

91. Area = Length · Width, so Width =  $\frac{\text{Area}}{\text{Length}}$ .

$$\text{Length} = \frac{2x^3 + 3x^2 - 6x - 9}{2x + 3} = x^2 - 3$$

$$\begin{array}{r} x^2 \quad - 3 \\ 2x + 3 \overline{) 2x^3 + 3x^2 - 6x - 9} \\ \underline{2x^3 + 3x^2} \phantom{- 9} \\ -6x - 9 \\ \underline{-6x - 9} \phantom{- 9} \\ \phantom{-6x - 9} \phantom{- 9} \phantom{- 9} \end{array}$$

95.  $x$  is not a factor of the numerator.

99. True. If  $\frac{n(x)}{d(x)} = q(x)$ , then  $n(x) = d(x) \cdot q(x)$ .

93. Volume = Area of triangle · Height (of prism)

$$\begin{aligned} \text{Area of triangle} &= \frac{\text{Volume}}{\text{Height (of prism)}} \\ &= \frac{x^3 + 18x^2 + 80x + 96}{x + 12} \\ &= x^2 + 6x + 8 \end{aligned}$$

$$\text{Area of triangle} = \frac{1}{2} \cdot \text{Base} \cdot \text{Height}$$

$$\begin{aligned} \text{Height} &= \frac{2(\text{Area of triangle})}{\text{Base}} \\ &= \frac{2(x^2 + 6x + 8)}{x + 2} \\ &= 2x + 8 \text{ or } 2(x + 4) \end{aligned}$$

97. A divisor divides evenly into a dividend when the remainder is 0 and the divisor is a factor of the dividend.

## Section 4.6 Solving Rational Equations

1. (a)  $x = 0$

$$\frac{0}{3} - \frac{0}{5} \stackrel{?}{=} \frac{4}{3}$$

$$0 \neq \frac{4}{3}$$

Not a solution

(b)  $x = -1$

$$\frac{-1}{3} - \frac{-1}{5} \stackrel{?}{=} \frac{4}{3}$$

$$\frac{-5}{15} - \frac{-3}{15} \stackrel{?}{=} \frac{20}{15}$$

$$\frac{-5}{15} + \frac{3}{15} \stackrel{?}{=} \frac{20}{15}$$

$$\frac{-2}{15} \neq \frac{20}{15}$$

Not a solution

(c)  $x = \frac{1}{8}$

$$\frac{1/8}{3} - \frac{1/8}{5} \stackrel{?}{=} \frac{4}{3}$$

$$\frac{1}{24} - \frac{1}{40} \stackrel{?}{=} \frac{4}{3}$$

$$\frac{5}{120} - \frac{3}{120} \stackrel{?}{=} \frac{160}{120}$$

$$\frac{2}{120} \neq \frac{160}{120}$$

Not a solution

(d)  $x = 10$

$$\frac{10}{3} - \frac{10}{5} \stackrel{?}{=} \frac{4}{3}$$

$$\frac{50}{15} - \frac{30}{15} \stackrel{?}{=} \frac{20}{15}$$

$$\frac{20}{15} = \frac{20}{15}$$

Solution

3. (a)  $x = -1$

$$\frac{-1}{4} + \frac{3}{4(-1)} \stackrel{?}{=} 1$$

$$\frac{-1}{4} + \frac{-3}{4} \stackrel{?}{=} 1$$

$$-1 \neq 1$$

Not a solution

(b)  $x = 1$

$$\frac{1}{4} + \frac{3}{4(1)} \stackrel{?}{=} 1$$

$$\frac{1}{4} + \frac{3}{4} \stackrel{?}{=} 1$$

$$1 = 1$$

Solution

(c)  $x = 3$

$$\frac{3}{4} + \frac{3}{4(3)} \stackrel{?}{=} 1$$

$$\frac{3}{4} + \frac{3}{12} \stackrel{?}{=} 1$$

$$\frac{3}{4} + \frac{1}{4} \stackrel{?}{=} 1$$

$$1 = 1$$

Solution

(d)  $x = 2$

$$\frac{2}{4} + \frac{3}{4(2)} \stackrel{?}{=} 1$$

$$\frac{4}{8} + \frac{3}{8} \stackrel{?}{=} 1$$

$$\frac{7}{8} \neq 1$$

Not a solution

5.  $\frac{x}{6} - 1 = \frac{2}{3}$

**Check:**  $\frac{10}{6} - 1 \stackrel{?}{=} \frac{2}{3}$

$6\left(\frac{x}{6} - 1\right) = \left(\frac{2}{3}\right)6$

$x - 6 = 4$

$x = 10$

$\frac{5}{3} - \frac{3}{3} \stackrel{?}{=} \frac{2}{3}$

$\frac{2}{3} = \frac{2}{3}$

7.  $\frac{z+2}{3} = 4 - \frac{z}{12}$

**Check:**  $\frac{8+2}{3} \stackrel{?}{=} 4 - \frac{8}{12}$

$12\left(\frac{z+2}{3}\right) = \left(4 - \frac{z}{12}\right)12$

$4(z+2) = 48 - z$

$4z + 8 = 48 - z$

$5z = 40$

$z = 8$

$\frac{10}{3} \stackrel{?}{=} \frac{12}{3} - \frac{2}{3}$

$\frac{10}{3} = \frac{10}{3}$

9.  $\frac{2y-9}{6} = 3y - \frac{3}{4}$

**Check:**  $\frac{2\left(-\frac{9}{32}\right) - 9}{6} \stackrel{?}{=} 3\left(-\frac{9}{32}\right) - \frac{3}{4}$

$(12)\left(\frac{2y-9}{6}\right) = \left(3y - \frac{3}{4}\right)(12)$

$\frac{1}{6}\left(-\frac{18}{32} - \frac{288}{32}\right) \stackrel{?}{=} -\frac{27}{32} - \frac{24}{32}$

