

CHAPTER 1

Linear Equations and Inequalities

Section 1.1 Linear Equations

Solutions to Odd-Numbered Exercises

1. (a) $x = 0$
 $3(0) - 7 \stackrel{?}{=} 2$
 $-7 \neq 2$
 No
- (b) $x = 3$
 $3(3) - 7 \stackrel{?}{=} 2$
 $9 - 7 = 2$
 $2 = 2$
 Yes
3. (a) $x = 4$
 $4 + 8 \stackrel{?}{=} 3(4)$
 $12 = 12$
 Yes
- (b) $x = -4$
 $-4 + 8 \stackrel{?}{=} 3(-4)$
 $4 \neq -12$
 No
5. (a) $x = -11$
 $3(-11) + 3 \stackrel{?}{=} 2(-11 - 4)$
 $-33 + 3 = 2(-15)$
 $-30 = -30$
 Yes
- (b) $x = 5$
 $3(5) + 3 \stackrel{?}{=} 2(5 - 4)$
 $15 + 3 = 2(1)$
 $18 \neq 2$
 No
7. (a) $x = -4$
 $\frac{1}{4}(-4) \stackrel{?}{=} 3$
 $-1 \neq 3$
 No
- (b) $x = 12$
 $\frac{1}{4}(12) \stackrel{?}{=} 3$
 $3 = 3$
 Yes
9. $3(x - 1) = 3x$ Original equation
 $3x - 3 = 3x$ Distributive Property
 $3x - 3 - 3x = 3x - 3x$ Subtract $3x$ from both sides.
 $-3 = 0$ Simplify.
 No solution since $-3 \neq 0$.
11. $5(x + 3) = 2x + 3(x + 5)$ Original equation
 $5x + 15 = 2x + 3x + 15$ Distributive Property
 $5x + 15 = 5x + 15$ Combine like terms.
 Identity since both sides equal.
13. $3x + 4 = 10$ is linear since variable has exponent 1.
15. $\frac{4}{x} - 3 = 5$ is not linear since variable has exponent -1
 not 1.
17. $3x + 15 = 0$ Original equation
 $3x + 15 - 15 = 0 - 15$ Subtract 15 from both sides.
 $3x = -15$ Combine like terms.
 $\frac{3x}{3} = \frac{-15}{3}$ Divide both sides by 3.
 $x = -5$ Simplify.

19. $-2x + 5 = 12$ Original equation
 $-2x + 5 - 5 = 12 - 5$ Subtract 5 from both sides.
 $-2x = 7$ Combine like terms.
 $\frac{-2x}{-2} = \frac{7}{-2}$ Divide both sides by -2 .
 $x = -\frac{7}{2}$ Simplify.

21. $x - 3 = 0$ **Check:** $3 - 3 \stackrel{?}{=} 0$
 $x - 3 + 3 = 0 + 3$ $0 = 0$
 $x = 3$

23. $3x = 12$ **Check:** $3(4) \stackrel{?}{=} 12$
 $\frac{3x}{3} = \frac{12}{3}$ $12 = 12$
 $x = 4$

25. $-6y = 4.2$ **Check:** $-6(-0.7) \stackrel{?}{=} 4.2$
 $\frac{-6y}{-6} = \frac{4.2}{-6}$ $4.2 = 4.2$
 $y = -0.7$

27. $6x + 4 = 0$ **Check:**
 $6x + 4 - 4 = 0 - 4$ $6\left(-\frac{2}{3}\right) + 4 \stackrel{?}{=} 0$
 $6x = -4$ $-4 + 4 \stackrel{?}{=} 0$
 $\frac{6x}{6} = \frac{-4}{6}$ $0 = 0$
 $x = -\frac{4}{6}$
 $x = -\frac{2}{3}$

29. $-2u + 5 = 7$ **Check:**
 $-2u + 5 - 5 = 7 - 5$ $-2(-1) + 5 \stackrel{?}{=} 7$
 $-2u = 2$ $2 + 5 \stackrel{?}{=} 7$
 $\frac{-2u}{-2} = \frac{2}{-2}$ $7 = 7$
 $u = -1$

