Section 3.5 Solving Polynomial Equations

1. $2x(x-8)=0$

$2x=0 \qquad x-8=0$

$x=0 \qquad x=8$

3. $(y-3)(y+10)=0$

$y-3=0 \qquad y+10=0$

$y=3 \qquad y=-10$

5. $25(a+4)(a-2)=0$

$a+4=0 \qquad a-2=0$

$a=-4 \qquad a=2$

7. $(2t+5)(3t+1)=0$

$2t+5=0 \qquad 3t+1=0$

$t=-\frac{5}{2} \qquad t=-\frac{1}{3}$

9. $4x(2x-3)(2x+25)=0$

$4x=0 \qquad 2x-3=0 \qquad 2x+25=0$

$x=0 \qquad x=\frac{3}{2} \qquad x=-\frac{25}{2}$

11. $(x-3)(2x+1)(x+4)=0$

$x-3=0 \qquad 2x+1=0 \qquad x+4=0$

$x=3 \qquad x=-\frac{1}{2} \qquad x=-4$

13. $5y-y^2=0$

$y(5-y)=0$

$y=0 \qquad 5-y=0$

$5=y$

15. $9x^2+15x=0$

$3x(3x+5)=0$

$3x=0 \qquad 3x+5=0$

$x=0 \qquad x=-\frac{5}{3}$

17. $x(x+2)-10(x+2)=0$

$(x+2)(x-10)=0$

$x+2=0 \qquad x-10=0$

$x=-2 \qquad x=10$

19. $u(u-3)+3(u-3)=0$

$(u-3)(u+3)=0$

$u-3=0 \qquad u+3=0$

$u=3 \qquad u=-3$

21. $x^2-25=0$

$(x+5)(x-5)=0$

$x+5=0 \qquad x-5=0$

$x=-5 \qquad x=5$

23. $3y^2-48=0$

$3(y^2-16)=0$

$3(y+4)(y-4)=0$

$y+4=0 \qquad y-4=0$

$y=-4 \qquad y=4$

25. $x^2-3x-10=0$

$(x-5)(x+2)=0$

$x-5=0 \qquad x+2=0$

$x=5 \qquad x=-2$

27. $x^2-10x+24=0$

$(x-6)(x-4)=0$

$x-6=0 \qquad x-4=0$

$x=6 \qquad x=4$

29. $4x^2+15x-25=0$

$(4x-5)(x+5)=0$

$4x-5=0 \qquad x+5=0$

$4x=5 \qquad x=-5$

$x=\frac{5}{4}$

31. $7 + 13x - 2x^2 = 0$

$(7 - x)(1 + 2x) = 0$

$7 - x = 0 \qquad 1 + 2x = 0$

$7 = x \qquad -\frac{1}{2} = x$

33. $m^2 - 8m + 18 = 2$

$m^2 - 8m + 16 = 0$

$(m - 4)^2 = 0$

$m - 4 = 0$

$m = 4$

35. $x^2 + 16x + 57 = -7$

$x^2 + 16x + 64 = 0$

$(x + 8)^2 = 0$

$x + 8 = 0$

$x = -8$

37. $4z^2 - 12z + 15 = 6$

$4z^2 - 12z + 9 = 0$

$(2z - 3)^2 = 0$

$2z - 3 = 0$

$z = \frac{3}{2}$

39. $x(x - 5) = 36$

$x^2 - 5x = 36$

$x^2 - 5x - 36 = 0$

$(x - 9)(x + 4) = 0$

$x - 9 = 0 \qquad x + 4 = 0$

$x = 9 \qquad x = -4$

41. $y(y + 6) = 72$

$y^2 + 6y - 72 = 0$

$(y + 12)(y - 6) = 0$

$y + 12 = 0 \qquad y - 6 = 0$

$y = -12 \qquad y = 6$

43. $t(2t - 3) = 35$

$2t^2 - 3t - 35 = 0$

$(2t + 7)(t - 5) = 0$

$2t + 7 = 0 \qquad t - 5 = 0$

$t = -\frac{7}{2} \qquad t = 5$

45. $(a + 2)(a + 5) = 10$

$a^2 + 7a + 10 - 10 = 0$

