

23. $(5x + 10)(2x + 1)$ $(5x - 10)(2x - 1)$
 $(5x + 1)(2x + 10)$ $(5x - 1)(2x - 10)$
 $(5x + 2)(2x + 5)$ $(5x - 2)(2x - 5)$
 $(5x + 5)(2x + 2)$ $(5x - 5)(2x - 2)$

24. Verbal model:

Area of shaded region

 =

Area of large triangle

 -

Area of small triangle

Equation:
$$\begin{aligned} A &= \frac{1}{2}(x + 2)^2 - \frac{1}{2}x^2 \\ &= \frac{1}{2}(x^2 + 4x + 4) - \frac{1}{2}x^2 \\ &= \frac{1}{2}x^2 + 2x + 2 - \frac{1}{2}x^2 \\ &= 2x + 2 \end{aligned}$$

25.
$$\begin{aligned} h(1) &= -16(1)^2 - 5(1) + 100 \\ &= -16 - 5 + 100 \\ &= 79 \text{ feet} \\ h(2) &= -16(2)^2 - 5(2) + 100 \\ &= -16(4) - 10 + 100 \\ &= -64 + 90 \\ &= 26 \text{ feet} \end{aligned}$$

26.
$$\begin{aligned} P(x) &= R(x) - C(x) \\ &= 19x - (5x + 2000) \\ &= 19x - 5x - 2000 \\ &= 14x - 2000 \\ P(1000) &= 14(1000) - 2000 = \$12,000 \end{aligned}$$

Section 3.4 Factoring Trinomials

1. $x^2 + 4x + 4 = x^2 + 2(2x) + 2^2 = (x + 2)^2$ 3. $a^2 - 12a + 36 = a^2 - 2(6a) + 6^2 = (a - 6)^2$
5. $25y^2 - 10y + 1 = (5y)^2 - 2(5y) + 1 = (5y - 1)^2$ 7. $9b^2 + 12b + 4 = (3b)^2 + 2(3b)(2) + 2^2 = (3b + 2)^2$
9. $u^2 + 8uv + 16v^2 = u^2 + 2(4uv) + (4v)^2 = (u + 4v)^2$
11. $36x^2 - 60xy + 25y^2 = (6x)^2 - 2(6x)(5y) + (5y)^2 = (6x - 5y)^2$
13. $5x^2 + 30x + 45 = 5(x^2 + 6x + 9) = 5[x^2 + 2(3)(x) + 3^2] = 5(x + 3)^2$
15. $2x^2 + 24x^2 + 72x = 2x(x^2 + 12x + 36) = 2x[x^2 + 2(6)x + 6^2] = 2x(x + 6)^2$
17. $20v^4 - 60v^3 + 45v^2 = 5v^2(4v^2 - 12v + 9) = 5v^2[(2v)^2 - 2(2v)(3) + 3^2] = 5v^2(2v - 3)^2$

$$\begin{aligned}
 19. \quad \frac{1}{4}x^2 - \frac{2}{3}x + \frac{4}{9} &= \left(\frac{1}{2}x\right)^2 - 2\left(\frac{1}{2}x\right)\left(\frac{2}{3}\right) + \left(\frac{2}{3}\right)^2 && \text{or} && = \frac{9}{36}x^2 - \frac{24}{36}x + \frac{16}{36} \\
 &= \left(\frac{1}{2}x - \frac{2}{3}\right)^2 && && = \frac{1}{36}(9x^2 - 24x + 16) \\
 &&& && = \frac{1}{36}[(3x)^2 - 2(3x)(4) + 4^2] \\
 &&& && = \frac{1}{36}(3x - 4)^2
 \end{aligned}$$

$$21. \quad x^2 + bx + 81 = x^2 + bx + 9^2$$

$$\begin{aligned}
 \text{(a) } b = 18 &&& \text{or} \\
 x^2 + 18x + 9^2 &= x^2 + 2(9x) + 9^2 \\
 &= (x + 9)^2
 \end{aligned}$$

$$\begin{aligned}
 \text{(b) } b = -18 &&& \\
 x^2 - 18x + 9^2 &= x^2 - 2(9x) + 9^2 \\
 &= (x - 9)^2
 \end{aligned}$$