$2(2y-9) = 36y - 9$

$\frac{1}{6}\left(-\frac{306}{32}\right) \stackrel{?}{=} -\frac{51}{32}$

$4y - 18 = 36y - 9$

$-\frac{51}{32} = -\frac{51}{32}$

$-9 = 32y$

$-\frac{9}{32} = y$

11.  $\frac{4t}{3} = 15 - \frac{t}{6}$

**Check:**  $\frac{4(10)}{3} \stackrel{?}{=} 15 - \frac{10}{6}$

$6\left(\frac{4t}{3}\right) = \left(15 - \frac{t}{6}\right)6$

$\frac{40}{3} \stackrel{?}{=} \frac{45}{3} - \frac{5}{3}$

$8t = 90 - t$

$\frac{40}{3} = \frac{40}{3}$

$9t = 90$

$t = 10$

13.  $\frac{5y-1}{12} + \frac{y}{3} = -\frac{1}{4}$

$12\left(\frac{5y-1}{12} + \frac{y}{3}\right) = \left(-\frac{1}{4}\right)12$

$5y - 1 + 4y = -3$

$9y = -2$

$y = -\frac{2}{9}$

15.  $\frac{h+2}{5} - \frac{h-1}{9} = \frac{2}{3}$

**Check:**  $\frac{\frac{7}{4} + 2}{5} - \frac{\frac{7}{4} - 1}{9} \stackrel{?}{=} \frac{2}{3}$

$45\left(\frac{h+2}{5} - \frac{h-1}{9}\right) = \left(\frac{2}{3}\right)45$

$\frac{1}{5}\left(\frac{7}{4} + \frac{8}{4}\right) - \frac{1}{9}\left(\frac{7}{4} - \frac{4}{4}\right) \stackrel{?}{=} \frac{2}{3}$

$9(h+2) - 5(h-1) = 30$

$\frac{1}{5}\left(\frac{15}{4}\right) - \frac{1}{9}\left(\frac{3}{4}\right) \stackrel{?}{=} \frac{2}{3}$

$9h + 18 - 5h + 5 = 30$

$\frac{3}{4} - \frac{1}{12} \stackrel{?}{=} \frac{2}{3}$

$4h + 23 = 30$

$4h = 7$

$\frac{9}{12} - \frac{1}{12} \stackrel{?}{=} \frac{2}{3}$

$h = \frac{7}{4}$

$\frac{8}{12} \stackrel{?}{=} \frac{2}{3}$

$\frac{2}{3} = \frac{2}{3}$

$$17. \quad \frac{x+5}{4} - \frac{3x-8}{3} = \frac{4-4}{12}$$

$$12\left(\frac{x+5}{4} - \frac{3x-8}{3}\right) = \left(\frac{4-4}{12}\right)12$$

$$3(x+5) - 4(3x-8) = 4-4$$

$$3x + 15 - 12x + 32 = 4 - 4$$

$$-9x + 47 = 4 - 4$$

$$-9x = -43$$

$$x = \frac{43}{9}$$

**Check:**

$$\frac{\frac{43}{9} + 5}{4} - \frac{3\left(\frac{43}{9}\right) - 8}{3} \stackrel{?}{=} \frac{4 - \left(\frac{43}{9}\right)}{12}$$

$$\frac{1\left(\frac{43}{9} + \frac{40}{9}\right) - \frac{1\left(129 - 64\right)}{3}}{4} \stackrel{?}{=} \frac{1\left(\frac{32}{9} - \frac{43}{9}\right)}{12}$$

$$\frac{1\left(\frac{83}{9}\right) - \frac{1\left(65\right)}{3}}{4} \stackrel{?}{=} \frac{1\left(-\frac{11}{9}\right)}{12}$$

$$\frac{1\left(\frac{83}{9} - \frac{65}{3}\right)}{4} \stackrel{?}{=} \frac{1\left(-\frac{11}{9}\right)}{12}$$

$$\frac{1\left(\frac{249}{9} - \frac{260}{9}\right)}{4} \stackrel{?}{=} \frac{1\left(-\frac{11}{9}\right)}{12}$$

$$\frac{1\left(-\frac{11}{9}\right)}{4} = \frac{1\left(-\frac{11}{9}\right)}{12}$$

$$19. \quad \frac{9}{25-y} = -\frac{1}{4}$$

$$4(25-y)\left(\frac{9}{25-y}\right) = \left(-\frac{1}{4}\right)4(25-y)$$

$$36 = -(25-y)$$

$$36 = -25 + y$$

$$61 = y$$

$$\text{Check: } \frac{9}{25-61} \stackrel{?}{=} -\frac{1}{4}$$

$$-\frac{9}{36} = -\frac{1}{4}$$

$$-\frac{1}{4} = -\frac{1}{4}$$

$$21. \quad 5 - \frac{12}{a} = \frac{5}{3}$$

$$3a\left(5 - \frac{12}{a}\right) = \left(\frac{5}{3}\right)3a$$

$$15a - 36 = 5a$$

$$10a = 36$$

$$a = \frac{36}{10}$$

$$a = \frac{18}{5}$$

$$\text{Check: } 5 - \frac{12}{\frac{18}{5}} \stackrel{?}{=} \frac{5}{3}$$

$$5 - \frac{60}{18} = \frac{5}{3}$$

$$\frac{15}{3} - \frac{10}{3} = \frac{5}{3}$$

$$\frac{5}{3} = \frac{5}{3}$$

$$23. \quad \frac{4}{x} - \frac{7}{5x} = -\frac{1}{2}$$

$$10x\left(\frac{4}{x} - \frac{7}{5x}\right) = \left(-\frac{1}{2}\right)10x$$

$$40 - 14 = -5x$$

$$26 = -5x$$

$$\frac{26}{-5} = x$$

$$\text{Check: } \frac{4}{\frac{26}{-5}} - \frac{7}{5\left(\frac{26}{-5}\right)} \stackrel{?}{=} -\frac{1}{2}$$