$a^2 + 7a = 0$

$a(a + 7) = 0$

$a = 0 \qquad a + 7 = 0$

$a = 0 \qquad a = -7$

47. $(x - 4)(x + 5) = 10$

$x^2 + x - 20 - 10 = 0$

$x^2 + x - 30 = 0$

$(x + 6)(x - 5) = 0$

$x + 6 = 0 \qquad x - 5 = 0$

$x = -6 \qquad x = 5$

49. $(t - 2)^2 - 16 = 0$

$(t - 2 + 4)(t - 2 - 4) = 0$

$(t + 2)(t - 6) = 0$

$t + 2 = 0 \qquad t - 6 = 0$

$t = -2 \qquad t = 6$

51. $(x + 2)^2 = 9$

$[(x + 2) - 3][(x + 2) + 3] = 0$

$(x - 1)(x + 5) = 0$

$(x - 1) = 0 \qquad (x + 5) = 0$

$x = 1 \qquad x = -5$

53. $x^3 - 19x^2 + 84x = 0$

$x(x^2 - 19x + 84) = 0$

$x(x - 12)(x - 7) = 0$

$x = 0 \quad x - 12 = 0 \qquad x - 7 = 0$

$x = 0 \qquad x = 12 \qquad x = 7$

55. $6t^3 - t^2 - t = 0$

$t(6t^2 - t - 1) = 0$

$t(3t + 1)(2t - 1) = 0$

$t = 0 \quad 3t + 1 = 0 \qquad 2t - 1 = 0$

$t = 0 \qquad t = -\frac{1}{3} \qquad t = \frac{1}{2}$

57. $z^2(z+2) - 4(z+2) = 0$

$(z+2)(z^2-4) = 0$

$(z+2)(z-2)(z+2) = 0$

$z+2=0 \quad z-2=0 \quad z+2=0$

$z=-2 \quad z=2 \quad z=-2$

59. $a^3 + 2a^2 - 9a - 18 = 0$

$(a^3+2a^2) + (-9a-18) = 0$

$a^2(a+2) - 9(a+2) = 0$

$(a+2)(a^2-9) = 0$

$(a+2)(a-3)(a+3) = 0$

$a+2=0 \quad a-3=0 \quad a+3=0$

$a=-2 \quad a=3 \quad a=-3$

61. $c^3 - 3c^2 - 9c + 27 = 0$

$c^2(c-3) - 9(c-3) = 0$

$(c-3)(c^2-9) = 0$

$(c-3)(c-3)(c+3) = 0$

$c-3=0 \quad c-3=0 \quad c+3=0$

$c=3 \quad c=3 \quad c=-3$

63. $x^4 - 3x^3 - x^2 + 3x = 0$

$x^3(x-3) - x(x-3) = 0$

$(x-3)(x^3-x) = 0$

$(x-3)\,x(x^2-1) = 0$

$(x-3)\,x(x-1)(x+1) = 0$

$x-3=0 \quad x=0 \quad x-1=0 \quad x+1=0$

$x=3 \quad x=1 \quad x=-1$

65. $8x^4 + 12x^3 - 32x^2 - 48x = 0$

$4x^3(2x+3) - 16x(2x+3) = 0$

$(2x+3)(4x^3-16x) = 0$

$(2x+3)\,4x(x^2-4) = 0$

$(2x+3)(4x)(x-2)(x+2) = 0$

$2x+3=0 \quad 4x=0 \quad x-2=0 \quad x+2=0$

$x=-\frac{3}{2} \quad x=0 \quad x=2 \quad x=-2$

67. From the graph, the x-intercepts are $(-3, 0)$ and $(3, 0)$. The solutions of the equation $0 = x^2 - 9$ are 3 and -3.

$0 = (x-3)(x+3)$

$0 = x-3 \quad 0 = x+3$

$3 = x \quad -3 = x$

69. From the graph, the x-intercepts are $(-1, 0)$ and $(3, 0)$. The solutions of the equation $0 = x^2 - 2x - 3$ are -1 and 3.

