

123. Yes, dividing both sides of an inequality by 5 is the same as multiplying both sides by  $\frac{1}{5}$ .

$$\begin{array}{rcl} 5x = 15 & & 5x = 15 \\ \frac{5x}{5} = \frac{15}{5} & & \frac{1}{5} \cdot 5x = 15 \cdot \frac{1}{5} \\ x = 3 & & x = 3 \end{array}$$

125. The multiplication and division properties differ. The inequality symbol is reversed if both sides of the inequality are multiplied or divided by a negative real number.

127. If  $-5 \leq t < 8$ , then  $(-1)(-5) \geq (-1)(t) > (-1)(8)$  and  $5 \geq -t > -8$  or  $-8 < -t \leq 5$ .

## Section 1.5 Absolute Value Equations and Inequalities

1.  $|4x + 5| = 10, x = -3$

$$|4(-3) + 5| \stackrel{?}{=} 10$$

$$|-12 + 5| \stackrel{?}{=} 10$$

$$|-7| \stackrel{?}{=} 10$$

$$7 \neq 10$$

No

3.  $|6 - 2w| = 2, w = 4$

$$|6 - 2(4)| \stackrel{?}{=} 2$$

$$|6 - 8| \stackrel{?}{=} 2$$

$$|-2| \stackrel{?}{=} 2$$

$$2 = 2$$

Yes

5.  $x - 10 = 17$  or  $x - 10 = -17$

7.  $4x + 1 = \frac{1}{2}$  or  $4x + 1 = -\frac{1}{2}$

9.  $|x| = 4$

$$x = 4 \text{ or } x = -4$$

11.  $|t| = -45$

No solution

13.  $|h| = 0$

$$h = 0$$

15.  $|5x| = 15$

$$5x = 15 \text{ or } 5x = -15$$

$$x = 3 \quad x = -3$$

17.  $|x - 16| = 5$

$$x - 16 = 5 \text{ or } x - 16 = -5$$

$$x = 21 \quad x = 11$$

19.  $|2s + 3| = 25$

$$2s + 3 = 25 \text{ or } 2s + 3 = -25$$

$$2s = 22 \quad 2s = -28$$

$$s = 11 \quad s = -14$$

21.  $|32 - 3y| = 16$

$$32 - 3y = 16 \text{ or } 32 - 3y = -16$$

$$-3y = -16 \quad -3y = -48$$

$$y = \frac{16}{3} \quad y = 16$$

23.  $|3x + 4| = -16$

No solution

25.  $|4 - 3x| = 0$

$$4 - 3x = 0$$

$$-3x = -4$$

$$x = \frac{4}{3}$$

27.  $|\frac{2}{3}x + 4| = 9$

$$\frac{2}{3}x + 4 = 9 \text{ or } \frac{2}{3}x + 4 = -9$$

$$\frac{2}{3}x = 5 \quad \frac{2}{3}x = -13$$

$$2x = 15 \quad 2x = -39$$

$$x = \frac{15}{2} \quad x = -\frac{39}{2}$$

29.  $|0.32x - 2| = 4$

$$0.32x - 2 = 4 \text{ or } 0.32x - 2 = -4$$

$$0.32x = 6 \quad 0.32x = -2$$

$$x = \frac{6}{0.32} \quad x = \frac{-2}{0.32}$$

$$x = 18.75 \quad x = -6.25$$

31.  $|5x - 3| + 8 = 22$

$|5x - 3| = 14$

$5x - 3 = 14$  or  $5x - 3 = -14$

$5x = 17$

$5x = -11$

$x = \frac{17}{5}$

$x = -\frac{11}{5}$

33.  $|3x + 9| - 12 = -8$

$|3x + 9| = 4$

$3x + 9 = 4$  or  $3x + 9 = -4$

$3x = -5$

$3x = -13$

$x = -\frac{5}{3}$

$x = -\frac{13}{3}$

35.  $-2|7 - 4x| = -16$

$|7 - 4x| = 8$

$7 - 4x = 8$  or  $7 - 4x = -8$

$-4x = 1$

$-4x = -15$

$x = -\frac{1}{4}$

$x = \frac{15}{4}$

37.  $3|2x - 5| + 4 = 7$

$3|2x - 5| = 3$

$|2x - 5| = 1$

$2x - 5 = 1$  or  $2x - 5 = -1$

$2x = 6$

$2x = 4$

$x = 3$

$x = 2$

39.  $|x + 8| = |2x + 1|$

$x + 8 = 2x + 1$  or  $x + 8 = -(2x + 1)$

$8 = x + 1$

$x + 8 = -2x - 1$

$7 = x$

$3x + 8 = -1$

$3x = -9$

$x = -3$

41.  $|x + 2| = |3x - 1|$

$x + 2 = 3x - 1$  or  $x + 2 = -(3x - 1)$

$3 = 2x$

$x + 2 = -3x + 1$

$\frac{3}{2} = x$

$4x = -1$

$x = -\frac{1}{4}$

43.  $|45 - 4x| = |32 - 3x|$

