

Section 4.3 Multiplying and Dividing Rational Expressions

1. (a) $x = 10$: $\frac{10 - 10}{4(10)} = \frac{0}{40} = 0$ (b) $x = 0$: $\frac{0 - 10}{4(0)} = \frac{-10}{0} = \text{undefined}$
- (c) $x = -2$: $\frac{-2 - 10}{4(-2)} = \frac{-12}{-8} = \frac{3}{2}$ (d) $x = 12$: $\frac{12 - 10}{4(12)} = \frac{2}{48} = \frac{1}{24}$
3. $\frac{7}{3y} = \frac{7x^2}{3y(x^2)}$, $x \neq 0$ 5. $\frac{3x}{x - 4} = \frac{3x(x + 2)^2}{(x - 4)(x + 2)^2}$, $x \neq -2$
7. $\frac{3u}{7v} = \frac{3u(u + 1)}{7v(u + 1)}$, $u \neq -1$ 9. $\frac{13x}{x - 2} = \frac{13x((-1)(2 + x))}{4 - x^2}$, $x \neq -2$
11. $\frac{45}{28} \cdot \frac{77}{60} = \frac{9 \cdot 5 \cdot 7 \cdot 11}{7 \cdot 4 \cdot 6 \cdot 10} = \frac{33}{16}$ 13. $7x \cdot \frac{9}{14x} = \frac{7x \cdot 3 \cdot 3}{7 \cdot 2 \cdot x} = \frac{9}{2}$
15. $\frac{8s^3}{9s} \cdot \frac{6s^2}{32s} = \frac{8s^3 \cdot 3 \cdot 2s \cdot s}{3 \cdot 3 \cdot s \cdot 8 \cdot 2 \cdot 2 \cdot s} = \frac{s^3}{6}$, $s \neq 0$ 17. $16u^4 \cdot \frac{12}{8u^2} = \frac{8 \cdot 2 \cdot u^2 \cdot u^2 \cdot 12}{8 \cdot u^2} = 24u^2$, $u \neq 0$
19. $\frac{8}{3 + 4x} \cdot (9 + 12x) = \frac{8 \cdot 3(3 + 4x)}{3 + 4x}$
 $= 24$, $x \neq -\frac{3}{4}$ 21. $\frac{8u^2v}{3u + v} \cdot \frac{u + v}{12u} = \frac{4 \cdot 2 \cdot u \cdot u \cdot v(u + v)}{(3u + v) \cdot 4 \cdot 3 \cdot u}$
 $= \frac{2uv(u + v)}{3(3u + v)}$, $u \neq 0$
23. $\frac{12 - r}{3} \cdot \frac{3}{r - 12} = \frac{-1(r - 12) \cdot 3}{3(r - 12)} = -1$, $r \neq 12$ 25. $\frac{(2x - 3)(x + 8)}{x^3} \cdot \frac{x}{3 - 2x} = \frac{(2x - 3)(x + 8)x}{x \cdot x^2 \cdot -1(2x - 3)}$
 $= \frac{x + 8}{-x^2}$, $x \neq \frac{3}{2}$
27. $\frac{4r - 12}{r - 2} \cdot \frac{r^2 - 4}{r - 3} = \frac{4(r - 3)(r - 2)(r + 2)}{(r - 2) \cdot (r - 3)} = 4(r + 2)$, $r \neq 3, r \neq 2$
29. $\frac{2t^2 - t - 15}{t + 2} \cdot \frac{t^2 - t - 6}{t^2 - 6t + 9} = \frac{(2t + 5)(t - 3)(t - 3)(t + 2)}{(t + 2)(t - 3)(t - 3)} = 2t + 5$, $t \neq 3, t \neq -2$
31. $(x^2 - 4y^2) \cdot \frac{xy}{(x - 2y)^2} = (x - 2y)(x + 2y) \cdot \frac{xy}{(x - 2y)^2} = \frac{(x + 2y)xy}{x - 2y}$
33. $\frac{x^2 + 2xy - 3y^2}{(x + y)^2} \cdot \frac{x^2 - y^2}{x + 3y} = \frac{(x + 3y)(x - y)}{(x + y)^2} \cdot \frac{(x - y)(x + y)}{x + 3y} = \frac{(x - y)^2}{x + y}$, $x \neq -3y$
35. $\frac{x + 5}{x - 5} \cdot \frac{2x^2 - 9x - 5}{3x^2 + x - 2} \cdot \frac{x^2 - 1}{x^2 + 7x + 10} = \frac{x + 5}{x - 5} \cdot \frac{(2x + 1)(x - 5)}{(3x - 2)(x + 1)} \cdot \frac{(x - 1)(x + 1)}{(x + 5)(x + 2)}$
 $= \frac{(x + 5)(2x + 1)(x - 5)(x - 1)(x + 1)}{(x - 5)(3x - 2)(x + 1)(x + 5)(x + 2)}$
 $= \frac{(2x + 1)(x - 1)}{(3x - 2)(x + 2)}$, $x \neq \pm 5, -1$

