

$$\begin{aligned}
 111. \frac{(5,000,000)^3(0.000037)^2}{(0.005)^4} &= \frac{(5.0 \times 10^6)^3(3.7 \times 10^{-5})^2}{(5.0 \times 10^{-3})^4} \\
 &= \frac{(5^3 \times 10^{18})(3.7^2 \times 10^{-10})}{5^4 \times 10^{-12}} \\
 &= \frac{(125)(13.69)}{625} \times 10^{18+(-10)-(-12)} \\
 &= 2.738 \times 10^{18-10+12} \\
 &= 2.738 \times 10^{20} \\
 &\approx 2.74 \times 10^{20}
 \end{aligned}$$

$$113. 93,000,000 = 9.3 \times 10^7$$

$$\begin{aligned}
 115. \frac{1.49 \times 10^{11}}{9.45 \times 10^{15}} &= \frac{1.49}{9.45} \times 10^{-4} \\
 &\approx 0.157672 \times 10^{-4} \\
 &\approx 1.58 \times 10^{-5} \\
 &\approx 8.3 \text{ minutes}
 \end{aligned}$$

$$\begin{aligned}
 117. \frac{1.99 \times 10^{30}}{5.975 \times 10^{24}} &= \frac{1.99}{5.975} \times 10^6 \\
 &\approx 0.3330544 \times 10^6 \\
 &\approx 3.33 \times 10^5
 \end{aligned}$$

$$\begin{aligned}
 119. \frac{\$5506 \text{ billion}}{270 \text{ million}} &= \frac{\$5,506,000,000,000}{270,000,000} \\
 &= \frac{5.506 \times 10^{12}}{2.7 \times 10^8} \\
 &\approx 2.03925 \times 10^4 \\
 &\approx \$20,393
 \end{aligned}$$

121. In $(3x)^4$, $3x$ is called the base and 4 is called the exponent.

123. You can “move” a factor from the numerator to the denominator by changing the sign of the exponent of the factor.

125. Scientific notation is an efficient way of writing and computing real numbers when the numbers are very large or very small.

Section 4.2 Rational Expressions and Functions

$$1. x - 8 \neq 0$$

$$x \neq 8$$

$$D = (-\infty, 8) \cup (8, \infty)$$

$$3. x + 4 \neq 0$$

$$x \neq -4$$

$$D = (-\infty, -4) \cup (-4, \infty)$$

$$5. 4 \neq 0$$

$$D = (-\infty, \infty)$$

$$7. D = (-\infty, \infty)$$

$$9. x^2 + 4 \neq 0$$

$$D = (-\infty, \infty)$$

$$11. y(y + 3) \neq 0$$

$$y \neq 0 \quad y \neq -3$$

$$D = (-\infty, -3) \cup (-3, 0) \cup (0, \infty)$$

13. $t^2 - 16 \neq 0$

$(t - 4)(t + 4) \neq 0$

$t \neq 4 \quad t \neq -4$

$D = (-\infty, -4) \cup (-4, 4) \cup (4, \infty)$

15. $y^2 - 3y \neq 0$

$y(y - 3) \neq 0$

$y \neq 0 \quad y \neq 3$

$D = (-\infty, 0) \cup (0, 3) \cup (3, \infty)$

17. $x^2 - 5x + 6 \neq 0$

$(x - 3)(x - 2) \neq 0$

$x \neq 3 \quad x \neq 2$

$D = (-\infty, 2) \cup (2, 3) \cup (3, \infty)$

19. $3u^2 - 2u - 5 \neq 0$

$(3u - 5)(u + 1) \neq 0$

$u \neq \frac{5}{3} \quad u \neq -1$

$D = (-\infty, -1) \cup (-1, \frac{5}{3}) \cup (\frac{5}{3}, \infty)$

21. (a) $f(1) = \frac{4(1)}{1+3} = \frac{4}{4} = 1$

(c) $f(-3) = \frac{4(-3)}{-3+3} = \frac{-12}{0}$

= not possible; undefined

(b) $f(-2) = \frac{4(-2)}{-2+3} = \frac{-8}{1} = -8$

(d) $f(0) = \frac{4(0)}{0+3} = \frac{0}{3} = 0$

23. (a) $g(0) = \frac{0^2 - 4(0)}{0^2 - 9} = 0$

(c) $g(3) = \frac{3^2 - 4(3)}{3^2 - 9} = \frac{9 - 12}{9 - 9} = \frac{-3}{0}$

= not possible; undefined

(b) $g(4) = \frac{4^2 - 4(4)}{4^2 - 9} = \frac{16 - 16}{16 - 9} = \frac{0}{7} = 0$