$0 = x^2 - 2x - 3$

$0 = (x-3)(x+1)$

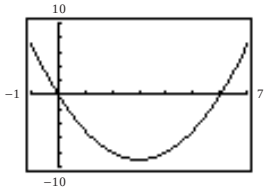
$0 = x-3 \quad x+1=0$

$3 = x \quad x=-1$

71. *Keystrokes:*

Y= X,T,θ x² − 6 X,T,θ GRAPH

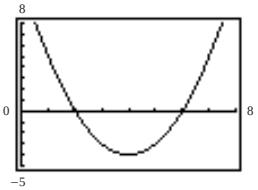
The x-intercepts are 0 and 6, so the solutions are 0 and 6.



73. *Keystrokes:*

Y= X,T,θ x² − 8 X,T,θ + 12 GRAPH

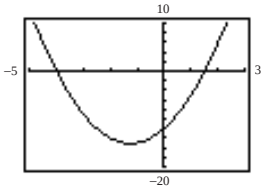
The x-intercepts are 2 and 6, so the solutions are 2 and 6.



75. *Keystrokes:*

[Y=] 2 [X,T,θ] [x²] [+] 5 [X,T,θ] [−] 12 [GRAPH]

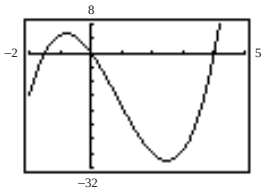
The x-intercepts are -4 and $\frac{3}{2}$, so the solutions are -4 and $\frac{3}{2}$.



77. *Keystrokes:*

[Y=] 2 [X,T,θ] [^] 3 [−] 5 [X,T,θ] [x²] [−] 12 [X,T,θ] [GRAPH]

The x-intercepts are $-\frac{3}{2}$, 0, and 4, so the solutions are $-\frac{3}{2}$, 0, and 4.



79. $ax^2 + bx = 0$

$x(ax + b) = 0$

$x = 0$ $\qquad ax + b = 0$

$ax = -b$

$x = -\frac{b}{a}$

81. $x = -3, x = 5$

$[x - (-3)](x - 5) = 0$

$(x + 3)(x - 5) = 0$

$x^2 - 2x - 15 = 0$

83. *Verbal model:* $\boxed{\text{Number}} + \boxed{\text{Its Square}} = \boxed{240}$

Labels: Number $= x$

Its square $= x^2$

Equation: $x + x^2 = 240$

$x^2 + x - 240 = 0$

$(x + 16)(x - 15) = 0$

$x + 16 = 0$ $\qquad x - 15 = 0$

$x = -16$ $\qquad x = 15$

reject

85. *Verbal model:* $\boxed{\begin{matrix}\text{First}\\\text{Integer}\end{matrix}} \cdot \boxed{\begin{matrix}\text{Second}\\\text{Integer}\end{matrix}} = \boxed{132}$

Labels: First integer $= x$

Second integer$x + 1$

Equation: $x \cdot (x + 1) = 132$

$x^2 + x - 132 = 0$

$(x + 12)(x - 11) = 0$

$x + 12 = 0$ $\qquad x - 11 = 0$

$x = -12$ $\qquad x = 11$ 1st integer

reject $\qquad x + 1 = 12$ 2nd integer

87. *Verbal model:* $\boxed{\text{Length}} \cdot \boxed{\text{Width}} = \boxed{\text{Area}}$

Labels: Length $= x + 7$

Width $= x$

Equation: $(x + 7) \cdot x = 330$

$x^2 + 7x = 330$

$x^2 + 7x - 330 = 0$

$(x + 22)(x - 15) = 0$

$x + 22 = 0$ $\qquad$ $x - 15 = 0$

$x = -22$ $\qquad$ $x = 15$ feet width

reject $\qquad$ $x + 7 = 22$ feet length

89. *Verbal model:* $\boxed{\frac{1}{2}} \cdot \boxed{\text{Base}} \cdot \boxed{\text{Height}} = \boxed{\text{Area}}$

Labels: Base $= x$

Height $= \frac{3}{2}x$

Equation: $\frac{1}{2} \cdot x \cdot \frac{3}{2}x = 48$

$\frac{3}{4}x^2 - 48 = 0$

$3x^2 - 192 = 0$

$3(x^2 - 64) = 0$

$(x + 8)(x - 8) = 0$

$x + 8 = 0$ $\qquad$ $x - 8 = 0$

$x = -8$ $\qquad$ $x = 8$ inches base

reject $\qquad$ $\frac{3}{2}x = 12$ inches height

91. (a) Volume $V =$ Length $\cdot$ Width $\cdot$ Height

$V = (5 - 2x)(4 - 2x)x$

(b) $0 = (5 - 2x)(4 - 2x)x$ $\qquad$ Domain: Each side must be positive.

