

111. Rewrite each fraction in terms of the lowest common denominator, combine the numerators, and place the result over the lowest common denominator.

$$\begin{aligned} 113. \frac{x-1}{x+4} - \frac{4x-11}{x+4} &= \frac{(x-1) - (4x-11)}{x+4} \\ &= \frac{x-1-4x+11}{x+4} \\ &= \frac{-3x+10}{x+4} \end{aligned}$$

The subtraction must be distributed to both terms of the numerator of the second fraction.

Section 4.5 Dividing Polynomials

$$1. \frac{6z+10}{2} = \frac{6z}{2} + \frac{10}{2} = 3z + 5$$

$$\begin{aligned} 3. \frac{10z^2+4z-12}{4} &= \frac{10z^2}{4} + \frac{4z}{4} - \frac{12}{4} \\ &= \frac{5z^2}{2} + z - 3 \end{aligned}$$

$$\begin{aligned} 5. (7x^3 - 2x^2) \div x &= \frac{7x^3 - 2x^2}{x} \\ &= \frac{7x^3}{x} - \frac{2x^2}{x} \\ &= 7x^2 - 2x, \quad x \neq 0 \end{aligned}$$

$$\begin{aligned} 7. \frac{m^4 + 2m^2 - 7}{m} &= \frac{m^4}{m} + \frac{2m^2}{m} - \frac{7}{m} \\ &= m^3 + 2m - \frac{7}{m}, \quad m \neq 0 \end{aligned}$$

$$\begin{aligned} 9. \frac{50z^3 + 30z}{-5z} &= \frac{50z^3}{-5z} + \frac{30z}{-5z} \\ &= -10z^2 - 6, \quad z \neq 0 \end{aligned}$$

$$\begin{aligned} 11. \frac{8z^3 + 3z^2 - 2z}{2z} &= \frac{8z^3}{2z} + \frac{3z^2}{2z} - \frac{2z}{2z} \\ &= 4z^2 + \frac{3}{2}z - 1, \quad z \neq 0 \end{aligned}$$

$$\begin{aligned} 13. (5x^2y - 8xy + 7xy^2) \div 2xy &= \frac{5x^2y - 8xy + 7xy^2}{2xy} \\ &= \frac{5x^2y}{2xy} - \frac{8xy}{2xy} + \frac{7xy^2}{2xy} \\ &= \frac{5x}{2} - 4 + \frac{7}{2}y, \quad x \neq 0, y \neq 0 \end{aligned}$$

$$\begin{array}{r} x - 5, \quad x \neq 3 \\ 15. \frac{x^2 - 8x + 15}{x - 3} = x - 3 \overline{)x^2 - 8x + 15} \\ \underline{x^2 - 3x} \\ -5x + 15 \\ \underline{-5x + 15} \\ 0 \end{array}$$

$$\begin{array}{r} x + 10, \quad x \neq -5 \\ 17. (x^2 + 15x + 50) \div (x + 5) = x + 5 \overline{)x^2 + 15x + 50} \\ \underline{x^2 + 5x} \\ 10x + 50 \\ \underline{10x + 50} \\ 0 \end{array}$$

$$\begin{array}{r} x - 3 + \frac{2}{x-2} \\ 19. x - 2 \overline{)x^2 - 5x + 8} \\ \underline{x^2 - 2x} \\ -3x + 8 \\ \underline{-3x + 6} \\ 2 \end{array}$$

$$\begin{array}{r} x + 7, \quad x \neq 3 \\ 21. -x + 3 \overline{)-x^2 - 4x + 21} \\ \underline{-x^2 + 3x} \\ -7x + 21 \\ \underline{-7x + 21} \\ 0 \end{array}$$

$$23. \begin{array}{r} 5x - 8 + \frac{19}{x+2} \\ x+2 \overline{)5x^2 + 2x + 3} \\ \underline{5x^2 + 10x} \\ -8x + 3 \\ \underline{-8x - 16} \\ 19 \end{array}$$