$$23. \quad 4x^2 + bx + 9 = (2x)^2 + bx + 3^2$$

$$\begin{aligned}
 \text{(a) } b = 12 &&& \text{or} \\
 (2x)^2 + 12x + 3^2 &= (2x)^2 + 2(2x)(3) + 3^2 \\
 &= (2x + 3)^2
 \end{aligned}$$

$$\begin{aligned}
 \text{(b) } b = -12 &&& \\
 (2x)^2 - 12x + 3^2 &= (2x)^2 - 2(2x)(3) + 3^2 \\
 &= (2x - 3)^2
 \end{aligned}$$

$$25. \quad c = 16$$

$$\begin{aligned}
 x^2 + 8x + c &= x^2 + 2(4x) + c \\
 &= x^2 + 2(4x) + 4^2 \\
 &= (x + 4)^2
 \end{aligned}$$

$$27. \quad c = 9$$

$$\begin{aligned}
 y^2 - 6y + c &= y^2 - 2(3y) + c \\
 &= y^2 - 2(3y) + 3^2 \\
 &= (y - 3)^2
 \end{aligned}$$

$$29. \quad x^2 + 5x + 4 = (x + 4)(x + 1)$$

$$31. \quad y^2 - y - 20 = (y + 4)(y - 5)$$

$$33. \quad x^2 - 2x - 24 = (x + 4)(x - 6)$$

$$35. \quad z^2 - 6z + 8 = (z - 4)(z - 2)$$

$$37. \quad x^2 + 4x + 3 = (x + 3)(x + 1)$$

$$39. \quad x^2 - 5x + 6 = (x - 3)(x - 2)$$

$$41. \quad y^2 + 7y - 30 = (y + 10)(y - 3)$$

$$43. \quad t^2 - 4t - 21 = (t - 7)(t + 3)$$

$$45. \quad x^2 - 20x + 96 = (x - 12)(x - 8)$$

$$47. \quad x^2 - 2xy - 35y^2 = (x - 7y)(x + 5y)$$

$$49. \quad x^2 + 30xy + 216y^2 = (x + 12y)(x + 18y)$$

$$51. \quad b = 19: \quad x^2 + 19x + 18 = (x + 18)(x + 1)$$

$$53. \quad b = 20: \quad x^2 + 20x - 21 = (x + 21)(x - 1)$$

$$b = -19: \quad x^2 - 19x + 18 = (x - 18)(x - 1)$$

$$b = -20: \quad x^2 - 20x - 21 = (x - 21)(x + 1)$$

$$b = 9: \quad x^2 + 9x + 18 = (x + 6)(x + 3)$$

$$b = 4: \quad x^2 + 4x - 21 = (x + 7)(x - 3)$$

$$b = -9: \quad x^2 - 9x + 18 = (x - 6)(x - 3)$$

$$b = -4: \quad x^2 - 4x - 21 = (x - 7)(x + 3)$$

$$b = 11: \quad x^2 + 11x + 18 = (x + 9)(x + 2)$$

$$b = -11: \quad x^2 - 11x + 18 = (x - 9)(x - 2)$$

$$55. \quad b = 36: \quad x^2 + 36x + 35 = (x + 35)(x + 1)$$

$$b = -36: \quad x^2 - 36x + 35 = (x - 35)(x - 1)$$

$$b = 12: \quad x^2 + 12x + 35 = (x + 7)(x + 5)$$

$$b = -12: \quad x^2 - 12x + 35 = (x - 7)(x - 5)$$

57. There are many possibilities, such as:

$$c = 5 \quad x^2 + 6x + 5 = (x + 5)(x + 1)$$

$$c = 8 \quad x^2 + 6x + 8 = (x + 4)(x + 2)$$

$$c = 9 \quad x^2 + 6x + 9 = (x + 3)(x + 3)$$

Also note that if $c =$ a negative number, there are many possibilities for c such as the following.