$$-\frac{20}{26} + \frac{7}{26} \stackrel{?}{=} -\frac{1}{2}$$

$$-\frac{13}{26} \stackrel{?}{=} -\frac{1}{2}$$

$$-\frac{1}{2} = -\frac{1}{2}$$

$$25. \quad \frac{12}{y+5} + \frac{1}{2} = 2$$

$$2(y+5)\left(\frac{12}{y+5} + \frac{1}{2}\right) = (2)2(y+5)$$

$$24 + y + 5 = 4(y+5)$$

$$y + 29 = 4y + 20$$

$$9 = 3y$$

$$3 = y$$

$$\text{Check: } \frac{12}{3+5} + \frac{1}{2} \stackrel{?}{=} 2$$

$$\frac{3}{2} + \frac{1}{2} = 2$$

$$\frac{4}{2} = 2$$

$$2 = 2$$

$$27. \quad \frac{5}{x} = \frac{25}{3(x+2)} \qquad \text{Check: } \frac{5}{3} \stackrel{?}{=} \frac{25}{3(3+2)}$$

$$3x(x+2)\left(\frac{5}{x}\right) = \left(\frac{25}{3(x+2)}\right)3x(x+2)$$

$$15(x+2) = 25x$$

$$15x + 30 = 25x$$

$$30 = 10x$$

$$3 = x$$

$$\frac{5}{3} = \frac{25}{15}$$

$$\frac{5}{3} = \frac{5}{3}$$

$$29. \quad \frac{8}{3x+5} = \frac{1}{x+2} \qquad \text{Check: } \frac{8}{3\left(-\frac{11}{5}\right)+5} \stackrel{?}{=} \frac{1}{-\frac{11}{5}+2}$$

$$(3x+5)(x+2)\left(\frac{8}{3x+5}\right) = \left(\frac{1}{x+2}\right)(3x+5)(x+2)$$

$$8(x+2) = 3x+5$$

$$8x+16 = 3x+5$$

$$5x = -11$$

$$x = -\frac{11}{5}$$

$$\frac{8}{-\frac{33}{5} + \frac{25}{5}} = \frac{1}{-\frac{11}{5} + \frac{10}{5}}$$

$$\frac{8}{-\frac{8}{5}} = \frac{1}{-\frac{1}{5}}$$

$$-\frac{8}{5} = -\frac{1}{5}$$

$$-5 = -5$$

$$31. \quad \frac{3}{x+2} - \frac{1}{x} = \frac{1}{5x} \qquad \text{Check: } \frac{1}{\frac{4}{3}+2} - \frac{1}{\frac{4}{3}} \stackrel{?}{=} \frac{1}{5\left(\frac{4}{3}\right)}$$

$$5x(x+2)\left(\frac{3}{x+2} - \frac{1}{x}\right) = \left(\frac{1}{5x}\right)5x(x+2)$$

$$15x - 5(x+2) = x+2$$

$$15x - 5x - 10 = x+2$$

$$10x - 10 = x+2$$

$$9x = 12$$

$$x = \frac{12}{9}$$

$$x = \frac{4}{3}$$

$$\frac{3}{\frac{10}{3}} - \frac{1}{\frac{4}{3}} = \frac{1}{\frac{20}{3}}$$

$$\frac{9}{10} - \frac{3}{4} = \frac{3}{20}$$

$$\frac{18}{20} - \frac{15}{20} = \frac{3}{20}$$

$$\frac{3}{20} = \frac{3}{20}$$

$$33. \quad \frac{1}{2} = \frac{18}{x^2} \qquad \text{Check: } \frac{1}{2} \stackrel{?}{=} \frac{18}{6^2} \qquad \frac{1}{2} \stackrel{?}{=} \frac{18}{(-6)^2}$$

$$2x^2\left(\frac{1}{2}\right) = \left(\frac{18}{x^2}\right)2x^2$$

$$x^2 = 36$$

$$x^2 - 36 = 0$$

$$(x-6)(x+6) = 0$$

$$x = 6 \quad x = -6$$

$$\frac{1}{2} = \frac{18}{36}$$

$$\frac{1}{2} = \frac{18}{36}$$

$$\frac{1}{2} = \frac{1}{2}$$

$$\frac{1}{2} = \frac{1}{2}$$

35.  $\frac{32}{t} = 2t$       **Check:**  $\frac{32}{4} \stackrel{?}{=} 2(4)$        $\frac{32}{-4} \stackrel{?}{=} 2(-4)$

$$t\left(\frac{32}{t}\right) = (2t)t$$

$$32 = 2t^2$$

$$16 = t^2$$

$$0 = t^2 - 16$$

$$0 = (t - 4)(t + 4)$$

$$t = 4 \quad t = -4$$

37.  $x + 1 = \frac{72}{x}$       **Check:**  $-9 + 1 \stackrel{?}{=} \frac{72}{-9}$        $8 + 1 \stackrel{?}{=} \frac{72}{8}$