$$\begin{aligned}
 37. \quad \frac{9-x^2}{2x+3} \cdot \frac{4x^2+8x-5}{4x^2-8x+3} \cdot \frac{6x^4-2x^3}{8x^2+4x} &= \frac{(3-x)(3+x)}{2x+3} \cdot \frac{(2x+5)(2x-1)}{(2x-3)(2x-1)} \cdot \frac{2x^3(3x-1)}{4x(2x+1)} \\
 &= \frac{-1(x-3)(x+3)(2x+5)x^2(3x-1)}{(2x+3)(2x-3)2(2x+1)} \\
 &= \frac{(x^2-9)(2x+5)x^2(3x-1)}{(2x+3)(3-2x)2(2x+1)}, \quad x \neq 0, \frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 39. \quad \frac{x^3+3x^2-4x-12}{x^3-3x^2-4x+12} \cdot \frac{x^2-9}{x} &= \frac{x^2(x+3)-4(x+3)}{x^2(x-3)-4(x-3)} \cdot \frac{(x+3)(x-3)}{x} \\
 &= \frac{(x+3)(x^2-4)(x+3)(x-3)}{(x-3)(x^2-4) \cdot x} \\
 &= \frac{(x+3)^2}{x}, \quad x \neq -2, 2, 3
 \end{aligned}$$

$$41. \quad \frac{-5}{12} \div \frac{45}{32} = \frac{-5}{12} \cdot \frac{32}{45} = \frac{-5 \cdot 8 \cdot 4}{4 \cdot 3 \cdot 5 \cdot 9} = \frac{-8}{27}$$

$$43. \quad x^2 \div \frac{3x}{4} = x^2 \cdot \frac{4}{3x} = \frac{4x}{3}, \quad x \neq 0$$

$$\begin{aligned}
 45. \quad \frac{7xy^2}{10u^2v} \div \frac{21x^3}{45uv} &= \frac{7xy^2}{10u^2v} \cdot \frac{45uv}{21x^3} \\
 &= \frac{7xy^2 \cdot 3 \cdot 3 \cdot 5 \cdot u \cdot v}{5 \cdot 2 \cdot u \cdot u \cdot v \cdot 7 \cdot 3x \cdot x^2} \\
 &= \frac{3y^2}{2ux^2}, \quad v \neq 0
 \end{aligned}$$

$$\begin{aligned}
 47. \quad \frac{3(a+b)}{4} \div \frac{(a+b)^2}{2} &= \frac{3(a+b)}{4} \cdot \frac{2}{(a+b)^2} \\
 &= \frac{3(a+b) \cdot 2}{2 \cdot 2 \cdot (a+b)(a+b)} \\
 &= \frac{3}{2(a+b)}
 \end{aligned}$$

$$\begin{aligned}
 49. \quad \frac{(x^3y)^2}{(x+2y)^2} \div \frac{x^2y}{(x+2y)^3} &= \frac{(x^3y)^2}{(x+2y)^2} \cdot \frac{(x+2y)^3}{x^2y} \\
 &= \frac{(x^3y)(x^3y)(x+2y)^2(x+2y)}{(x+2y)^2x^2y} \\
 &= \frac{(x^3y)(x^2 \cdot xy)(x+2y)}{x^2y} \\
 &= x^4y(x+2y), \quad x \neq 0, y \neq 0, x \neq -2y
 \end{aligned}$$