$45 - 4x = 32 - 3x$  or  $45 - 4x = -(32 - 3x)$

$45 = 32 + x$

$45 - 4x = -32 + 3x$

$13 = x$

$45 = -32 + 7x$

$77 = 7x$

$11 = x$

45.  $|4x - 10| = 2|2x + 3|$

$4x - 10 = 2(2x + 3)$  or  $4x - 10 = -2(2x + 3)$

$4x - 10 = 4x + 6$

$4x - 10 = -4x - 6$

$-10 = 6$

$8x - 10 = -6$

No solution

$8x = 4$

$x = \frac{4}{8} = \frac{1}{2}$

47.  $|x - 5| = 3$

49. (a)  $x = 2$

$|2| < 3$

$2 < 3$

Yes

(b)  $x = -4$

$|-4| < 3$

$4 < 3$

No

(c)  $x = 4$

$|4| < 3$

$4 < 3$

No

(d)  $x = -1$

$|-1| < 3$

$1 < 3$

Yes

51. (a)  $x = 9$

$|9 - 7| \geq 3$

$|2| \geq 3$

$2 \geq 3$

No

(b)  $x = -4$

$|-4 - 7| \geq 3$

$|-11| \geq 3$

$11 \geq 3$

Yes

(c)  $x = 11$

$|11 - 7| \geq 3$

$|4| \geq 3$

$4 \geq 3$

Yes

(d)  $x = 6$

$|6 - 7| \geq 3$

$|-1| \geq 3$

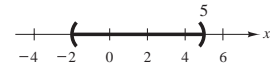
$1 \geq 3$

No

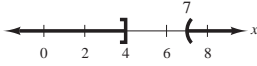
53.  $|y + 5| < 3$   
 $-3 < y + 5 < 3$

55.  $|7 - 2h| \geq 9$   
 $7 - 2h \geq 9$  or  $7 - 2h \leq -9$

57. "All  $x$  greater than  $-2$  and less than  $5$ ."



59. "All  $x$  less than or equal to 4 or greater than 7."



61.  $|y| < 4$   
 $-4 < y < 4$

63.  $|x| \geq 6$   
 $x \geq 6$  or  $x \leq -6$

65.  $|2x| < 14$   
 $-14 < 2x < 14$   
 $-7 < x < 7$

67.  $\left|\frac{y}{3}\right| \leq 3$   
 $-3 \leq \frac{y}{3} \leq 3$   
 $-9 \leq y \leq 9$

69.  $|y - 2| \leq 4$   
 $-4 \leq y - 2 \leq 4$   
 $-2 \leq y \leq 6$

71.  $|x + 6| > 10$   
 $x + 6 > 10$  or  $x + 6 < -10$   
 $x > 4$                        $x < -16$

73.  $|2x - 1| \leq 7$   
 $-7 \leq 2x - 1 \leq 7$   
 $-6 \leq 2x \leq 8$   
 $-3 \leq x \leq 4$

75.  $|6t + 15| \geq 30$   
 $6t + 15 \leq -30$  or  $6t + 15 \geq 30$   
 $6t \leq -45$                        $6t \geq 15$   
 $t \leq -\frac{45}{6}$                        $t \geq \frac{15}{6}$   
 $t \leq -\frac{15}{2}$                        $t \geq \frac{5}{2}$

77.  $|2 - 5x| > -8$   
 $-\infty < x < \infty$   
 Absolute value is always positive.

79.  $|3x + 10| < -1$   
 No solution  
 Absolute value is never negative.

81.  $\frac{|x + 2|}{10} \leq 8$   
 $|x + 2| \leq 80$   
 $-80 \leq x + 2 \leq 80$   
 $-82 \leq x \leq 78$

83.  $\frac{|y - 16|}{4} < 30$   
 $|y - 16| < 120$   
 $-120 < y - 16 < 120$   
 $-104 < y < 136$

85.  $\left|\frac{z}{10} - 3\right| > 8$   
 $\frac{z}{10} - 3 < -8$  or  $\frac{z}{10} - 3 > 8$   
 $\frac{z}{10} < -5$                        $\frac{z}{10} > 11$   
 $z < -50$                        $z > 110$

87.  $|0.2x - 3| < 4$   
 $-4 < 0.2x - 3 < 4$   
 $-1 < 0.2x < 7$   
 $-\frac{1}{0.2} < x < \frac{7}{0.2}$   
 $-5 < x < 35$

89.  $\left|6 - \frac{3}{5}x\right| \leq 0.4$   
 $-0.4 \leq 6 - \frac{3}{5}x \leq 0.4$   
 $-2 \leq 30 - 3x \leq 2$   
 $-32 \leq -3x \leq -28$   
 $\frac{32}{3} \geq x \geq \frac{28}{3}$   
 $\frac{28}{3} \leq x \leq \frac{32}{3}$

91.  $-2|3x + 6| < 4$   
 $|3x + 6| > -2$   
 $-\infty < x < \infty$   
 Absolute value is always positive.