$$25. \begin{array}{r} 4x + 3 + \frac{-11}{3x+2} \\ 3x+2 \overline{)12x^2 + 17x - 5} \\ \underline{12x^2 + 8x} \\ 9x - 5 \\ \underline{9x + 6} \\ -11 \end{array}$$

$$27. \begin{array}{r} 6t - 5, \quad t \neq \frac{5}{2} \\ 2t-5 \overline{)12t^2 - 40t + 25} \\ \underline{12t^2 - 30t} \\ -10t + 25 \\ \underline{-10t + 25} \end{array}$$

$$29. \begin{array}{r} y + 3, \quad y \neq -\frac{1}{2} \\ 2y+1 \overline{)2y^2 + 7y + 3} \\ \underline{2y^2 + y} \\ 6y + 3 \\ \underline{6y + 3} \end{array}$$

$$31. \begin{array}{r} x^2 + 4, \quad x \neq 2 \\ x-2 \overline{)x^3 - 2x^2 + 4x - 8} \\ \underline{x^3 - 2x^2} \\ 4x - 8 \\ \underline{4x - 8} \end{array}$$

$$33. \begin{array}{r} 2x^2 + x + 4 + \frac{6}{x-3} \\ x-3 \overline{)2x^3 - 5x^2 + x - 6} \\ \underline{2x^3 - 6x^2} \\ x^2 + x - 6 \\ \underline{x^2 - 3x} \\ 4x - 6 \\ \underline{4x - 12} \\ 6 \end{array}$$

$$35. \begin{array}{r} 2 + \frac{5}{x+2} \\ x+2 \overline{)2x + 9} \\ \underline{2x + 4} \\ 5 \end{array}$$

$$37. \begin{array}{r} x - 4 + \frac{32}{x+4} \\ x+4 \overline{)x^2 + 0x + 16} \\ \underline{x^2 + 4x} \\ -4x + 16 \\ \underline{-4x - 16} \\ 32 \end{array}$$

$$39. \begin{array}{r} \frac{6}{5}z + \frac{41}{25} + \frac{41}{25(5z-1)} \\ 5z-1 \overline{)6z^2 + 7z + 0} \\ \underline{6z^2 - \frac{6}{5}z} \\ \frac{41}{5}z + 0 \\ \underline{\frac{41}{5}z - \frac{41}{25}} \\ \frac{41}{25} \end{array}$$

$$41. \begin{array}{r} 4x - 1, \quad x \neq -\frac{1}{4} \\ 4x+1 \overline{)16x^2 + 0x - 1} \\ \underline{16x^2 + 4x} \\ -4x - 1 \\ \underline{-4x - 1} \end{array}$$

$$\begin{array}{r}
 x^2 - 5x + 25, \quad x \neq -5 \\
 43. \quad x + 5 \overline{)x^3 + 0x^2 + 0x + 125} \\
 \underline{x^3 + 5x^2} \\
 -5x^2 + 0x \\
 \underline{-5x^2 - 25x} \\
 25x + 125 \\
 \underline{25x + 125} \\
 0
 \end{array}$$

$$\begin{array}{r}
 x + 2 \\
 45. \quad x^2 + 2x + 3 \overline{)x^3 + 4x^2 + 7x + 6} \\
 \underline{x^3 + 2x^2 + 3x} \\
 2x^2 + 4x + 6 \\
 \underline{2x^2 + 4x + 6} \\
 0
 \end{array}$$

$$\begin{array}{r}
 4x^2 + 12x + 25 + \frac{52x - 55}{x^2 - 3x + 2} \\
 47. \quad x^2 - 3x + 2 \overline{)4x^4 + 0x^3 - 3x^2 + x - 5} \\
 \underline{4x^4 - 12x^3 + 8x^2} \\
 12x^3 - 11x^2 + x \\
 \underline{12x^3 - 36x^2 + 24x} \\
 25x^2 - 23x - 5 \\
 \underline{25x^2 - 75x + 50} \\
 52x - 55
 \end{array}$$