31. $4x - 7 = -11$ **Check:**
 $4x - 7 + 7 = -11 + 7$ $4(-1) - 7 \stackrel{?}{=} -11$
 $4x = -4$ $-4 - 7 \stackrel{?}{=} -11$
 $\frac{4x}{4} = \frac{-4}{4}$ $-11 = -11$
 $x = -1$

33. $23x - 4 = 42$ **Check:**
 $23x - 4 + 4 = 42 + 4$ $23(2) - 4 \stackrel{?}{=} 42$
 $23x = 46$ $46 - 4 \stackrel{?}{=} 42$
 $\frac{23x}{23} = \frac{46}{23}$ $42 = 42$
 $x = 2$

35. $3t + 8 = -2$ **Check:**
 $3t + 8 - 8 = -2 - 8$ $3\left(-\frac{10}{3}\right) + 8 \stackrel{?}{=} -2$
 $3t = -10$ $-10 + 8 \stackrel{?}{=} -2$
 $\frac{3t}{3} = \frac{-10}{3}$ $-2 = -2$
 $t = -\frac{10}{3}$

37. $8 - 5t = 20 + t$

$8 - 5t + 5t = 20 + t + 5t$

$8 = 20 + 6t$

$8 - 20 = 20 + 6t - 20$

$-12 = 6t$

$\frac{-12}{6} = \frac{6t}{6}$

$-2 = t$

Check:

$8 - 5(-2) \stackrel{?}{=} 20 + (-2)$

$8 + 10 \stackrel{?}{=} 18$

$18 = 18$

39. $4x - 5 = 2x - 1$

$4x - 2x - 5 = 2x - 2x - 1$

$2x - 5 = -1$

$2x - 5 + 5 = -1 + 5$

$2x = 4$

$\frac{2x}{2} = \frac{4}{2}$

$x = 2$

Check:

$4(2) - 5 \stackrel{?}{=} 2(2) - 1$

$8 - 5 \stackrel{?}{=} 4 - 1$

$3 = 3$

41. $7 - 8x = 13x$

$7 - 8x + 8x = 13x + 8x$

$7 = 21x$

$\frac{7}{21} = \frac{21x}{21}$

$\frac{1}{3} = x$

Check:

$7 - 8\left(\frac{1}{3}\right) \stackrel{?}{=} 13\left(\frac{1}{3}\right)$

$7 - \frac{8}{3} \stackrel{?}{=} \frac{13}{3}$

$\frac{21}{3} - \frac{8}{3} \stackrel{?}{=} \frac{13}{3}$

$\frac{13}{3} = \frac{13}{3}$

43. $4y - 3 = 4y$

$4y - 3 + 3 = 4y + 3$

$4y = 4y + 3$

$4y - 4y = 4y + 3 - 4y$

$0 = 3$

$0 \neq 3$

No solution

45. $-8t = -16t$

$-8t + 16t = -16t + 16t$

$8t = 0$

$\frac{8t}{8} = \frac{0}{8}$

$t = 0$

Check:

$-8(0) \stackrel{?}{=} -16(0)$

$0 = 0$

47. $-9y - 4 = -9y$

$-9y + 9y - 4 = -9y + 9y$

$-4 = 0$

$-4 \neq 0$

No solution

49. $8(x - 8) = 24$

$8x - 64 = 24$

$8x - 64 + 64 = 24 + 64$

$8x = 88$

$\frac{8x}{8} = \frac{88}{8}$

$x = 11$

Check:

$8(11 - 8) \stackrel{?}{=} 24$

$8(3) \stackrel{?}{=} 24$

$24 = 24$

51. $-4(t + 2) = 0$

$-4t - 8 = 0$

$-4t - 8 + 8 = 0 + 8$

$-4t = 8$

$\frac{-4t}{-4} = \frac{8}{-4}$

$t = -2$

Check:

$-4[(-2) + 2] \stackrel{?}{=} 0$

$-4[0] \stackrel{?}{=} 0$

$0 = 0$

$$\begin{aligned}
 53. \quad & 3(x - 4) = 7x + 6 \\
 & 3x - 12 = 7x + 6 \\
 & 3x - 7x - 12 = 7x - 7x + 6 \\
 & -4x - 12 = 6 \\
 & -4x - 12 + 12 = 6 + 12 \\
 & -4x = 18 \\
 & \frac{-4x}{-4} = \frac{18}{-4} \\
 & x = -\frac{9}{2}
 \end{aligned}$$