$$x(x + 1) = \left(\frac{72}{x}\right)x$$

$$x^2 + x = 72$$

$$x^2 + x - 72 = 0$$

$$(x + 9)(x - 8) = 0$$

$$x = -9 \quad x = 8$$

39.  $1 = \frac{16}{y} - \frac{39}{y^2}$       **Check:**  $1 \stackrel{?}{=} \frac{16}{13} - \frac{39}{13^2}$        $1 \stackrel{?}{=} \frac{16}{3} - \frac{39}{3^2}$

$$y^2(1) = \left[\frac{16}{y} - \frac{39}{y^2}\right]y^2$$

$$y^2 = 16y - 39$$

$$y^2 - 16y + 39 = 0$$

$$(y - 13)(y - 3) = 0$$

$$y = 13 \quad y = 3$$

41.  $\frac{2x}{3x - 10} - \frac{5}{x} = 0$

$$x(3x - 10)\left(\frac{2x}{3x - 10} - \frac{5}{x}\right) = (0)x(3x - 10)$$

$$2x^2 - 5(3x - 10) = 0$$

$$2x^2 - 15x + 50 = 0$$

No real solution

43.  $5x\left(\frac{2x}{5}\right) = \left(\frac{x^2 - 5x}{5x}\right)5x$       **Check:**  $x = 0$        $x = -5$

$$2x^2 = x^2 - 5x$$

$$x^2 + 5x = 0$$

$$x(x + 5) = 0$$

$$x = 0 \quad x + 5 = 0$$

$$x = -5$$

$$\frac{2(0)}{5} \stackrel{?}{=} \frac{0^2 - 5(0)}{5(0)} \quad \frac{2(-5)}{5} \stackrel{?}{=} \frac{(-5)^2 - 5(-5)}{5(-5)}$$

$$0 \neq \text{undefined} \quad \frac{-10}{5} = \frac{25 + 25}{-25}$$

so  $x = 0$  is extraneous.

$$-2 = -2$$

$$45. \quad \frac{2}{6q+5} - \frac{3}{4(6q+5)} = \frac{1}{28}$$

$$28(6q+5)\left(\frac{2}{6q+5} - \frac{3}{4(6q+5)}\right) = \left(\frac{1}{28}\right)28(6q+5)$$

$$28(2) - 7(3) = 6q + 5$$

$$56 - 21 = 6q + 5$$

$$35 = 6q + 5$$

$$30 = 6q$$

$$5 = q$$

$$\text{Check: } \frac{2}{6(5)+5} - \frac{3}{4[6(5)+5]} \stackrel{?}{=} \frac{1}{28}$$

$$\frac{2}{30+5} - \frac{3}{4(30+5)} \stackrel{?}{=} \frac{1}{28}$$

$$\frac{2}{35} - \frac{3}{4(35)} \stackrel{?}{=} \frac{1}{28}$$

$$\frac{8}{140} - \frac{3}{140} \stackrel{?}{=} \frac{1}{28}$$

$$\frac{5}{140} \stackrel{?}{=} \frac{1}{28}$$

$$\frac{1}{28} = \frac{1}{28}$$

$$47. \quad \frac{4}{2x+3} + \frac{17}{5x-3} = 3$$

$$(5x-3)(2x+3)\left(\frac{4}{2x+3} + \frac{17}{5x-3}\right) = (3)(2x+3)(5x-3)$$

$$4(5x-3) + 17(2x+3) = 3(10x^2 + 9x - 9)$$

$$20x - 12 + 34x + 51 = 30x^2 + 27x - 27$$

$$0 = 30x^2 - 27x - 66$$

$$0 = 10x^2 - 9x - 22$$

$$0 = (10x+11)(x-2)$$

$$x = -\frac{11}{10} \quad x = 2$$

$$\text{Check: } \frac{4}{2\left(-\frac{11}{10}\right)+3} + \frac{17}{5\left(-\frac{11}{10}\right)-3} \stackrel{?}{=} 3$$

$$\frac{4}{-\frac{22}{10}+\frac{30}{10}} + \frac{17}{-\frac{55}{10}-\frac{30}{10}} \stackrel{?}{=} 3$$

$$\frac{4}{\frac{8}{10}} + \frac{17}{-\frac{85}{10}} \stackrel{?}{=} 3$$

$$\frac{4}{\frac{4}{5}} + \frac{17}{-\frac{17}{2}} \stackrel{?}{=} 3$$

$$5 + -2 \stackrel{?}{=} 3$$

$$3 = 3$$

$$\text{Check: } \frac{4}{2(2)+3} + \frac{17}{5(2)-3} \stackrel{?}{=} 3$$

$$\frac{4}{7} + \frac{17}{7} \stackrel{?}{=} 3$$

$$3 = 3$$

$$49. \quad \frac{2}{x-10} - \frac{3}{x-2} = \frac{6}{x^2-12x+20}$$

$$\frac{2}{x-10} - \frac{3}{x-2} = \frac{6}{(x-10)(x-2)}$$

$$(x-10)(x-2)\left(\frac{2}{x-10} - \frac{3}{x-2}\right) = \left(\frac{6}{(x-10)(x-2)}\right)(x-10)(x-2)$$

$$2(x-2) - 3(x-10) = 6$$

$$2x - 4 - 3x + 30 = 6$$

$$-x + 26 = 6$$

$$-x = -20$$

$$x = 20$$

$$\text{Check: } \frac{2}{20-10} - \frac{3}{20-2} \stackrel{?}{=} \frac{6}{(20)^2-12(20)+20}$$