$$\begin{array}{r}
 x^5 + x^4 + x^3 + x^2 + x + 1, \quad x \neq 1 \\
 49. \quad x - 1 \overline{)x^6 - 1} \\
 \underline{x^6 - x^5} \\
 x^5 \\
 \underline{x^5 - x^4} \\
 x^4 \\
 \underline{x^4 - x^3} \\
 x^3 \\
 \underline{x^3 - x^2} \\
 x^2 \\
 \underline{x^2 - x} \\
 x - 1 \\
 \underline{x - 1} \\
 0
 \end{array}$$

$$\begin{array}{r}
 x^3 - x + \frac{x}{x^2 + 1} \\
 51. \quad x^2 + 1 \overline{)x^5} \\
 \underline{x^5 + x^3} \\
 -x^3 \\
 \underline{-x^3 - x} \\
 x
 \end{array}$$

$$53. \quad \frac{4x^4}{x^3} - 2x = 4x - 2x = 2x, \quad x \neq 0$$

$$\begin{aligned}
 55. \quad \frac{8u^2v}{2u} + \frac{3(uv)^2}{uv} &= 4uv + \frac{3u^2v^2}{uv} \\
 &= 4uv + 3uv \\
 &= 7uv, \quad u \neq 0, v \neq 0
 \end{aligned}$$

$$\begin{array}{r}
 57. \quad \frac{x^3 - 5x^2 + 3x - 4}{x - 2} \\
 2 \left| \begin{array}{cccc}
 1 & -5 & 3 & -4 \\
 & 2 & -6 & -6 \\
 \hline
 1 & -3 & -3 & -10
 \end{array} \right.
 \end{array}$$

$$\frac{x^3 - 5x^2 + 3x - 4}{x - 2} = x^2 - 3x - 3 - \frac{10}{x - 2}$$

$$\begin{array}{r}
 59. \quad \frac{x^3 + 3x^2 - 1}{x + 4} \\
 -4 \left| \begin{array}{cccc}
 1 & 3 & 0 & -1 \\
 & -4 & 4 & -16 \\
 \hline
 1 & -1 & 4 & -17
 \end{array} \right.
 \end{array}$$

$$\frac{x^3 + 3x^2 - 1}{x + 4} = x^2 - x + 4 + \frac{-17}{x + 4}$$

61.
$$\frac{x^4 - 4x^3 + x + 10}{x - 2}$$

$$2 \left| \begin{array}{cccc|c} 1 & -4 & 0 & 1 & 10 \\ & 2 & -4 & -8 & -14 \\ \hline & 1 & -2 & -4 & -7 & -4 \end{array} \right.$$

$$\frac{x^4 - 4x^3 + x + 10}{x - 2} = x^3 - 2x^2 - 4x - 7 + \frac{-4}{x - 2}$$

65.
$$\frac{10x^4 - 50x^3 - 800}{x - 6}$$

$$6 \left| \begin{array}{cccc|c} 10 & -50 & 0 & 0 & -800 \\ & 60 & 60 & 360 & 2160 \\ \hline 10 & 10 & 60 & 360 & 1360 \end{array} \right.$$

$$\frac{10x^4 - 50x^3 - 800}{x - 6} = 10x^3 + 10x^2 + 60x + 360 + \frac{1360}{x - 6}$$

69.
$$3 \left| \begin{array}{ccc|c} 1 & 0 & -13 & 12 \\ & 3 & 9 & -12 \\ \hline 1 & 3 & -4 & 0 \end{array} \right.$$

$$x^2 + 3x - 4 = (x + 4)(x - 1)$$

$$x^3 - 13x + 12 = (x - 3)(x + 4)(x - 1)$$

73.
$$-5 \left| \begin{array}{ccc|c} 9 & 45 & -4 & -20 \\ & -45 & 0 & 20 \\ \hline 9 & 0 & -4 & 0 \end{array} \right.$$