$$\begin{aligned}
 51. \quad \frac{\left(\frac{x^2}{12}\right)}{\left(\frac{5x}{18}\right)} &= \frac{x^2}{12} \div \frac{5x}{18} \\
 &= \frac{x^2}{12} \cdot \frac{18}{5x} \\
 &= \frac{x^2 \cdot 3 \cdot 3 \cdot 2}{2 \cdot 2 \cdot 3 \cdot 5 \cdot x} \\
 &= \frac{3x}{10}, \quad x \neq 0
 \end{aligned}$$

$$\begin{aligned}
 53. \quad \frac{\left(\frac{25x^2}{x-5}\right)}{\left(\frac{10x}{5+4x-x^2}\right)} &= \frac{25x^2}{x-5} \div \frac{10x}{5+4x-x^2} \\
 &= \frac{25x^2}{x-5} \cdot \frac{5+4x-x^2}{10x} \\
 &= \frac{5 \cdot 5 \cdot x \cdot x \cdot (-1)(x^2-4x-5)}{(x-5) \cdot 5 \cdot 2 \cdot x} \\
 &= \frac{5 \cdot x \cdot (-1)(x-5)(x+1)}{(x-5)2} \\
 &= \frac{-5x(x+1)}{2}, \quad x \neq 0, 5, -1
 \end{aligned}$$

$$\begin{aligned}
 55. \quad \frac{16x^2 + 8x + 1}{3x^2 + 8x - 3} \div \frac{4x^2 - 3x - 1}{x^2 + 6x + 9} &= \frac{16x^2 + 8x + 1}{3x^2 + 8x - 3} \cdot \frac{x^2 + 6x + 9}{4x^2 - 3x - 1} \\
 &= \frac{(4x + 1)(4x + 1)}{(3x - 1)(x + 3)} \cdot \frac{(x + 3)(x + 3)}{(4x + 1)(x - 1)} \\
 &= \frac{(4x + 1)(4x + 1)(x + 3)(x + 3)}{(3x - 1)(x + 3)(4x + 1)(x - 1)} \\
 &= \frac{(4x + 1)(x + 3)}{(3x - 1)(x - 1)}, \quad x \neq -3, -\frac{1}{4}
 \end{aligned}$$

$$\begin{aligned}
 57. \quad \frac{x^2 + 3x - 2x - 6}{x^2 - 4} \div \frac{x + 3}{x^2 + 4x + 4} &= \frac{x(x + 3) - 2(x + 3)}{x^2 - 4} \cdot \frac{x^2 + 4x + 4}{x + 3} \\
 &= \frac{(x + 3)(x - 2)}{(x - 2)(x + 2)} \cdot \frac{(x + 2)(x + 2)}{x + 3} \\
 &= \frac{(x + 3)(x - 2)(x + 2)(x + 2)}{(x - 2)(x + 2)(x + 3)} \\
 &= (x + 2), \quad x \neq \pm 2, -3
 \end{aligned}$$

$$\begin{aligned}
 59. \quad \frac{\left(\frac{x^2 - 3x - 10}{x^2 - 4x + 4}\right)}{\left(\frac{21 + 4x - x^2}{x^2 - 5x - 14}\right)} &= \frac{x^2 - 3x - 10}{x^2 - 4x + 4} \div \frac{21 + 4x - x^2}{x^2 - 5x - 14} \\
 &= \frac{x^2 - 3x - 10}{x^2 - 4x + 4} \cdot \frac{x^2 - 5x - 14}{1(x^2 - 4x - 21)} \\
 &= \frac{(x - 5)(x + 2)}{(x - 2)(x - 2)} \cdot \frac{(x - 7)(x + 2)}{-1(x - 7)(x + 3)} \\
 &= -\frac{(x^2 - 3x - 10)(x + 2)}{(x^2 - 4x + 4)(x + 3)}, \quad x \neq \pm 2, 7
 \end{aligned}$$

$$\begin{aligned}
 61. \quad \left[\frac{x^2}{9} \cdot \frac{3(x + 4)}{x^2 + 2x}\right] \div \frac{x}{x + 2} &= \frac{x^2}{9} \cdot \frac{3(x + 4)}{x(x + 2)} \cdot \frac{x + 2}{x} \\
 &= \frac{x + 4}{3}, \quad x \neq -2, 0
 \end{aligned}$$

$$63. \quad \left[\frac{xy + y}{4x} \div (3x + 3)\right] \div \frac{y}{3x} = \frac{y(x + 1)}{4x} \cdot \frac{1}{3(x + 1)} \cdot \frac{3x}{y} = \frac{1}{4}, \quad x \neq -1, 0, y \neq 0$$