(d) $g(-3) = \frac{(-3)^2 - 4(-3)}{(-3)^2 - 9} = \frac{9 + 12}{9 - 9} = \frac{21}{0}$

= not possible; undefined

25. (a) $h(10) = \frac{10^2}{10^2 - 10 - 2} = \frac{100}{88} = \frac{25}{22}$

(c) $h(-1) = \frac{(-1)^2}{(-1)^2 - (-1) - 2} = \frac{1}{1 + 1 - 2} = \frac{1}{0}$

= not possible; undefined

(b) $h(0) = \frac{0^2}{0^2 - 0 - 2} = \frac{0}{-2} = 0$

(d) $h(2) = \frac{2^2}{2^2 - 2 - 2} = \frac{4}{4 - 2 - 2} = \frac{4}{0}$

= not possible; undefined

27. Since length must be positive,
 $x \geq 0$. Since $\frac{500}{x}$ must be defined,
 $x \neq 0$. Therefore, the domain is
 $x > 0$ or $(0, \infty)$.

29. $x =$ units of a product
 $D = \{1, 2, 3, 4, \dots\}$

31. Since p is the percent of air pollutants in the stack emission of a utility, $0 \leq p \leq 100$. Since

$$\frac{80,000p}{100 - p}$$

must be defined, $p \neq 100$.

Therefore, the domain is $[0, 100)$.

33. $\frac{5}{6} = \frac{5(x+3)}{6(x+3)}, \quad x \neq -3$

35. $\frac{x}{2} = \frac{3x(x+16)^2}{2(3(x+16)^2)}, \quad x \neq -16$

37. $\frac{x+5}{3x} = \frac{(x+5)(x(x-2))}{3x^2(x-2)}, \quad x \neq 2$

39. $\frac{8x}{x-5} = \frac{8x(x+2)}{x^2-3x-10}, \quad x \neq -2$

41. $\frac{5x}{25} = \frac{5x}{5 \cdot 5} = \frac{x}{5}$

43. $\frac{12y^2}{2y} = \frac{2 \cdot 6 \cdot y \cdot y}{2 \cdot y}$
 $= 6y, \quad y \neq 0$

$$45. \frac{18x^2y}{15xy^4} = \frac{3 \cdot 6 \cdot x \cdot x \cdot y}{3 \cdot 5 \cdot x \cdot y \cdot y^3}$$

$$= \frac{6x}{5y^3}, \quad x \neq 0$$

$$51. \frac{2x-3}{4x-6} = \frac{2x-3}{2(2x-3)} = \frac{1}{2}, \quad x \neq \frac{3}{2}$$

$$57. \frac{x^2-7x}{x^2-14x+49} = \frac{x(x-7)}{(x-7)(x-7)}$$

$$= \frac{x}{x-7}$$

$$63. \frac{3x^2-7x-20}{12+x-x^2} = \frac{(3x+5)(x-4)}{-1(x^2-x-12)}$$

$$= \frac{(3x+5)(x-4)}{-1(x-4)(x+3)}$$

$$= -\frac{3x+5}{x+3}, \quad x \neq 4$$

$$67. \frac{15x^2+7x-4}{25x^2-16} = \frac{(5x+4)(3x-1)}{(5x+4)(5x-4)}$$

$$= \frac{3x-1}{5x-4}, \quad x \neq -\frac{4}{5}$$

$$71. \frac{y^2-64x^2}{5(3y+24x)} = \frac{(y-8x)(y+8x)}{15(y+8x)}$$

$$= \frac{y-8x}{15}, \quad y \neq -8x$$

$$75. \frac{u^2-4v^2}{u^2+uv-2v^2} = \frac{(u-2v)(u+2v)}{(u-v)(u+2v)}$$

$$= \frac{u-2v}{u-v}, \quad u \neq -2v$$

$$79. \frac{x-4}{4} \neq x-1$$

$$\frac{10-4}{4} \neq 10-1$$

$$\frac{6}{4} \neq 9$$

Choose a value such as 10
for x and evaluate both sides.