$$\frac{2}{10} - \frac{3}{18} \stackrel{?}{=} \frac{6}{400-240+20}$$

$$\frac{1}{5} - \frac{1}{6} \stackrel{?}{=} \frac{6}{180}$$

$$\frac{6}{30} - \frac{5}{30} = \frac{1}{30}$$

$$\frac{1}{30} = \frac{1}{30}$$

$$51. \quad \frac{x+3}{x^2-9} + \frac{4}{3-x} - 2 = 0$$

$$\frac{\cancel{x+3}}{(x-3)\cancel{(x+3)}} - \frac{4}{x-3} - 2 = 0$$

$$(x-3)\left(\frac{1}{x-3} - \frac{4}{x-3} - 2\right) = 0(x-3)$$

$$1 - 4 - 2(x-3) = 0$$

$$-3 - 2x + 6 = 0$$

$$-2x = -3$$

$$x = \frac{3}{2}$$

$$\text{Check: } \frac{\frac{3}{2} + 3}{\left(\frac{3}{2}\right)^2 - 9} + \frac{4}{3 - \frac{3}{2}} - 2 \stackrel{?}{=} 0$$

$$\frac{\frac{3}{2} + \frac{6}{2}}{\frac{9}{4} - \frac{36}{4}} + \frac{4}{\frac{6}{2} - \frac{3}{2}} - 2 \stackrel{?}{=} 0$$

$$\frac{\frac{9}{2}}{-\frac{27}{4}} + \frac{4}{\frac{3}{2}} - 2 \stackrel{?}{=} 0$$

$$-\frac{2}{3} + \frac{8}{3} - \frac{6}{3} \stackrel{?}{=} 0$$

$$0 = 0$$

$$53. \quad \frac{x}{x-2} + \frac{3x}{x-4} = \frac{-2(x-6)}{x^2-6x+8}$$

$$(x-2)(x-4)\left(\frac{x}{x-2} + \frac{3x}{x-4}\right) = \left(\frac{-2(x-6)}{(x-4)(x-2)}\right)(x-2)(x-4)$$

$$x(x-4) + 3x(x-2) = -2(x-6)$$

$$x^2 - 4x + 3x^2 - 6x = -2x + 12$$

$$4x^2 - 8x - 12 = 0$$

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

$$x = 3 \quad x = -1$$

$$\text{Check: } \frac{3}{3-2} + \frac{3(3)}{3-4} \stackrel{?}{=} \frac{-2(3-6)}{3^2-6(3)+8}$$

$$3 + \frac{9}{-1} \stackrel{?}{=} \frac{6}{-1}$$

$$-6 = -6$$

Check:

$$\frac{-1}{-1-2} + \frac{3(-1)}{-1-4} \stackrel{?}{=} \frac{-2(-1-6)}{(-1)^2-6(-1)+8}$$

$$\frac{1}{3} + \frac{3}{5} \stackrel{?}{=} \frac{14}{15}$$

$$\frac{5}{15} + \frac{9}{15} \stackrel{?}{=} \frac{14}{15}$$

$$\frac{14}{15} = \frac{14}{15}$$

$$55. \quad \frac{2(x+7)}{x+4} - 2 = \frac{2x+20}{2x+8}$$

$$2(x+4)\left(\frac{2(x+7)}{x+4} - 2\right) = \left(\frac{2x+20}{2(x+4)}\right)2(x+4)$$

$$4(x+7) - 2 \cdot 2(x+4) = 2x+20$$

$$4x+28-4x-16 = 2x+20$$

$$12 = 2x+20$$

$$-8 = 2x$$

$$-4 = x$$

$$\text{Check: } \frac{2[-4+7]}{-4+4} - 2 \stackrel{?}{=} \frac{2(-4)+20}{2(-4)+8}$$

$$\frac{6}{0} - 2 \neq \frac{12}{0}$$

Division by zero is undefined. Solution is extraneous, so equation has no solution.

57. 
$$\frac{x}{2} = \frac{2 - \frac{3}{x}}{1 - \frac{1}{x}}$$

$$\frac{x}{2} = \frac{2 - \frac{3}{x}}{1 - \frac{1}{x}} \cdot \frac{x}{x}$$

$$2(x-1)\left(\frac{x}{2}\right) = \left(\frac{2x-3}{x-1}\right)2(x-1)$$

$$x(x-1) = (2x-3)2$$

$$x^2 - x = 4x - 6$$

$$x^2 - 5x + 6 = 0$$

$$(x-3)(x-2) = 0$$

$$x = 3 \quad x = 2$$

Check:  $\frac{3}{2} \stackrel{?}{=} \frac{2 - \frac{3}{3}}{1 - \frac{1}{3}}$        $\frac{2}{2} \stackrel{?}{=} \frac{2 - \frac{3}{2}}{1 - \frac{1}{2}}$

$$\frac{3}{2} = \frac{1}{\frac{2}{3}}$$

$$1 = \frac{1}{\frac{1}{2}}$$

$$\frac{3}{2} = \frac{3}{2}$$

$$1 = 1$$

59.  $x$ -intercept:  $(-2, 0)$

$$0 = \frac{x+2}{x-2}$$

$$(x-2)(0) = \left(\frac{x+2}{x-2}\right)(x-2)$$

$$0 = x+2$$

$$-2 = x$$

61.  $x$ -intercepts:  $(-1, 0)$  and  $(1, 0)$

$$0 = x - \frac{1}{x}$$

$$x(0) = \left(x - \frac{1}{x}\right)x$$

$$0 = x^2 - 1$$

$$0 = (x-1)(x+1)$$

$$x-1 = 0 \quad x+1 = 0$$

$$x = 1 \quad x = -1$$

63. (a) Keystrokes:

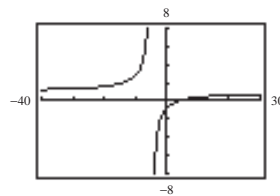
$\boxed{Y=}$   $\boxed{C}$   $\boxed{X,T,\theta}$   $\boxed{-}$   $\boxed{4}$   $\boxed{D}$   $\boxed{\div}$   $\boxed{C}$   $\boxed{X,T,\theta}$   $\boxed{+}$   $\boxed{5}$   $\boxed{D}$   $\boxed{\text{GRAPH}}$

$x$ -intercept:  $(4, 0)$

(b)  $0 = \frac{x-4}{x+5}$

$$0 = x - 4$$

$$4 = x$$



65. Keystrokes:

$\boxed{Y=}$   $\boxed{1}$   $\boxed{\div}$   $\boxed{X,T,\theta}$   $\boxed{+}$   $\boxed{4}$   $\boxed{\div}$   $\boxed{C}$   $\boxed{X,T,\theta}$   $\boxed{-}$   $\boxed{5}$   $\boxed{D}$   $\boxed{\text{GRAPH}}$

$x$ -intercept:  $(1, 0)$

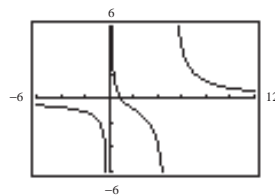
$$0 = \frac{1}{x} + \frac{4}{x-5}$$

$$x(x-5)(0) = \left(\frac{1}{x} + \frac{4}{x-5}\right)x(x-5)$$

$$0 = x - 5 + 4x$$

$$5 = 5x$$

$$1 = x$$



67. Keystrokes:

$\boxed{Y=}$   $\boxed{C}$   $\boxed{X,T,\theta}$   $\boxed{+}$   $\boxed{1}$   $\boxed{)}$   $\boxed{-}$   $\boxed{6}$   $\boxed{\div}$   $\boxed{X,T,\theta}$   $\boxed{GRAPH}$

x-intercepts:  $(-3, 0)$  and  $(2, 0)$

$$0 = (x + 1) - \frac{6}{x}$$

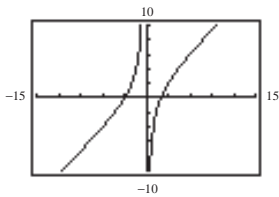
$$x(0) = \left[ (x + 1) - \frac{6}{x} \right] x$$

$$0 = x^2 + x - 6$$

$$0 = (x + 3)(x - 2)$$

$$x + 3 = 0 \quad x - 2 = 0$$

$$x = -3 \quad x = 2$$



69. (a) Keystrokes:

$\boxed{Y=}$   $\boxed{C}$   $\boxed{X,T,\theta}$   $\boxed{-}$   $\boxed{1}$   $\boxed{)}$   $\boxed{-}$   $\boxed{12}$   $\boxed{\div}$   $\boxed{X,T,\theta}$   $\boxed{GRAPH}$

x-intercepts:  $(-3, 0)$  and  $(4, 0)$

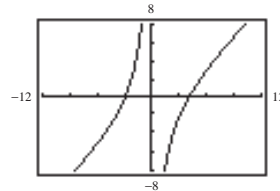
$$(b) \quad 0 = (x - 1) - \frac{12}{x}$$

$$0 = x^2 - x - 12$$

$$0 = (x - 4)(x + 3)$$

$$x - 4 = 0 \quad x + 3 = 0$$

$$x = 4 \quad x = -3$$



71. Verbal Model:

$$\boxed{\text{Number}} + \boxed{\text{Reciprocal}} = \boxed{\frac{65}{8}}$$

Labels:

$$\text{Number} = x$$

$$\text{Reciprocal} = \frac{1}{x}$$

Equation:

$$x + \frac{1}{x} = \frac{65}{8}$$

$$8x\left(x + \frac{1}{x}\right) = \left(\frac{65}{8}\right)8x$$

$$8x^2 + 8 = 65x$$

$$8x^2 - 65x + 8 = 0$$

$$(8x - 1)(x - 8) = 0$$

$$x = \frac{1}{8} \quad x = 8$$

73. Verbal Model:

$$\boxed{\text{Distance}} \div \boxed{\text{Rate}} = \boxed{\text{Time}}$$

$$\frac{\boxed{\text{Distance}}}{\boxed{\text{Rate}}} = \boxed{\text{Time}}$$

$$\boxed{\text{Time traveled with wind}} = \boxed{\text{Time traveled without wind}}$$

Labels:

$$\text{Speed of the wind} = x$$

Equation:

$$\frac{680}{300 + x} = \frac{520}{300 - x}$$

$$(300 + x)(300 - x)\left(\frac{680}{300 + x}\right) = \left(\frac{520}{300 - x}\right)(300 + x)(300 - x)$$

$$680(300 - x) = 520(300 + x)$$

$$204,000 - 680x = 156,000 + 520x$$

$$-1200x = -48,000$$

$$x = 40 \text{ miles per hour}$$

75. Verbal Model:  $\boxed{\text{Distance}} = \boxed{\text{Rate}} \cdot \boxed{\text{Time}}$

$$\frac{\boxed{\text{Distance person 1}}}{\boxed{\text{Rate person 1}}} = \frac{\boxed{\text{Distance person 2}}}{\boxed{\text{Rate person 2}}}$$