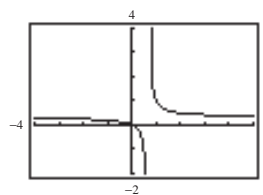
$$\begin{aligned}
 65. \quad \frac{2x^2 + 5x - 25}{3x^2 + 5x + 2} \cdot \frac{3x^2 + 2x}{x + 5} \div \left(\frac{x}{x + 1}\right)^2 &= \frac{(2x - 5)(x + 5)}{(3x + 2)(x + 1)} \cdot \frac{x(3x + 2)}{x + 5} \cdot \left(\frac{x + 1}{x}\right)^2 \\
 &= \frac{(2x - 5)(x + 5)x(3x + 2)(x + 1)(x + 1)}{(3x + 2)(x + 1)(x + 5)x \cdot x} \\
 &= \frac{(2x - 5)(x + 1)}{x}, \quad x \neq -1, -5, -\frac{2}{3}
 \end{aligned}$$

$$\begin{aligned}
 67. \quad x^3 \cdot \frac{x^{2n} - 9}{x^{2n} + 4x^n + 3} \div \frac{x^{2n} - 2x^n - 3}{x} &= x^3 \cdot \frac{(x^n - 3)(x^n + 3)}{(x^n + 3)(x^n + 1)} \cdot \frac{x}{(x^n - 3)(x^n + 1)} \\
 &= \frac{x^4}{(x^n + 1)^2}, \quad x^n \neq -3, 3, 0
 \end{aligned}$$

69. Keystrokes:

$$y_1 \text{ [Y=] [C] [C] 3 [X,T,\theta] [+] 2 [)] [\div] [X,T,\theta] [)] [\times]$$

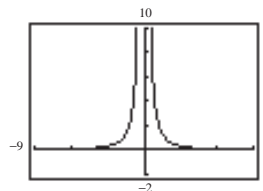
$$[C] [X,T,\theta] [x^2] [\div] [C] 9 [X,T,\theta] [x^2] [-] 4 [)] [)] [\text{ENTER}]$$

$$y_2 \text{ [X,T,\theta] [\div] [C] 3 [X,T,\theta] [-] 2 [)] [\text{GRAPH}]$$


71. Keystrokes:

$$y_1 \text{ [Y=] [C] [C] 3 [X,T,\theta] [+] 15 [)] [\div] [X,T,\theta] [^] 4 [)] [\div]$$

$$[C] [C] [X,T,\theta] [+] 5 [)] [\div] [X,T,\theta] [x^2] [)] [\text{ENTER}]$$

$$y_2 \text{ 3 [\div] [X,T,\theta] [x^2] [\text{GRAPH}]}$$


73.
$$\text{Area} = \left(\frac{2w+3}{3}\right)\left(\frac{w}{2}\right) = \frac{(2w+3)w}{6}$$

75.
$$\frac{\text{Unshaded Area}}{\text{Total Area}} = \frac{\frac{x}{2} \cdot \frac{x}{2}}{x(2x+1)} = \left[\frac{x}{2} \cdot \frac{x}{2}\right] \div [x(2x+1)]$$

$$= \frac{x}{2} \cdot \frac{x}{2} \cdot \frac{1}{x(2x+1)}$$

$$= \frac{x}{4(2x+1)}$$

77.
$$\frac{\text{Unshaded Area}}{\text{Total Area}} = \frac{x \cdot \frac{x}{2}}{x(2x+1)} = \frac{x}{2(2x+1)}$$

79. (a)
$$\frac{20 \text{ pages}}{1 \text{ minute}} = \frac{20 \text{ pages}}{60 \text{ seconds}} = \frac{1 \text{ page}}{3 \text{ seconds}},$$

$$t = 3 \text{ seconds or } \frac{1}{20} \text{ minutes}$$

(b)
$$\frac{3 \text{ seconds}}{1 \text{ page}} \cdot x \text{ pages} = 3x \text{ seconds or } \frac{x}{20} \text{ minutes}$$