$$47. \frac{3x^2-9x}{12x^2} = \frac{3x(x-3)}{12x^2} = \frac{(x-3)}{4x}$$

$$53. \frac{5-x}{3x-15} = \frac{-1(x-5)}{3(x-5)}$$

$$= -\frac{1}{3}, \quad x \neq 5$$

$$59. \frac{y^3-4y}{y^2+4y-12} = \frac{y(y^2-4)}{(y+6)(y-2)}$$

$$= \frac{y(y-2)(y+2)}{(y+6)(y-2)}$$

$$= \frac{y(y+2)}{y+6}, \quad y \neq 2$$

$$49. \frac{x^2(x-8)}{x(x-8)} = \frac{x \cdot x(x-8)}{x(x-8)}$$

$$= x, \quad x \neq 0, x \neq 8$$

$$55. \frac{a+3}{a^2+6a+9} = \frac{a+3}{(a+3)(a+3)}$$

$$= \frac{1}{a+3}$$

$$61. \frac{x^3-4x}{x^2-5x+6} = \frac{x(x^2-4)}{(x-3)(x-2)}$$

$$= \frac{x(x-2)(x+2)}{(x-3)(x-2)}$$

$$= \frac{x(x+2)}{x-3}, \quad x \neq 2$$

$$65. \frac{2x^2+19x+24}{2x^2-3x-9} = \frac{(2x+3)(x+8)}{(2x+3)(x-3)}$$

$$= \frac{x+8}{x-3}, \quad x \neq -\frac{3}{2}$$

$$69. \frac{3xy^2}{xy^2+x} = \frac{3xy^2}{x(y^2+1)}$$

$$= \frac{3y^2}{y^2+1}, \quad x \neq 0$$

$$73. \frac{5xy+3x^2y^2}{xy^3} = \frac{xy(5+3xy)}{xy \cdot y^2}$$

$$= \frac{5+3xy}{y^2}, \quad x \neq 0$$

$$77. \frac{3m^2-12n^2}{m^2+4mn+4n^2} = \frac{3(m^2-4n^2)}{(m+2n)(m+2n)}$$

$$= \frac{3(m-2n)(m+2n)}{(m+2n)(m+2n)}$$

$$= \frac{3(m-2n)}{m+2n}$$

$$81. \frac{3x+2}{4x+2} \neq \frac{3}{4}$$

$$\frac{3(0)+2}{4(0)+2} \neq \frac{3}{4}$$

$$1 \neq \frac{3}{4}$$

Choose a value such as 0 for
 x and evaluate both sides.

83. x	-2	-1	0	1	2	3	4
$\frac{x^2 - x - 2}{x - 2}$	-1	0	1	2	Undefined	4	5
$x + 1$	-1	0	1	2	3	4	5

Domain of $\frac{x^2 - x - 2}{x - 2}$ is $(-\infty, 2) \cup (2, \infty)$.

Domain of $x + 1$ is $(-\infty, \infty)$.

The two expressions are equal for all replacements of the variable x except 2.

85. $\frac{\text{Area of shaded portion}}{\text{Area of total figure}} = \frac{x(x+1)}{(x+1)(x+3)} = \frac{x}{x+3}, \quad x > 0$

87. (a) Verbal Model: $\boxed{\text{Total cost}} = \boxed{\text{Number of units}} \cdot \boxed{\text{Cost per unit}} + \boxed{\text{Initial cost}}$

Labels: Total cost = C

Number of units = x

Equation: $2500 + 9.25x = C$

(b) Verbal Model: $\boxed{\text{Average cost}} = \boxed{\text{Total cost}} \div \boxed{\text{Number of units}}$

Label: Average cost = \bar{C}

Equation: $\bar{C} = \frac{2500 + 9.25x}{x}$

(c) Domain = $\{1, 2, 3, 4, \dots\}$

(d) $\frac{2500 + 9.25(100)}{100} = \34.25

89. (a) Verbal Model: $\boxed{\text{Distance}} = \boxed{\text{Rate}} \cdot \boxed{\text{Time}}$

Van: $45(t + 3)$

Car: $60t$

(b) Distance between van and car = d

$$= 45(t + 3) - 60t$$

$$= 45t + 135 - 60t$$

$$= 135 - 15t$$

$$= 15(9 - t)$$

(c) $\frac{\text{Distance of car}}{\text{Distance of van}} = \frac{60t}{45(t + 3)} = \frac{4t}{3(t + 3)}$

91. $\frac{\text{Circular pool volume}}{\text{Rectangular pool volume}} = \frac{\pi(3d)^2(d + 2)}{d(3d)(3d + 6)}$

$$= \frac{\pi(3d)^2(d + 2)}{3d^2 \cdot 3(d + 2)}$$

$$= \frac{\pi(3d)^2(d + 2)}{(3d)^2(d + 2)}$$

$$= \pi$$

93. Average cost of Medicare per person = $\frac{107.30 + 15.09t \text{ billion}}{34.26 + 0.65t \text{ million}} = \frac{(10,730 + 1509t)1000}{3426 + 65t}$

95. Let u and v be polynomials. The algebraic expression u/v is a rational expression.

97. The rational expression is in simplified form if the numerator and denominator have no factors in common (other than ± 1).

99. You can cancel only common factors.