$$9x^2 - 4 = (3x - 2)(3x + 2)$$

$$9x^3 + 45x^2 - 4x - 20 = (x + 5)(3x - 2)(3x + 2)$$

77.
$$\frac{15x^2 - 2x - 8}{x - \frac{4}{5}}$$

$$\frac{4}{5} \left| \begin{array}{cc|c} 15 & -2 & -8 \\ & 12 & 8 \\ \hline 15 & 10 & 0 \end{array} \right.$$

$$15x^2 - 2x - 8 = (15x + 10)\left(x - \frac{4}{5}\right) \\ = 5(3x + 2)\left(x - \frac{4}{5}\right)$$

81. Keystrokes:

$$y_1 \text{ [Y=] [(] [X,T,\theta] [+] 4 [)] [\div] 2 [X,T,\theta] [ENTER]}$$

$$y_2 \text{ [(] 1 [\div] 2 [)] [+] [(] 2 [\div] [X,T,\theta] [)] [GRAPH]}$$

$$\frac{x + 4}{2x} = \frac{x}{2x} + \frac{4}{2x} = \frac{1}{2} + \frac{2}{x}$$

 So, $y_1 = y_2$.

63.
$$\frac{5x^3 - 6x^2 + 8}{x - 4}$$

$$4 \left| \begin{array}{ccc|c} 5 & -6 & 0 & 8 \\ & 20 & 56 & 224 \\ \hline 5 & 14 & 56 & 232 \end{array} \right.$$

$$\frac{5x^3 - 6x^2 + 8}{x - 4} = 5x^2 + 14x + 56 + \frac{232}{x - 4}$$

67.
$$\frac{0.1x^2 + 0.8x + 1}{x - 0.2}$$

$$0.2 \left| \begin{array}{ccc|c} 0.1 & 0.8 & 1 \\ & 0.02 & 0.164 \\ \hline 0.1 & 0.82 & 1.164 \end{array} \right.$$

$$\frac{0.1x^2 + 0.8x + 1}{x - 0.2} = 0.1x + 0.82 + \frac{1.164}{x - 0.2}$$

71.
$$1 \left| \begin{array}{ccc|c} 6 & -13 & 9 & -2 \\ & 6 & -7 & 2 \\ \hline 6 & -7 & 2 & 0 \end{array} \right.$$

$$6x^2 - 7x + 2 = (2x - 1)(3x - 2)$$

$$6x^3 - 13x^2 + 9x - 2 = (x - 1)(2x - 1)(3x - 2)$$

75.
$$-3 \left| \begin{array}{cccc|c} 1 & 7 & 3 & -63 & -108 \\ & -3 & -12 & 27 & 108 \\ \hline 1 & 4 & -9 & -36 & 0 \end{array} \right.$$

$$x^3 + 4x^2 - 9x - 36 = x^2(x + 4) - 9(x + 4)$$

$$= (x + 4)(x^2 - 9)$$

$$= (x + 4)(x - 3)(x + 3)$$

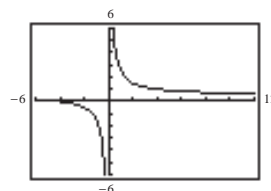
$$x^4 + 7x^3 + 3x^2 - 63x - 108 = (x + 3)^2(x + 4)(x - 3)$$

79.
$$\frac{x^3 - 2x^2 - 4x + c}{x - 2}$$

$$2 \left| \begin{array}{ccc|c} 1 & 2 & -4 & c \\ & 2 & 8 & 8 \\ \hline 1 & 4 & 4 & 0 \end{array} \right.$$

$$c + 8 = 0$$

$$c = -8$$

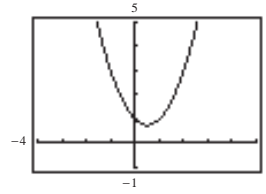


83. Keystrokes:

$$y_1 \text{ (Y=) (((X,T,\theta) \wedge 3 + 1)) \div ((X,T,\theta) + 1) (ENTER)}$$