$5 - 2x = 0$ $\qquad$ $4 - 2x = 0$ $\qquad$ $x = 0$ $\qquad$ $x > 0$ $\qquad$ $5 - 2x > 0$ $\qquad$ $4 - 2x > 0$ so $0 < x < 2$

$x = \frac{5}{2}$ $\qquad$ $x = 2$ $\qquad$ $x = 0$ $\qquad$ $x < \frac{5}{2}$ $\qquad$ $x < 2$

(c)

x	0.25	0.50	0.75	1.00	1.25	1.50	1.75
V	3.94	6	6.56	6	4.69	3	1.31

(d) If $V = 3$, then $x = 1.5$.

$3 = [5 - 2(1.5)][4 - 2(1.5)](1.5)$

$3 = (5 - 3)(4 - 3)(1.5)$

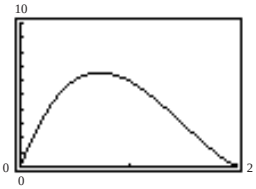
$3 = (2)(1)(1.5)$

$3 = 3$

(e) *Keystrokes:*

Y= (5 − 2 X,T,θ) (4 − 2 X,T,θ) X,T,θ GRAPH

$x = 0.74$ yields the box of greatest volume.



93. $-16t^2 + 6400 = 0$

$-16(t^2 - 400) = 0$

$-16(t - 20)(t + 20) = 0$

$t - 20 = 0$ $\quad$ $t + 20 = 0$

$t = 20$ $\quad$ $t = -20$

reject

The object reaches the ground after 20 seconds.

95. *Verbal model:* $\boxed{\text{Revenue}} = \boxed{\text{Cost}}$

Equation: $90x - x^2 = 200 + 60x$

$0 = x^2 - 30x + 200$

$0 = (x - 20)(x - 10)$

$x - 20 = 0$ $\quad$ $x - 10 = 0$

$x = 20$ units $\quad$ $x = 10$ units

97. (a) $2(x + 3)^2 + (x + 3) - 15 = 0$

$2u^2 + u - 15 = 0$

$(2u - 5)(u + 3) = 0$

$2u - 5 = 0$ $\quad$ $u + 3 = 0$

$2u = 5$ $\quad$ $u = -3$

$u = \frac{5}{2}$

$u = x + 3$ $\quad$ $u = x + 3$

$\frac{5}{2} = x + 3$ $\quad$ $-3 = x + 3$

$-\frac{1}{2} = x$ $\quad$ $-6 = x$

(b) $2(x + 3)^2 + (x + 3) - 15 = 0$

$2(x^2 + 6x + 9) + (x + 3) - 15 = 0$

$2x^2 + 12x + 18 + x + 3 - 15 = 0$

$2x^2 + 13x + 6 = 0$

$(2x + 1)(x + 6) = 0$

$2x + 1 = 0$ $\quad$ $x + 6 = 0$

$2x = -1$ $\quad$ $x = -6$

$x = -\frac{1}{2}$

(c) Answers will vary.

99. (d) *Verbal model:* $\boxed{\text{Area}} = \boxed{\text{Length}} \cdot \boxed{\text{Width}}$