Labels: Rate person 1 =  $x + 2$

Rate person 2 =  $x$

Equation:  $\frac{5}{x + 2} = \frac{4}{x}$

$$x(x + 2)\left(\frac{5}{x + 2}\right) = \left(\frac{4}{x}\right)x(x + 2)$$

$$5x = 4(x + 2)$$

$$5x = 4x + 8$$

$$x = 8 \text{ mph person 2}$$

$$x + 2 = 10 \text{ mph person 1}$$

77. Verbal Model:  $\boxed{\text{Distance}} = \boxed{\text{Rate}} \cdot \boxed{\text{Time}}$

$$\frac{\boxed{\text{Distance}}}{\boxed{\text{Rate}}} = \boxed{\text{Time}}$$

$$\boxed{\text{Time traveled upstream}} + \boxed{\text{Time traveled downstream}} = \boxed{\text{Total time}}$$

Labels: Speed of the current =  $x$

Equation:  $\frac{48}{20 - x} + \frac{48}{20 + x} = 5$

$$(20 - x)(20 + x)\left(\frac{48}{20 - x} + \frac{48}{20 + x}\right) = (5)(20 - x)(20 + x)$$

$$48(20 + x) + 48(20 - x) = 5(400 - x^2)$$

$$960 + 48x + 960 - 48x = 2000 - 5x^2$$

$$1920 = 2000 - 5x^2$$

$$5x^2 = 80$$

$$x^2 = 16$$

$$x = 4 \text{ mph}$$

79. Verbal Model:  $\boxed{\text{Cost per person current group}} - \boxed{\text{Cost per person new group}} = \boxed{4000}$

Labels: Persons in current group =  $x$

Persons in new group =  $x + 2$

Equation:  $\frac{240,000}{x} - \frac{240,000}{x + 2} = 4000$

$$x(x + 2)\left(\frac{240,000}{x} - \frac{240,000}{x + 2}\right) = (4000)x(x + 2)$$

$$240,000(x + 2) - 240,000x = 4000(x^2 + 2x)$$

$$240,000x + 480,000 - 240,000x = 4000x^2 + 8000x$$

$$0 = 4000x^2 + 8000x - 480,000$$

$$0 = x^2 + 2x - 120$$

$$0 = (x + 12)(x - 10)$$

$$x + 12 = 0 \quad x - 10 = 0$$

$$x = -12 \quad x = 10 \text{ people}$$

81. Verbal Model:  $\boxed{\text{Cost per person original group}} - \boxed{\text{Cost per person new group}} = \boxed{1300}$

Labels: Persons in current group =  $x$   
 Persons in new group =  $x + 3$

Equation: 
$$\frac{78,000}{x} - \frac{78,000}{x + 3} = 1300$$

$$x(x + 3)\left(\frac{78,000}{x} - \frac{78,000}{x + 3}\right) = (1300)x(x + 3)$$

$$78,000(x + 3) - 78,000x = 1300x(x + 3)$$

$$78,000x + 234,000 - 78,000x = 1300x^2 + 3900x$$

$$0 = 1300x^2 + 3900x - 234,000$$

$$0 = x^2 + 3x - 180$$

$$0 = (x + 15)(x - 12)$$

$$x = -15 \quad x = 12 \text{ persons}$$

83. (a) Keystrokes:

$\boxed{Y=}$   $\boxed{C}$   $\boxed{120,000}$   $\boxed{X,T,\theta}$   $\boxed{)}$   $\boxed{\div}$   $\boxed{C}$   $\boxed{100}$   $\boxed{-}$   $\boxed{X,T,\theta}$   $\boxed{)}$   $\boxed{\text{GRAPH}}$

(b) Verbal Model:  $\boxed{\text{Cost}} = \boxed{\frac{120,000p}{100 - p}}$

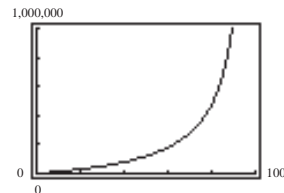
Equation: 
$$680,000 = \frac{120,000p}{100 - p}$$

$$(100 - p)(680,000) = \left(\frac{120,000p}{100 - p}\right)(100 - p)$$

$$68,000,000 - 680,000p = 120,000p$$

$$68,000,000 = 800,000p$$

$$85\% = p$$



85.  $\frac{1}{6} + \frac{1}{6} = \frac{1}{t}$

$$t + t = 6$$

$$2t = 6$$

$$t = 3 \text{ hours}$$

$$\frac{1}{3} + \frac{1}{5} = \frac{1}{t}$$

$$5t + 3t = 15$$

$$8t = 15$$

$$t = \frac{15}{8} \text{ minutes}$$

$$\frac{1}{5} + \frac{1}{2\frac{1}{2}} = \frac{1}{t}$$

$$\frac{1}{5} + \frac{1}{\frac{5}{2}} = \frac{1}{t}$$

$$\frac{1}{5} + \frac{2}{5} = \frac{1}{t}$$

$$t + 2t = 5$$

$$3t = 5$$

$$t = \frac{5}{3} \text{ hours}$$

Person #1	Person #2	Together
6 hours	6 hours	3 hours
3 minutes	5 minutes	$\frac{15}{8}$ minutes
5 hours	$2\frac{1}{2}$ hours	$\frac{5}{3}$ hours

87. Verbal Model:  $\boxed{\text{Rate Person 1}} + \boxed{\text{Rate Person 2}} = \boxed{\text{Rate Together}}$

Labels: Second landscaper's time =  $x$   
 First landscaper's time =  $\frac{3}{2}x$