(c)
$$\frac{3 \text{ seconds}}{1 \text{ page}} \cdot 35 \text{ pages} = 3 \cdot 35 \text{ seconds}$$

$$= 105 \text{ seconds or } \frac{7}{4} \text{ minutes}$$

81. (a) Keystrokes:

$$y_1 \text{ [Y=] 6357 [+] 1070 [X,T,\theta] [x^2] [\text{ENTER}]}$$

$$y_2 \text{ 6115.2 [+] 590.7 [X,T,\theta] [x^2] [\text{GRAPH}]}$$

(b) Verbal Model:

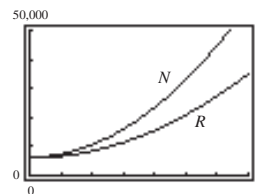
Average monthly bill	=	Revenue Number of subscribers
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Equation:
$$AMB = \frac{(6115.2 + 590.7t^2)1000}{(6357 + 1070t^2)12} = \frac{6115200 + 590700t^2}{(6357 + 1070t^2)12}$$

 (c)

Year, t	0	2	4	6
Monthly bill	\$80.16	\$66.40	\$55.21	\$50.70

(d) The number of subscribers was increasing at a faster rate than the revenue.



83. Divide a rational expression by a polynomial by multiplying the rational expression by the reciprocal of the polynomial.

85. Invert the divisor, not the dividend.

$$16. (x^2 + 2x) \cdot \frac{5}{x^2 - 4} = \frac{x(x+2)5}{(x-2)(x+2)}$$

$$= \frac{5x}{x-2}$$

$$17. \frac{4}{3(x-1)} \cdot \frac{12x}{6(x^2+2x-3)} = \frac{4(12x)}{3(x-1)6(x+3)(x-1)}$$

$$= \frac{8x}{3(x-1)^2(x+3)}$$

$$= \frac{8x}{3(x-1)(x^2+2x-3)}$$

$$18. \frac{5u}{3(u+v)} \cdot \frac{2(u^2-v^2)}{3v} \div \frac{25u^2}{18(u-v)} = \frac{5u \cdot 2(u-v)(u+v) \cdot 18(u-v)}{3(u+v)(3v)(25u^2)}$$

$$= \frac{4(u-v)^2}{5uv}$$

$$19. \frac{\frac{9t^2}{3-t}}{6t} \cdot \frac{t-3}{t-3} = \frac{-9t^2}{6t} = -\frac{3t}{2}$$

$$20. \frac{\frac{10}{x^2+2x}}{15} = \frac{\frac{10}{x(x+2)}}{15} \cdot \frac{x(x+2)(x+1)}{x(x+2)(x+1)}$$

$$= \frac{10(x+1)}{15x}$$

$$= \frac{2(x+1)}{3x}$$

21. (a) Verbal Model:

Average cost

 =

Total cost

 \div

Number of units

Equation: Average cost = $\frac{6000 + 10.50x}{x}$

(b) Average cost when $x = 500$ units are produced = $\frac{6000 + 10.50(500)}{500} = \22.50

Section 4.4 Adding and Subtracting Rational Expressions

$$1. \frac{5}{8} + \frac{7}{8} = \frac{5+7}{8} = \frac{12}{8} = \frac{3}{2}$$

$$3. \frac{5x}{8} - \frac{7x}{8} = \frac{-2x}{8} = \frac{-x}{4}$$

$$5. \frac{2}{3a} - \frac{11}{3a} = \frac{2-11}{3a} = \frac{-9}{3a} = \frac{-3}{a}$$

$$7. \frac{x}{9} - \frac{x+2}{9} = \frac{x-(x+2)}{9} = \frac{x-x-2}{9} = \frac{-2}{9}$$

$$9. \frac{z^2}{3} + \frac{z^2-2}{3} = \frac{z^2+z^2-2}{3} = \frac{2z^2-2}{3}$$

$$11. \frac{2x+5}{3} + \frac{1-x}{3} = \frac{2x+5+1-x}{3} = \frac{x+6}{3}$$

$$13. \frac{3y}{3} - \frac{3y-3}{3} - \frac{7}{3} = \frac{3y-(3y-3)-7}{3}$$

$$= \frac{3y-3y+3-7}{3}$$

$$= \frac{-4}{3}$$

$$15. \frac{3y-22}{y-6} - \frac{2y-16}{y-6} = \frac{3y-22-(2y-16)}{y-6}$$

$$= \frac{3y-22-2y+16}{y-6}$$

$$= \frac{y-6}{y-6}$$

$$= 1, y \neq 6$$