Check:

$$\begin{aligned}
 & 3\left(-\frac{9}{2} - 4\right) \stackrel{?}{=} 7\left(-\frac{9}{2}\right) + 6 \\
 & 3\left(-\frac{9}{2} - \frac{8}{2}\right) \stackrel{?}{=} -\frac{63}{2} + \frac{12}{2} \\
 & 3\left(-\frac{17}{2}\right) \stackrel{?}{=} -\frac{51}{2} \\
 & -\frac{51}{2} = -\frac{51}{2}
 \end{aligned}$$

$$\begin{aligned}
 55. \quad & 8x - 3(x - 2) = 12 \\
 & 8x - 3x + 6 = 12 \\
 & 5x + 6 = 12 \\
 & 5x + 6 - 6 = 12 - 6 \\
 & 5x = 6 \\
 & \frac{5x}{5} = \frac{6}{5} \\
 & x = \frac{6}{5}
 \end{aligned}$$

Check:

$$\begin{aligned}
 & 8\left(\frac{6}{5}\right) - 3\left(\frac{6}{5} - 2\right) \stackrel{?}{=} 12 \\
 & \frac{48}{5} - 3\left(\frac{6}{5} - \frac{10}{5}\right) \stackrel{?}{=} 12 \\
 & \frac{48}{5} - 3\left(-\frac{4}{5}\right) \stackrel{?}{=} 12 \\
 & \frac{48}{5} + \frac{12}{5} \stackrel{?}{=} 12 \\
 & \frac{60}{5} \stackrel{?}{=} 12 \\
 & 12 = 12
 \end{aligned}$$

$$\begin{aligned}
 57. \quad & 5 - (2y - 4) = 15 \\
 & 5 - 2y + 4 = 15 \\
 & -2y + 9 = 15 \\
 & -2y + 9 - 9 = 15 - 9 \\
 & -2y = 6 \\
 & \frac{-2y}{-2} = \frac{6}{-2} \\
 & y = -3
 \end{aligned}$$

Check:

$$\begin{aligned}
 & 5 - [2(-3) - 4] \stackrel{?}{=} 15 \\
 & 5 - [-6 - 4] \stackrel{?}{=} 15 \\
 & 5 - [-10] \stackrel{?}{=} 15 \\
 & 5 + 10 \stackrel{?}{=} 15 \\
 & 15 = 15
 \end{aligned}$$

$$\begin{aligned}
 59. \quad & 12(x + 3) = 7(x + 3) \\
 & 12x + 36 = 7x + 21 \\
 & 12x + 36 - 7x = 7x + 21 - 7x \\
 & 5x + 36 = 21 \\
 & 5x + 36 - 36 = 21 - 36 \\
 & 5x = -15 \\
 & \frac{5x}{5} = \frac{-15}{5} \\
 & x = -3
 \end{aligned}$$

Check:

$$\begin{aligned}
 & 12[(-3) + 3] \stackrel{?}{=} 7[(-3) + 3] \\
 & 12[0] \stackrel{?}{=} 7[0] \\
 & 0 = 0
 \end{aligned}$$

$$\begin{aligned}
 61. \quad & 2(x + 7) - 9 = 5(x - 4) \\
 & 2x + 14 - 9 = 5x - 20 \\
 & 2x + 5 = 5x - 20 \\
 & 2x + 5 - 2x = 5x - 20 - 2x \\
 & 5 = 3x - 20 \\
 & 5 + 20 = 3x - 20 + 20 \\
 & 25 = 3x \\
 & \frac{25}{3} = \frac{3x}{3} \\
 & \frac{25}{3} = x
 \end{aligned}$$

