

Part II Texas Instruments TI-86 Graphics Calculator

II.1 Systems of Linear Equations

II.1.1 Basics: Press the ON key to begin using your TI-86 calculator. If you need to adjust the display contrast, first press 2nd and then press and hold ▲ (the up arrow key) to increase the contrast or ▼ (the down arrow key) to decrease the contrast. As you press and hold ▲ or ▼, an integer between 0 (lightest) and 9 (darkest) appears in the upper right corner of the display. When you have finished with the calculator, turn it off to conserve battery power by pressing 2nd and then OFF.

Check the TI-86's settings by pressing 2nd MODE. If necessary, use the arrow keys to move the blinking cursor to a setting you want to change. Press ENTER to select a new setting. To start, select the options along the left side of the MODE menu as illustrated in Figure II.1: normal display, floating decimals, radian measure, rectangular coordinates, function graphs, decimal number system, rectangular vectors, and differentiation type. Details on alternative options will be given later in this guide. For now, leave the MODE menu by pressing EXIT or 2nd QUIT or CLEAR.

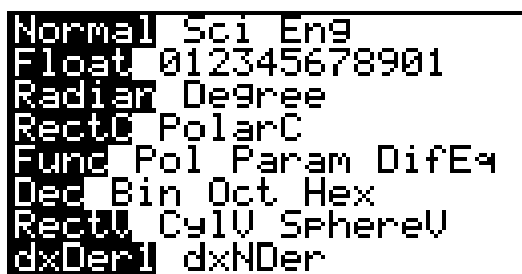


Figure II.1: MODE menu

II.1.2 Key Functions: Most keys on the TI-86 offer access to more than one function, just as the keys on a computer keyboard can produce more than one letter (“g” and “G”) or even quite different characters (“5” and “%”). The primary function of a key is indicated on the key itself, and you access that function by a simple press on the key.

To access the *second* function indicated to the *left* above a key, first press 2nd (the cursor changes to a blinking ↑) and *then* press the key. For example, to calculate $\sqrt{25}$, press 2nd $\sqrt{\quad}$ 25 ENTER.

When you want to use a capital letter or other character printed to the *right* above a key, first press ALPHA (the cursor changes to a blinking A) and then the key. For example to use the letter *K* in a formula, press ALPHA K. If you need several letters in a row, press ALPHA twice in succession, which is like CAPS LOCK on a computer keyboard, and then press all the letters you want. Remember to press ALPHA when you are finished and want to restore the keys to their primary functions. To type lowercase letters, press 2nd alpha (the cursor changes to a blinking a). To lock in lowercase letters, press 2nd alpha 2nd alpha or 2nd alpha ALPHA. To unlock from lowercase, press ALPHA ALPHA (you’ll see the cursor change from blinking a to blinking A and then to the standard blinking rectangle).

Technology Tip: Pressing x-VAR is just the same as pressing 2nd alpha X: both keys produce the lower-case *x*.

II.1.3 Algebraic Expressions and Memory: Your calculator can evaluate expressions such as $\frac{N(N+1)}{2}$ after you have entered a value for *N*. Suppose you want *N* = 200. Press 200 STO ► N ENTER to store the value 200 in memory location *N*. (The STO ► key prepares the TI-86 for an alphabetical entry, so it is *not* necessary to press ALPHA also.) Whenever you use *N* in an expression, the calculator will substitute the value 200 until you make a change by storing *another* number in *N*. Next enter the expression $\frac{N(N+1)}{2}$ by typing ALPHA N (ALPHA N + 1) ÷ 2 ENTER. For *N* = 200, you will find that $\frac{N(N+1)}{2} = 20100$.

Technology Tip: The current value of any variable may be revealed by typing just its name and then ENTER. Simply press ALPHA N ENTER to see the current value of the variable N . And the TI-86 retains memorized values even when it is turned off, so long as its batteries are good.

Now a variable name in the TI-86 can be a single letter, or a string of up to eight characters that begins with a letter followed by other letters, numerals, and various symbols. Variable names are case sensitive, which means that length and Length and LENGTH may represent *different* quantities.