Equation: 
$$\frac{1}{x} + \frac{1}{\frac{3}{2}x} = \frac{1}{9}$$

$$9x\left(\frac{1}{x} + \frac{2}{3x}\right) = \left(\frac{1}{9}\right)9x$$

$$9 + 6 = x$$

$$15 \text{ hours} = x$$

$$22\frac{1}{2} \text{ hours} = \frac{45}{2} = \frac{3}{2}x$$

89. Verbal Model:  $\boxed{\text{Rate Pipe 1}} + \boxed{\text{Rate Pipe 2}} = \boxed{\text{Rate Together}}$

Labels: Second pipe's time =  $x$   
 First pipe's time =  $\frac{5}{4}x$

Equation: 
$$\frac{1}{x} + \frac{1}{\frac{5}{4}x} = \frac{1}{5}$$

$$5x\left(\frac{1}{x} + \frac{4}{5x}\right) = \left(\frac{1}{5}\right)5x$$

$$5 + 4 = x$$

$$9 \text{ hours} = x$$

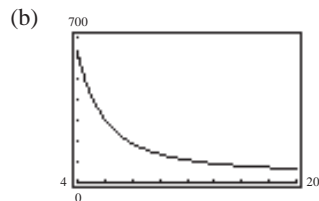
$$11\frac{1}{4} \text{ hours} = \frac{45}{4} = \frac{5}{4}x$$

91.  $y = \frac{87,709 - 1236(0)}{1000 - 93(0)} \approx 87.7$        $y = \frac{87,709 - 1236(3)}{1000 - 93(3)} \approx 116.5$

$y = \frac{87,709 - 1236(1)}{1000 - 93(1)} \approx 95.3$        $y = \frac{87,709 - 1236(4)}{1000 - 93(4)} \approx 131.8$

$y = \frac{87,709 - 1236(2)}{1000 - 93(2)} \approx 104.7$        $y = \frac{87,709 - 1236(5)}{1000 - 93(5)} \approx 152.4$

93. (a) Domain =  $\{4, 6, 8, 10, \dots\}$



(c)  $135 = 43.4 + \frac{9353}{x^2}$

$$91.6 = \frac{9353}{x^2}$$

$$x^2 = \frac{9353}{91.6}$$

$$x \approx 10d$$

$$\begin{aligned}
 95. \text{ (d)} \quad t(x) &= \frac{10}{5-x} + \frac{10}{5+x} \\
 6\frac{1}{4} &= \frac{10}{5-x} + \frac{10}{5+x} \\
 4(5-x)(5+x)\left(\frac{25}{4}\right) &= \left(\frac{10}{5-x} + \frac{10}{5+x}\right)4(5-x)(5+x) \\
 25(25-x^2) &= 40(5+x) + 40(5-x) \\
 625 - 25x^2 &= 200 + 40x + 200 - 40x \\
 0 &= 25x^2 - 225 \\
 0 &= x^2 - 9 \\
 x &= 3 \text{ miles per hour}
 \end{aligned}$$

$$\begin{aligned}
 \text{(e)} \quad t(x) &= \frac{10}{5-4} + \frac{10}{5+4} \\
 t(x) &= 10 + \frac{10}{9} = \frac{90}{9} + \frac{10}{9} = \frac{100}{9} \\
 t(x) &= 11\frac{1}{9} \text{ or } 11.1 \text{ hours} \\
 &\text{Yes}
 \end{aligned}$$

97. Solve a rational equation by multiplying both sides of the equation by the lowest common denominator. Then solve the resulting equation, checking for any extraneous solutions.

99. (a) Simplify each side by removing symbols of grouping, combining like terms, and reducing fractions on one or both sides.

(b) Add (or subtract) the same quantity to (from) both sides of the equation.

(c) Multiply (or divide) both sides of the equation by the same nonzero real number.

(d) Interchange the two sides of the equation.

101. When the equation involves only two fractions, one on each side of the equation, the equation can be solved by cross-multiplication.

## Review Exercises for Chapter 4

$$\begin{aligned}
 1. \quad (2^3 \cdot 3^2)^{-1} &= (8 \cdot 9)^{-1} \\
 &= 72^{-1} \\
 &= \frac{1}{72}
 \end{aligned}$$

$$\begin{aligned}
 3. \quad \left(\frac{2}{5}\right)^{-3} &= \left(\frac{5}{2}\right)^3 \\
 &= \frac{125}{8}
 \end{aligned}$$

$$\begin{aligned}
 5. \quad (6 \times 10^3)^2 &= 6^2 \times 10^6 \\
 &= 36 \times 10^6 \\
 &= 36,000,000
 \end{aligned}$$

$$\begin{aligned}
 7. \quad \frac{3.5 \times 10^7}{7 \times 10^4} &= \frac{3.5}{7} \times 10^{7-4} \\
 &= 0.5 \times 10^3 \\
 &= 5 \times 10^2 \\
 &= 500
 \end{aligned}$$

$$9. \quad 0.0000538 = 5.38 \times 10^{-5}$$

$$11. \quad 4.833 \times 10^8 = 483,300,000$$

$$\begin{aligned}
 13. \quad (6y^4)(2y^{-3}) &= 12y^{4+(-3)} \\
 &= 12y^1 \\
 &= 12y
 \end{aligned}$$

$$\begin{aligned}
 15. \quad \frac{4x^{-2}}{2x} &= 2x^{-2-1} \\
 &= 2x^{-3} \\
 &= \frac{2}{x^3}
 \end{aligned}$$

$$\begin{aligned}
 17. \quad (x^3y^{-4})^2 &= x^6y^{-8} \\
 &= \frac{x^6}{y^8}
 \end{aligned}$$