$$y_2 \text{ (X,T,\theta) (x^2) (-) (X,T,\theta) (+) 1 (GRAPH)}$$

$$\frac{x^3 + 1}{x + 1} = \frac{(x + 1)(x^2 - x + 1)}{x + 1} = x^2 - x + 1, \quad x \neq -1$$



$$\begin{array}{r} x^{2n} + x^n + 4, \quad x^n \neq -2 \\ 85. \quad x^n + 2 \overline{) x^{3n} + 3x^{2n} + 6x^n + 8} \\ \quad \underline{x^{3n} + 2x^{2n}} \\ \quad \quad x^{2n} + 6x^n \\ \quad \quad \underline{x^{2n} + 2x^n} \\ \quad \quad \quad 4x^n + 8 \\ \quad \quad \quad \underline{4x^n + 8} \\ \quad \quad \quad \quad 0 \end{array}$$

87. Dividend = Divisor \cdot Quotient + Remainder

$$\begin{aligned} &= (x - 6) \cdot (x^2 - x + 1) - 4 \\ &= x^3 + x^2 + x - 6x^2 - 6x - 6 - 4 \\ &= x^3 - 5x^2 - 5x - 10 \end{aligned}$$

89.

x	Polynomial value	Divisor	Remainder
-2	-8	$x + 2$	-8
-1	0	$x + 1$	0
0	0	x	0
$\frac{1}{2}$	$-\frac{9}{8}$	$x - \frac{1}{2}$	$-\frac{9}{8}$
1	-2	$x - 1$	-2
2	0	$x - 2$	0

$$\begin{aligned} f(-1) &= (-1)^3 - (-1)^2 - 2(-1) \\ &= -1 - 1 + 2 \\ &= 0 \end{aligned}$$

$$\begin{array}{r|rrrr} -1 & 1 & -1 & -2 & 0 \\ & & -1 & 2 & 0 \\ \hline & 1 & -2 & 0 & 0 \end{array}$$

$$\begin{aligned} f(0) &= 0^3 - 0^2 - 2(0) \\ &= 0 \end{aligned}$$

$$\begin{array}{r|rrrr} 0 & 1 & -1 & -2 & 0 \\ & & 0 & 0 & 0 \\ \hline & 1 & -1 & -2 & 0 \end{array}$$

$$\begin{aligned} f\left(\frac{1}{2}\right) &= \left(\frac{1}{2}\right)^3 - \left(\frac{1}{2}\right)^2 - 2\left(\frac{1}{2}\right) \\ &= \frac{1}{8} - \frac{1}{4} - 1 \\ &= \frac{1}{8} - \frac{2}{8} - \frac{8}{8} \\ &= -\frac{9}{8} \end{aligned}$$

$$\begin{array}{r|rrrr} \frac{1}{2} & 1 & -1 & -2 & 0 \\ & & \frac{1}{2} & -\frac{1}{4} & -\frac{9}{8} \\ \hline & 1 & -\frac{1}{2} & -\frac{9}{4} & -\frac{9}{8} \end{array}$$

$$\begin{aligned} f(1) &= 1^3 - 1^2 - 2(1) \\ &= 1 - 1 - 2 \\ &= -2 \end{aligned}$$

$$\begin{array}{r|rrrr} 1 & 1 & -1 & -2 & 0 \\ & & 1 & 0 & -2 \\ \hline & 1 & 0 & -2 & -2 \end{array}$$

$$\begin{aligned} f(2) &= 2^3 - 2^2 - 2(2) \\ &= 8 - 4 - 4 \\ &= 0 \end{aligned}$$

$$\begin{array}{r|rrrr} 2 & 1 & -1 & -2 & 0 \\ & & 2 & 2 & 0 \\ \hline & 1 & 1 & 0 & 0 \end{array}$$

The polynomial values equal the remainders.