Technology Tip: Because variable names may be more than one character in length, multiplication between variables must always be *expressed*. So for the product ab , you *must* enter 2nd alpha A \times 2nd alpha B *with* the multiplication key. With a *numerical* coefficient, however, the multiplication does *not* need to be expressed; hence for $4ab$ you may enter 4 2nd alpha A \times 2nd alpha B.

II.1.4 The MATH Menu: Operators and functions associated with a scientific calculator are available either immediately from the keys of the TI-86 or by 2nd keys. You have direct key access to common arithmetic operations (x^2 , 2nd $\sqrt{\quad}$, 2nd x^{-1} , and \wedge), trigonometric functions (SIN, COS, TAN) and their inverses (2nd SIN^{-1} , 2nd COS^{-1} , 2nd TAN^{-1}), exponential and logarithmic functions (LOG, 2nd 10^x , LN, 2nd e^x), and a famous constant (2nd π).

Note that the TI-86 distinguishes between *subtraction* and the *negative* sign. If you wish to enter a negative number, it is necessary to use the (-) key. For example, you would evaluate $-5(4 \cdot -3)$ by pressing (-) 5 - (4 \times (-) 3) ENTER to get 7.



Figure II.2: *Subtraction and the Negative Sign*

A significant difference between the TI-86 and many scientific calculators is that the TI-86 requires the argument of a function *after* the function, as you would see a formula written in your textbook. For example, on the TI-86 you calculate $\sqrt{16}$ by pressing the 2nd $\sqrt{\quad}$ 16 in that order.

Here are keystrokes for basic mathematical operations. Try them for practice on your TI-86.

Expressions	Keystrokes	Display
$\sqrt{3^2 + 4^2}$	2nd $\sqrt{\quad}$ (3 x^2 + 4 x^2) ENTER	5
$2\frac{1}{3}$	2 + 3 2nd x^{-1} ENTER or 2 + (1 \div 3) ENTER	2.333333333
$\log 200$	LOG 200 ENTER	2.30102999566
$2.34 \cdot 10^5$	2.34 \times 2nd 10^x 5 ENTER or 2.34 \times 10 \wedge 5 ENTER	234000

Additional mathematical operations and functions are available from the MATH menu (Figure II.3). Press 2nd MATH to see the various options that are listed across the bottom of the screen. These options are activated by pressing corresponding menu keys , F1 through F5.

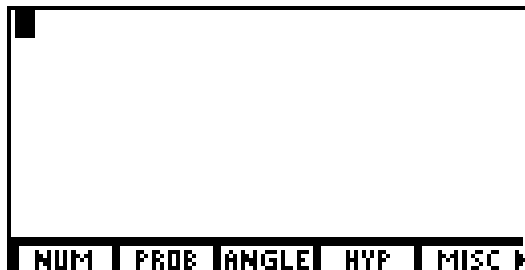


Figure II.3: Basic MATH Menu

For example, F1 brings up the NUM menu of numerical functions. You will learn in your mathematics textbook how to apply many of them. Note that the basic MATH menu items have moved up a line; these options are now available by pressing 2nd M1 through 2nd M5. As an example, determine $|-5|$ by pressing 2nd MATH F1 and then F5 (-) 5 ENTER (see Figure II.4).

Next calculate $\sqrt[3]{7}$ by pressing 2nd MATH F5 (when the MATH NUM menu is displayed, as in Figure II.4, press just 2nd M5) to access the MISC menu of miscellaneous mathematical functions.

Technology Tip: The arrow at the right end of a menu (as in Figure II.4) indicates there are more items that you can access.

You may press the MORE key repeatedly to move down the row of options and back again. To calculate $\sqrt[3]{7}$, press 2nd MATH F5 MORE 3 F4 7 ENTER to see 1.9129 (Figure II.5). To leave the MATH menu or any other menu and take no further action, press EXIT a couple of times.

