1} + \frac{1}{x-1} = \frac{2}{x^2-1}$$

$$x-1 + x+1 = 2$$

$$2x = 2$$

$$x = 1$$

Check:

$$\frac{1}{1+1} + \frac{1}{1-1} \neq \frac{2}{1-1}$$

Division by zero is undefined. Solution is extraneous, so equation has no solution.

$$27. \quad \begin{array}{l} \text{Verbal} \\ \text{Model:} \end{array} \quad \boxed{\text{Rate of painter 1}} + \boxed{\text{Rate of painter 2}} = \boxed{\text{Rate together}}$$

$$\text{Labels:} \quad \text{Time of painter 1} = x$$

$$\text{Time of painter 2} = \frac{3}{2}x$$

$$\text{Equation:} \quad \frac{1}{x} + \frac{1}{\frac{3}{2}x} = \frac{1}{4}$$

$$12x\left(\frac{1}{x} + \frac{2}{3x}\right) = \frac{1}{4}(12x)$$

$$12 + 8 = 3x$$

$$20 = 3x$$

$$\frac{20}{3} = 6\frac{2}{3} \text{ hours} = x$$

$$10 \text{ hours} = \frac{3}{2}x$$