$$c = -7 \quad x^2 + 6x - 7 = (x + 7)(x - 1)$$

$$c = -16 \quad x^2 + 6x - 16 = (x + 8)(x - 2)$$

$$c = -27 \quad x^2 + 6x - 27 = (x + 9)(x - 3)$$

59. There are many possibilities, such as:

$$c = 2 \quad x^2 - 3x + 2 = (x - 2)(x - 1)$$

$$c = -4 \quad x^2 - 3x - 4 = (x - 4)(x + 1)$$

$$c = -10 \quad x^2 - 3x - 10 = (x - 5)(x + 2)$$

$$c = -18 \quad x^2 - 3x - 18 = (x - 6)(x + 3)$$

There are more possibilities.

$$61. 5x^2 + 18x + 9 = (x + 3)(5x + 3) \quad 63. 5a^2 + 12a - 9 = (a + 3)(5a - 3) \quad 65. 2y^2 - 3y - 27 = (y + 3)(2y - 9)$$

$$67. 3x^2 + 4x + 1 = (3x + 1)(x + 1) \quad 69. 7x^2 + 15x + 2 = (7x + 1)(x + 2) \quad 71. 2x^2 - 9x + 9 = (2x - 3)(x - 3)$$

$$73. 6x^2 - 11x + 3 = (3x - 1)(2x - 3) \quad 75. 3t^2 - 4t - 10 = \text{prime} \quad 77. 6b^2 + 19b - 7 = (3b - 1)(2b + 7)$$

$$79. 18y^2 + 35y + 12 = (2y + 3)(9y + 4)$$

$$81. -2x^2 - x + 6 = -1(2x^2 + x - 6) = -1(2x - 3)(x + 2)$$

$$\begin{aligned} 83. 1 - 11x - 60x^2 &= -60x^2 - 11x + 1 \\ &= -1(60x^2 + 11x - 1) \\ &= -1(15x - 1)(4x + 1) \end{aligned}$$

$$\begin{aligned} 85. 6x^2 - 3x - 84 &= 3(2x^2 - x - 28) \\ &= 3(2x + 7)(x - 4) \end{aligned}$$

$$\begin{aligned} 87. 60y^3 + 35y^2 - 50y &= 5y(12y^2 + 7y - 10) \\ &= 5y(3y - 2)(4y + 5) \end{aligned}$$

$$89. 10a^2 + 23ab + 6b^2 = (a + 2b)(10a + 3b)$$

$$91. 24x^2 - 14xy - 3y^2 = (6x + y)(4x - 3y)$$

$$\begin{aligned} 93. 3x^2 + 10x + 8 &= 3x^2 + 6x + 4x + 8 \\ &= (3x^2 + 6x) + (4x + 8) \\ &= 3x(x + 2) + 4(x + 2) \\ &= (3x + 4)(x + 2) \end{aligned}$$

$$\begin{aligned} 95. 6x^2 + x - 2 &= 6x^2 + 4x - 3x - 2 \\ &= (6x^2 + 4x) + (-3x - 2) \\ &= 2x(3x + 2) - 1(3x + 2) \\ &= (2x - 1)(3x + 2) \end{aligned}$$

$$\begin{aligned} 97. 15x^2 - 11x + 2 &= 15x^2 - 6x - 5x + 2 \\ &= (15x^2 - 6x) + (-5x + 2) \\ &= 3x(5x - 2) - 1(5x - 2) \\ &= (3x - 1)(5x - 2) \end{aligned}$$

$$99. 3x^4 - 12x^3 = 3x^3(x - 4)$$

$$\begin{aligned} 101. 10t^3 + 2t^2 - 36t &= 2t(5t^2 + t - 18) \\ &= 2t(5t - 9)(t + 2) \end{aligned}$$

$$\begin{aligned} 103. 54x^3 - 2 &= 2(27x^3 - 1) \\ &= 2(3x - 1)(9x^2 + 3x + 1) \end{aligned}$$

$$\begin{aligned} 105. 27a^3b^4 - 9a^2b^3 - 18ab^2 &= 9ab^2(3a^2b^2 - ab - 2) \\ &= 9ab^2(3ab + 2)(ab - 1) \end{aligned}$$