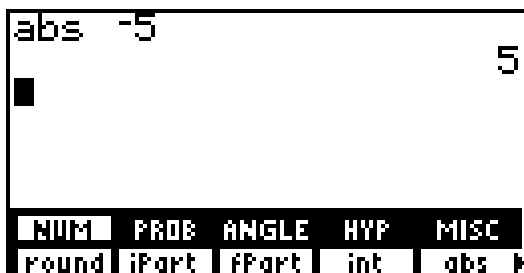


Figure II.4: MATH NUM Menu

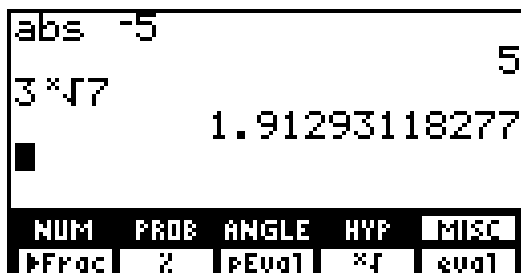


Figure II.5: MATH MISC Menu

The *factorial* of a non-negative integer is the *product* of all the integers from 1 up to the given integer. The symbol for factorial is the exclamation point. So $4!$ (pronounced *four factorial*) is $1 \cdot 2 \cdot 3 \cdot 4 = 24$. You will learn more about applications of factorials in your textbook, but for now use the TI-86 to calculate $4!$. Press these keystrokes: 2nd MATH F2 [PROB] 4 F1 [!] ENTER.

II.1.5 Graphing Linear Functions: The TI-86 can store many different functions in rectangular, polar, and parametric form, then plot their graphs.

Here is how to graph $y = -x + 3$. First press the GRAPH key and then F1 to select $y(x) =$. This gives you access to the function editing screen (Figure II.6). Press F4 [DEL] as many times as necessary to delete any functions that may be there already. Then, with the cursor on the top line to the right of $y1 =$, press (-) F1 + 3 to enter the function.



Figure II.6: Function editing screen

Technology Tip: The TI-86 uses lower-case letters for its graphing variables, just like your mathematics textbook.

Technology Tip: Note that pressing F1 in *this* menu is the same as pressing either x-VAR or 2nd alpha X. Now press 2nd M5 [GRAPH] and the TI-86 changes to a window with the graph of $y = -x + 3$ (Figure II.7)

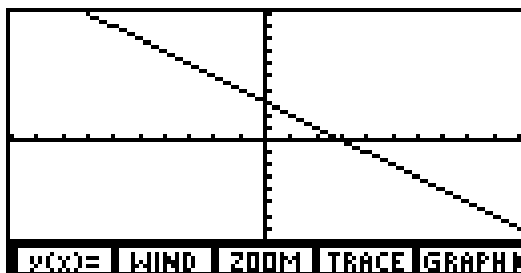


Figure II.7: Graph of $y = -x + 3$

Technology Tip: While the TI-86 is calculating coordinates for a plot, it displays a busy indicator at the top right of the graph window.

The viewing rectangle in Figure II.8 shows the part of the graph that extends horizontally from -10 to 10 and vertically from -10 to 10 . Press F2 [WIND] to see information about your viewing rectangle. Figure II.8 shows the WINDOW screen that corresponds to the viewing rectangle in Figure II.7. This is the *standard* viewing rectangle for the TI-86.

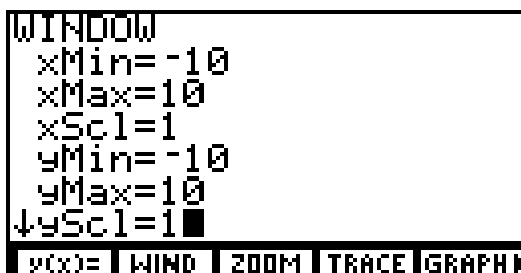


Figure II.8: Standard WINDOW

The variables xMin and xMax are the minimum and maximum x -values of the viewing rectangle; yMin and yMax are its minimum and maximum y -values.

xScl and yScl set the spacing between tick marks on the axes.

xRes sets the resolution.

Technology Tip: To set the range quickly to standard values (see Figure II.8) press F3 [ZOOM] F4 [ZSTD].

II.1.6 Graphing Parametric Functions: The TI-86 plots parametric functions as easily as it plots functions. Just use the MODE menu (Figure II.1), go to the fifth line from the top, and change the setting from Func for function graphs to Param for parametric graphs. Be sure, if the independent parameter is an angle measure, that MODE is also set to whichever you need, Radian or Degree.