Check:

$$\begin{aligned}
 & 2\left(\frac{25}{3} + 7\right) - 9 \stackrel{?}{=} 5\left(\frac{25}{3} - 4\right) \\
 & 2\left(\frac{25}{3} + \frac{21}{3}\right) - 9 \stackrel{?}{=} 5\left(\frac{25}{3} - \frac{12}{3}\right) \\
 & 2\left(\frac{46}{3}\right) - 9 \stackrel{?}{=} 5\left(\frac{13}{3}\right) \\
 & \frac{92}{3} - \frac{27}{3} \stackrel{?}{=} \frac{65}{3} \\
 & \frac{65}{3} = \frac{65}{3}
 \end{aligned}$$

$$63. \quad \frac{u}{5} = 10 \quad \text{Check:}$$

$$5\left(\frac{u}{5}\right) = (10)5 \quad \frac{50}{5} \stackrel{?}{=} 10$$

$$u = 50 \quad 10 = 10$$

$$65. \quad t - \frac{2}{5} = \frac{3}{2} \quad \text{Check:}$$

$$10\left(t - \frac{2}{5}\right) = \left(\frac{3}{2}\right)10 \quad \frac{19}{10} - \frac{2}{5} \stackrel{?}{=} \frac{3}{2}$$

$$10t - 4 = 15 \quad \frac{19}{10} - \frac{4}{10} \stackrel{?}{=} \frac{15}{10}$$

$$10t - 4 + 4 = 15 + 4 \quad \frac{15}{10} = \frac{15}{10}$$

$$10t = 19$$

$$\frac{10t}{10} = \frac{19}{10}$$

$$t = \frac{19}{10}$$

$$67. \quad \frac{t}{5} - \frac{t}{2} = 1 \quad \text{Check:}$$

$$10\left(\frac{t}{5} - \frac{t}{2}\right) = (1)10 \quad \frac{-\frac{10}{3}}{5} - \frac{-\frac{10}{3}}{2} \stackrel{?}{=} 1$$

$$2t - 5t = 10 \quad \frac{10}{-15} + \frac{10}{6} \stackrel{?}{=} 1$$

$$-3t = 10 \quad -\frac{2}{3} + \frac{5}{3} \stackrel{?}{=} 1$$

$$\frac{-3t}{-3} = \frac{10}{-3} \quad \frac{3}{3} \stackrel{?}{=} 1$$

$$t = \frac{10}{-3} \quad 1 = 1$$

$$69. \quad \frac{8x}{5} - \frac{x}{4} = -3 \quad \text{Check:}$$

$$20\left(\frac{8x}{5} - \frac{x}{4}\right) = (-3)20 \quad \frac{8}{5}\left(-\frac{20}{9}\right) - \frac{1}{4}\left(-\frac{20}{9}\right) \stackrel{?}{=} -3$$

$$4(8x) - 5x = -60 \quad -\frac{32}{9} + \frac{5}{9} \stackrel{?}{=} -3$$

$$32x - 5x = -60 \quad -\frac{27}{9} \stackrel{?}{=} -3$$

$$27x = -60 \quad -3 = -3$$

$$\frac{27x}{27} = \frac{-60}{27}$$

$$x = -\frac{20}{9}$$

$$71. \quad \frac{1}{3}x + 1 = \frac{1}{12}x - 4 \quad \text{Check:}$$

$$12\left(\frac{1}{3}x + 1\right) = \left(\frac{1}{12}x - 4\right)12 \quad \frac{1}{3}(-20) + 1 \stackrel{?}{=} \frac{1}{12}(-20) - 4$$

$$4x + 12 = x - 48 \quad \frac{-20}{3} + \frac{3}{3} \stackrel{?}{=} \frac{-20}{12} - 4$$

$$4x + 12 - x = x - 48 - x \quad \frac{-17}{3} \stackrel{?}{=} \frac{-5}{3} - \frac{12}{3}$$

$$3x + 12 = -48 \quad \frac{17}{3} = \frac{17}{3}$$

$$3x + 12 - 12 = -48 - 12$$

$$3x = -60$$

$$\frac{3x}{3} = \frac{-60}{3}$$

$$x = -20$$

$$\begin{aligned}
 73. \quad \frac{25 - 4u}{3} &= \frac{5u + 12}{4} + 6 \\
 12\left(\frac{25 - 4u}{3}\right) &= \left(\frac{5u + 12}{4} + 6\right)12 \\
 4(25 - 4u) &= 3(5u + 12) + 72 \\
 100 - 16u &= 15u + 36 + 72 \\
 100 - 16u &= 15u + 108 \\
 100 - 16u + 16u &= 15u + 108 + 16u \\
 100 &= 31u + 108 \\
 100 - 108 &= 31u + 108 - 108 \\
 -8 &= 31u \\
 \frac{-8}{31} &= \frac{31u}{31} \\
 -\frac{8}{31} &= u
 \end{aligned}$$