91. Area = Length · Width, so Width = $\frac{\text{Area}}{\text{Length}}$.

$$\text{Length} = \frac{2x^3 + 3x^2 - 6x - 9}{2x + 3} = x^2 - 3$$

$$\begin{array}{r} x^2 \quad - 3 \\ 2x + 3 \overline{) 2x^3 + 3x^2 - 6x - 9} \\ \underline{2x^3 + 3x^2} \\ -6x - 9 \\ \underline{-6x - 9} \\ 0 \end{array}$$

95. x is not a factor of the numerator.

99. True. If $\frac{n(x)}{d(x)} = q(x)$, then $n(x) = d(x) \cdot q(x)$.

93. Volume = Area of triangle · Height (of prism)

$$\begin{aligned} \text{Area of triangle} &= \frac{\text{Volume}}{\text{Height (of prism)}} \\ &= \frac{x^3 + 18x^2 + 80x + 96}{x + 12} \\ &= x^2 + 6x + 8 \end{aligned}$$

$$\text{Area of triangle} = \frac{1}{2} \cdot \text{Base} \cdot \text{Height}$$

$$\begin{aligned} \text{Height} &= \frac{2(\text{Area of triangle})}{\text{Base}} \\ &= \frac{2(x^2 + 6x + 8)}{x + 2} \\ &= 2x + 8 \text{ or } 2(x + 4) \end{aligned}$$

97. A divisor divides evenly into a dividend when the remainder is 0 and the divisor is a factor of the dividend.

Section 4.6 Solving Rational Equations

1. (a) $x = 0$

$$\frac{0}{3} - \frac{0}{5} \stackrel{?}{=} \frac{4}{3}$$

$$0 \neq \frac{4}{3}$$

Not a solution

(b) $x = -1$

$$\frac{-1}{3} - \frac{-1}{5} \stackrel{?}{=} \frac{4}{3}$$

$$\frac{-5}{15} - \frac{-3}{15} \stackrel{?}{=} \frac{20}{15}$$

$$\frac{-5}{15} + \frac{3}{15} \stackrel{?}{=} \frac{20}{15}$$

$$\frac{-2}{15} \neq \frac{20}{15}$$

Not a solution

(c) $x = \frac{1}{8}$

$$\frac{1/8}{3} - \frac{1/8}{5} \stackrel{?}{=} \frac{4}{3}$$

$$\frac{1}{24} - \frac{1}{40} \stackrel{?}{=} \frac{4}{3}$$

$$\frac{5}{120} - \frac{3}{120} \stackrel{?}{=} \frac{160}{120}$$

$$\frac{2}{120} \neq \frac{160}{120}$$

Not a solution

(d) $x = 10$

$$\frac{10}{3} - \frac{10}{5} \stackrel{?}{=} \frac{4}{3}$$

$$\frac{50}{15} - \frac{30}{15} \stackrel{?}{=} \frac{20}{15}$$

$$\frac{20}{15} = \frac{20}{15}$$

Solution

3. (a) $x = -1$

$$\frac{-1}{4} + \frac{3}{4(-1)} \stackrel{?}{=} 1$$

$$\frac{-1}{4} + \frac{-3}{4} \stackrel{?}{=} 1$$

$$-1 \neq 1$$

Not a solution

(b) $x = 1$

$$\frac{1}{4} + \frac{3}{4(1)} \stackrel{?}{=} 1$$

$$\frac{1}{4} + \frac{3}{4} \stackrel{?}{=} 1$$

$$1 = 1$$

Solution

(c) $x = 3$

$$\frac{3}{4} + \frac{3}{4(3)} \stackrel{?}{=} 1$$

$$\frac{3}{4} + \frac{3}{12} \stackrel{?}{=} 1$$

$$\frac{3}{4} + \frac{1}{4} \stackrel{?}{=} 1$$

$$1 = 1$$

Solution

(d) $x = 2$

$$\frac{2}{4} + \frac{3}{4(2)} \stackrel{?}{=} 1$$

$$\frac{4}{8} + \frac{3}{8} \stackrel{?}{=} 1$$

$$\frac{7}{8} \neq 1$$

Not a solution