Labels: Area $= 30$

Length $= 2x - 6$

Width $= x - 1$

Equation: $30 = (2x - 6)(x - 1)$

$30 = 2x^2 - 8x + 6$

$0 = 2x^2 - 8x - 24$

$0 = x^2 - 4x - 12$

$0 = (x - 6)(x + 2)$

$0 = x - 6$ $\quad$ $x + 2 = 0$

$6 = x$ $\quad$ $x = -2$ reject

Length $= 2(6) - 6$ $\quad$ Width $= 6 - 1$ $\quad$ Height $= 6 - 3$

$= 6$ feet $\quad$ $= 5$ feet $\quad$ $= 3$ feet

(e) Volume $= V_S(x) = \frac{7}{3}x^3 + \frac{53}{3}x^2 - 20x + 6$

$V_S(6) = \frac{7}{3}(6)^3 + \frac{53}{3}(6)^2 - 20(6) + 6$

$= \frac{7}{3}(216) + \frac{53}{3}(36) - 120 + 6$

$= 504 + 636 - 120 + 6$

$= 1026$ cubic feet

(f) Domain of $V_B(x)$

101. False. This is not an application of the Zero Factor Property because there are unlimited number of factors whose product is 1.

103. The maximum number of solutions of an n^{th} degree polynomial equation is n. The third-degree equation $(x + 1)^3 = 0$ has only one solution, $x = -1$.

Review Exercises for Chapter 3

1. $x^2 + 2 + 3x^{1/2}$ is not a polynomial because the exponent of a variable must be a natural number.

3. Standard form: $-x^4 + 6x^3 + 5x^2 - 4x$

Leading coefficient: -1

Degree: 4

5. Standard form: $-7x^3 + 3x^2 - 6x + 14$

Leading coefficient: -7

Degree: 3

7. Binomial of degree 4: $3x^4 - 2$

9. Monomial of degree 3 and leading coefficient 5: $5x^3$

11. $(5x + 3x^2) + (6 - x - 4x^2) = (3x^2 - 4x^2) + (5x - x) + 6 = -x^2 + 4x + 6$

13. $(5x^3 - 6x + 11) + (5 + 6x - x^2 - 8x^3) = (5x^3 - 8x^3) - x^2 + (-6x + 6x) + (11 + 5) = -3x^3 - x^2 + 16$

15. $(3t - 5) - (t^2 - t - 5) = (3t - 5) + (-t^2 + t + 5) = -t^2 + (3t + t) + (-5 + 5) = -t^2 + 4t$

17. $$\begin{aligned}(3x^5 + 4x^2 - 8x + 12) - (2x^5 + x) + (3x^2 - 4x^3 - 9) &= (3x^5 - 2x^5) - 4x^3 + (4x^2 + 3x^2) + (-8x - x) + (12 - 9)\\ &= x^5 - 4x^3 + 7x^2 - 9x + 3\end{aligned}$$

19. $$\begin{aligned}(-x^3 - 3x) - 4(2x^3 - 3x + 1) &= -x^3 - 3x - 8x^3 + 12x - 4\\ &= (-x^3 - 8x^3) + (-3x + 12x) + (-4)\\ &= -9x^3 + 9x - 4\end{aligned}$$

21. $$\begin{aligned}3y^2 - [2y + 3(y^2 + 5)] &= 3y^2 - [2y + 3y^2 + 15]\\ &= 3y^2 - 2y - 3y^2 - 15\\ &= (3y^2 - 3y^2) - 2y - 15\\ &= -2y - 15\end{aligned}$$

23. $x^2 \cdot x^3 = x^{2+3} = x^5$

25. $(u^2)^3 = u^{2 \cdot 3} = u^6$

27. $$\begin{aligned}(-2z)^3 &= (-2)^3 z^3\\ &= -8z^3\end{aligned}$$

29. $$\begin{aligned}-(u^2v)^2(-4u^3v) &= -(u^4v^2)(-4u^3v)\\ &= 4u^{4+3}v^{2+1}\\ &= 4u^7v^3\end{aligned}$$

31. $\dfrac{12z^5}{6z^2} = \left(\dfrac{12}{6}\right) \cdot z^{5-2} = 2z^3$

33. $$\begin{aligned}\frac{120u^5v^3}{15u^3v} &= \frac{120}{15} \cdot \frac{u^5}{u^3} \cdot \frac{v^3}{v}\\ &= 8u^2v^2\end{aligned}$$

35. $$\begin{aligned}\left(\frac{72x^4}{6x^2}\right)^2 &= (12x^{4-2})^2\\ &= (12x^2)^2\\ &= 144x^4\end{aligned}$$