Check:

$$\begin{aligned}
 \frac{25 - 4\left(-\frac{8}{31}\right)}{3} &\stackrel{?}{=} \frac{5\left(-\frac{8}{31}\right) + 12}{4} + 6 \\
 \frac{25 + \frac{32}{31}}{3} &\stackrel{?}{=} \frac{-\frac{40}{31} + 12}{4} + 6 \\
 \frac{25}{3} + \frac{32}{93} &\stackrel{?}{=} \frac{-10}{31} + 3 + 6 \\
 \frac{775}{93} + \frac{32}{93} &\stackrel{?}{=} \frac{-10}{31} + \frac{93}{31} + \frac{186}{31} \\
 \frac{807}{93} &\stackrel{?}{=} \frac{269}{31} \\
 \frac{269}{31} &= \frac{269}{31}
 \end{aligned}$$

$$\begin{aligned}
 75. \quad 0.3x + 1.5 &= 8.4 \\
 10(0.3x + 1.5) &= (8.4)10 \\
 3x + 15 &= 84 \\
 3x + 15 - 15 &= 84 - 15 \\
 3x &= 69 \\
 \frac{3x}{3} &= \frac{69}{3} \\
 x &= 23
 \end{aligned}$$

Check:

$$\begin{aligned}
 0.3(23) + 1.5 &\stackrel{?}{=} 8.4 \\
 6.9 + 1.5 &\stackrel{?}{=} 8.4 \\
 8.4 &= 8.4
 \end{aligned}$$

$$77. \quad 1.2(x - 3) = 10.8$$

Check:

$$\begin{aligned}
 1.2x - 3.6 &= 10.8 \\
 10(1.2x - 3.6) &= (10.8)10 \\
 12x - 36 &= 108 \\
 12x - 36 + 36 &= 108 + 36 \\
 12x &= 144 \\
 \frac{12x}{12} &= \frac{144}{12} \\
 x &= 12
 \end{aligned}$$

$$\begin{aligned}
 79. \quad \frac{2}{3}(2x - 4) &= \frac{1}{2}(x + 3) - 4 \\
 6\left[\frac{2}{3}(2x - 4)\right] &= \left[\frac{1}{2}(x + 3) - 4\right]6 \\
 4(2x - 4) &= 3(x + 3) - 24 \\
 8x - 16 &= 3x + 9 - 24 \\
 8x - 16 &= 3x - 15 \\
 8x - 3x - 16 &= 3x - 3x - 15 \\
 5x - 16 &= -15 \\
 5x - 16 + 16 &= -15 + 16 \\
 5x &= 1 \\
 \frac{5x}{5} &= \frac{1}{5} \\
 x &= \frac{1}{5}
 \end{aligned}$$

Check:

$$\begin{aligned}
 \frac{2}{3}\left[2\left(\frac{1}{5}\right) - 4\right] &\stackrel{?}{=} \frac{1}{2}\left(\frac{1}{5} + 3\right) - 4 \\
 \frac{2}{3}\left(\frac{2}{5} - \frac{20}{5}\right) &\stackrel{?}{=} \frac{1}{2}\left(\frac{1}{5} + \frac{15}{5}\right) - 4 \\
 \frac{2}{3}\left(-\frac{18}{5}\right) &\stackrel{?}{=} \frac{1}{2}\left(\frac{16}{5}\right) - 4 \\
 -\frac{12}{5} &\stackrel{?}{=} \frac{8}{5} - \frac{20}{5} \\
 -\frac{12}{5} &= -\frac{12}{5}
 \end{aligned}$$

81. Verbal Model: $\boxed{\text{First integer}} + \boxed{\text{Second integer}} = 251$

Labels: $n = \text{first integer}$
 $n + 1 = \text{second integer}$

Equation: $n + (n + 1) = 251$
 $2n + 1 = 251$
 $2n + 1 - 1 = 251 - 1$
 $2n = 250$
 $\frac{2n}{2} = \frac{250}{2}$
 $n = 125$
 $n + 1 = 126$