93.  $\left|9 - \frac{x}{2}\right| - 7 \leq 4$   
 $\left|9 - \frac{x}{2}\right| \leq 11$   
 $-11 \leq 9 - \frac{x}{2} \leq 11$   
 $-20 \leq -\frac{x}{2} \leq 2$   
 $40 \geq x \geq -4$   
 $-4 \leq x \leq 40$

95.  $|3x + 2| < 4$

 Keystrokes:  $\boxed{Y=}$   $\boxed{ABS}$   $\boxed{(}$   $\boxed{3}$   $\boxed{X,T,\theta}$   $\boxed{+}$   $\boxed{2}$   $\boxed{)}$   $\boxed{<}$   $\boxed{4}$   $\boxed{GRAPH}$ 

$$-2 < x < \frac{2}{3}$$

97.  $|2x + 3| > 9$

 Keystrokes:  $\boxed{Y=}$   $\boxed{ABS}$   $\boxed{(}$   $\boxed{2}$   $\boxed{X,T,\theta}$   $\boxed{+}$   $\boxed{3}$   $\boxed{)}$   $\boxed{>}$   $\boxed{9}$   $\boxed{GRAPH}$ 

$$x < -6 \text{ or } x > 3$$

99.  $|x - 5| + 3 \leq 5$

 Keystrokes:  $\boxed{Y=}$   $\boxed{ABS}$   $\boxed{(}$   $\boxed{X,T,\theta}$   $\boxed{-}$   $\boxed{5}$   $\boxed{)}$   $\boxed{+}$   $\boxed{3}$   $\boxed{\leq}$   $\boxed{5}$   $\boxed{GRAPH}$ 

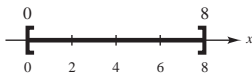
$$3 \leq x \leq 7$$

101. Matches graph (d).

$$|x - 4| \leq 4$$

$$-4 \leq x - 4 \leq 4$$

$$0 \leq x \leq 8$$



103. Matches graph (b).

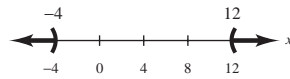
$$\frac{1}{2}|x - 4| > 4$$

$$|x - 4| > 8$$

$$x - 4 > 8 \quad \text{or} \quad x - 4 < -8$$

$$x > 12$$

$$x < -4$$


 105.  $[-2, 2]$ 

$$|x| \leq 2$$

107. (16, 22)

$$16 < x < 22$$

$$-3 < x - 19 < 3$$

$$|x - 19| < 3$$

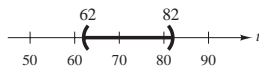
 109.  $|x| < 3$ 

 111.  $|x - 5| > 6$ 

 113.  $|t - 72| < 10$ 

$$-10 < t - 72 < 10$$

$$62 < t < 82$$


 115. (a)  $|s - x| \leq \frac{3}{16}$ 

(b)  $\left|5\frac{1}{8} - x\right| \leq \frac{3}{16}$

$$-\frac{3}{16} \leq 5\frac{1}{8} - x \leq \frac{3}{16}$$

$$-\frac{3}{16} \leq \frac{41}{8} - x \leq \frac{3}{16}$$

$$-\frac{85}{16} \leq -x \leq -\frac{79}{16}$$

$$\frac{85}{16} \geq x \geq \frac{79}{16}$$

117. The absolute value of a real number measures the distance of the real number from zero.

 119. The solutions of  $|x| = a$  are  $x = a$  and  $x = -a$ . For example, to solve  $|x - 3| = 5$ :

$$x - 3 = 5 \quad \text{or} \quad x - 3 = -5$$

$$x = 8 \quad \quad \quad x = -2$$

 121. The graph of  $|x - 4| < 1$  can be described as all real numbers less than one unit from 4.

 123.  $|x - 3| \leq 3$  since

$$-3 \leq x - 3 \leq 3$$

$$0 \leq x \leq 6.$$

## Review Exercises for Chapter 1

1. (a)  $45 - 7(3) = 3$

$$45 - 21 = 3$$

$$24 = 3$$

Not a solution

(b)  $45 - 7(6) = 3$

$$45 - 42 = 3$$

$$3 = 3$$

Solution