$$\begin{aligned}
 107. \quad x^3 + 2x^2 - 16x - 32 &= (x^3 + 2x^2) + (-16x - 32) \\
 &= x^2(x + 2) - 16(x + 2) \\
 &= (x + 2)(x^2 - 16) \\
 &= (x + 2)(x - 4)(x + 4)
 \end{aligned}$$

$$\begin{aligned}
 109. \quad 36 - (z + 3)^2 &= [6 - (z + 3)][6 + (z + 3)] \\
 &= [6 - z - 3][6 + z + 3] \\
 &= (3 - z)(9 + z)
 \end{aligned}$$

$$\begin{aligned}
 111. \quad x^2 - 10x + 25 - y^2 &= (x - 5)^2 - y^2 \\
 &= [(x - 5) + y][(x - 5) - y] \\
 &= (x - 5 + y)(x - 5 - y)
 \end{aligned}$$

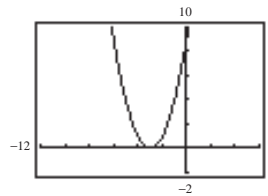
$$\begin{aligned}
 113. \quad x^8 - 1 &= (x^4)^2 - 1^2 = (x^4 - 1)(x^4 + 1) \\
 &= [(x^2)^2 - 1^2](x^4 + 1) \\
 &= (x^2 - 1)(x^2 + 1)(x^4 + 1) \\
 &= (x - 1)(x + 1)(x^2 + 1)(x^4 + 1)
 \end{aligned}$$

115. Keystrokes:

$$y_1 \text{ [Y=] [X,T,\theta] [x^2] [+] 6 [X,T,\theta] [+] 9 [ENTER]$$

$$y_2 \text{ [C] [X,T,\theta] [+] 3 [)] [x^2] [GRAPH]$$

$$y_1 = y_2$$

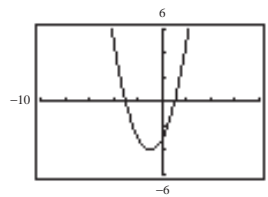


117. Keystrokes:

$$y_1 \text{ [Y=] [X,T,\theta] [x^2] [+] 2 [X,T,\theta] [-] 3 [ENTER]$$

$$y_2 \text{ [C] [X,T,\theta] [-] 1 [)] [C] [X,T,\theta] [+] 3 [)] [GRAPH]$$

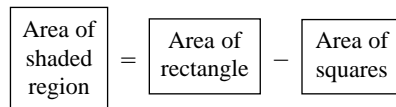
$$y_1 = y_2$$



119. $a^2 - b^2 = (a + b)(a - b)$ matches graph (c).

121. $a^2 + 2ab + b^2 = (a + b)^2$ matches graph (b).

123. Verbal model:



$$\begin{aligned}
 \text{Equation:} \quad \text{Area} &= (8 \cdot 18) - 4 \cdot x^2 \\
 &= 144 - 4x^2 \\
 &= 4(36 - x^2) \\
 &= 4(6 + x)(6 - x)
 \end{aligned}$$

$$\begin{aligned}
 125. \quad (a) \quad 8n^3 - 8n &= 2n(4n^2 - 4) \\
 &= 2n[(2n)^2 - 2^2] \\
 &= 2n(2n - 2)(2n + 2)
 \end{aligned}$$

$$\begin{aligned}
 (b) \quad \text{If } n = 10, \quad 2n &= 2(10) = 20 \\
 2n - 2 &= 2(10) - 2 = 18 \\
 2n + 2 &= 2(10) + 2 = 22
 \end{aligned}$$

127. To factor $x^2 - 5x + 6$ begin by finding the factors of 6 whose sum is -5 . They are -2 and -3 . The factorization is $(x - 2)(x - 3)$.

129. Check the factors of a trinomial by multiplication. The factors of $x^2 - 5x + 6$ are $x - 2$ and $x - 3$ because

$$(x - 2)(x - 3) = x^2 - 5x + 6.$$

131. No, $x(x + 2) - 2(x + 2)$ is not in factored form. It is not yet a product. $x(x + 2) - 2(x + 2) = (x + 2)(x - 2)$