83. Verbal Model: $\boxed{\text{First even integer}} + \boxed{\text{Second even integer}} = 166$

Labels: $n = \text{first even integer}$
 $n + 2 = \text{second even integer}$

Equation: $n + (n + 2) = 166$
 $2n + 2 = 166$
 $2n + 2 - 2 = 166 - 2$
 $2n = 164$
 $\frac{2n}{2} = \frac{164}{2}$
 $n = 82$
 $n + 2 = 84$

85. Verbal Model: $162 + 32 \cdot \boxed{\text{Number of hours}} = 210$

Label: $n = \text{number of hours for the repair}$

Equation: $162 + 32n = 210$
 $162 - 162 + 32n = 210 - 162$
 $32n = 48$
 $\frac{32n}{32} = \frac{48}{32}$
 $n = \frac{3}{2}$

The repairs took 1.5 hours to complete.

87. The fountain reaches its maximum height when the velocity of the stream of water is zero.

$0 = 48 - 32t$
 $0 + 32t = 48 - 32t + 32t$
 $32t = 48$
 $\frac{32t}{32} = \frac{48}{32}$
 $t = \frac{3}{2} \text{ seconds} = 1.5 \text{ seconds}$

89. $\frac{t}{10} + \frac{t}{15} = 1$

$\frac{3t}{30} + \frac{2t}{30} = 1$

$\frac{5t}{30} = 1$

$\frac{t}{6} = 1$

$6\left(\frac{t}{6}\right) = 6(1)$

$t = 6 \text{ hours}$

91. (a)

t	1	1.5	2	3	4	5
Width	300	240	200	150	120	100
Length	300	360	400	450	480	500
Area	90,000	86,400	80,000	67,500	57,600	50,000

—CONTINUED—

91. (a) —CONTINUED—

If $t = 1$: $1200 = 2w + 2w$

$1200 = 4w$

$300 = w$

$300 = l$

If $t = 3$: $1200 = 2w + 2(3w)$

$1200 = 8w$

$150 = w$

$450 = l$

If $t = 1.5$: $1200 = 2w + 2(1.5w)$

$1200 = 5w$

$240 = w$

$360 = l$

If $t = 4$: $1200 = 2w + 2(4w)$

$1200 = 10w$

$120 = w$

$480 = l$

If $t = 2$: $1200 = 2w + 2(2w)$

$1200 = 6w$

$200 = w$

$400 = l$

If $t = 5$: $1200 = 2w + 2(5w)$

$1200 = 12w$

$100 = w$

$500 = l$

(b) Since the length is t times the width and the perimeter is fixed, as t gets larger, the length gets larger and the area gets smaller. The maximum area occurs when the length and width are equal.

93. $5500 = 207t + 4962$

$5500 - 4962 = 207t + 4962 - 4962$

$538 = 207t$

$\frac{538}{207} = \frac{207t}{207}$

$2.6 \approx t$

From the graph, 1993 is the year in which expenditures reached \$5500.

95. A conditional equation is an equation whose solution set is not the entire set of real numbers. An identity is an equation whose solution set is all real numbers.

97. Evaluating an expression means finding its value when its variables are replaced by real numbers. Solving an equation means finding all values of the variable for which the equation is true.

99. Equivalent equations have the same solution set. For example, $3x + 4 = 10$ and $3x - 6 = 0$ are equivalent.

101. False. Multiplying both sides of an equation by zero does not yield an equivalent equation.

Section 1.2 Linear Equations and Problem Solving

1. Verbal Model: $\boxed{\text{Number}} + \boxed{30} = \boxed{82}$

Label: Number = x

Equation: $x + 30 = 82$

$$x + 30 - 30 = 82 - 30$$

$$x = 52$$

3. Verbal Model: $\boxed{\text{Annual salary}} = 26 \cdot \boxed{\text{Amount of each paycheck}} + \boxed{\text{Bonus}}$

Labels: Annual salary = 30,500

Amount of each paycheck = x

Bonus = 2300

Equation: $30,500 = 26x + 2300$

$$30,500 - 2300 = 26x + 2300 - 2300$$

$$28,200 = 26x$$

$$\frac{28,200}{26} = \frac{26x}{26}$$

$$\$1084.62 = x$$