For example, here are the keystrokes needed to graph the parametric equations $x = \cos^3 t$ and $y = \sin^3 t$. First check that angles are currently being measured in radians. Change to parametric mode and press GRAPH F1 to examine the new parametric equation menu E(t) = (Figure II.9). Enter the two parametric equations by pressing (COS F1) ^ 3 ENTER (SIN F1) ^ 3 ENTER.



Figure II.9: Parametric E(t)= menu

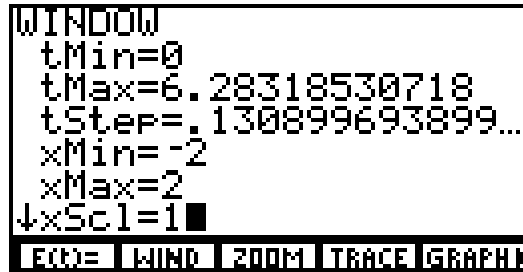


Figure II.10: Parametric RANGE menu

Technology Tip: In *this* menu (Figure II.9), pressing F1 is the same as pressing 2nd alpha T for the lower-case letter t .

Also, look at the new parametric WINDOW menu (Figure II.10). In the standard viewing rectangle, the values of t go from 0 to 2π in steps of $\frac{\pi}{24} \approx .1309$, with the view from -10 to 10 in both directions. But here the viewing rectangle has been changed to extend from -2 to 2 in both directions. Press F5 [GRAPH] to see the parametric graph (Figure II.11).

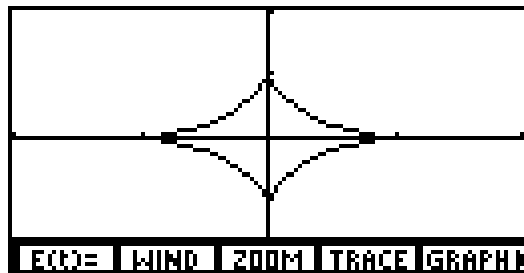


Figure II.11: Parametric graph of $x = \cos^3 t$ and $y = \sin^3 t$.

II.1.7 Solving Linear Systems: The solutions to a system of equations correspond to the points of intersection of their graphs. As an example, let's graph and solve the system $\begin{cases} 3x - y = 1 \\ 2x - y = 0 \end{cases}$.

First transform each equation by solving for y : $\begin{cases} y = 3x - 1 \\ y = 2x \end{cases}$. Then press GRAPH F1 [$y(x)=$] and enter $3x - 1$ for y_1 and $2x$ for y_2 (Figure II.12) by pressing 3 F1 - 1 ENTER 2 F1.

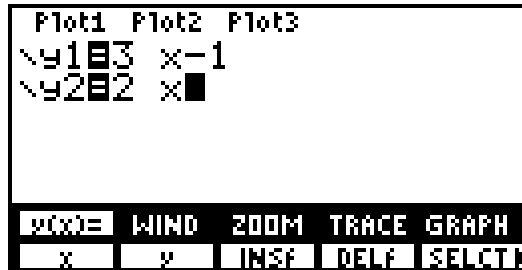


Figure II.12: $3x - 1$ for y_1 and $2x$ for y_2

Find the coordinates of a point of intersection of two graphs by pressing **GRAPH MORE F1 [MATH] MORE F3 [ISECT]**. The cursor will be blinking on one of the curves given as the first curve. Press **ENTER**. The cursor moves to the next curve given as the second curve. Press **ENTER**. (Figure II.13). Trace with the cursor keys **◀** or **▶** near the point of intersection and press **ENTER**. Coordinates of the intersection will be displayed at the bottom of the window.

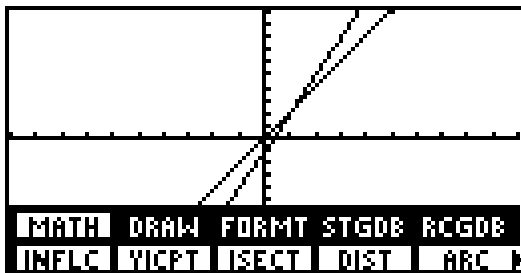


Figure II.13: MATH ISECT

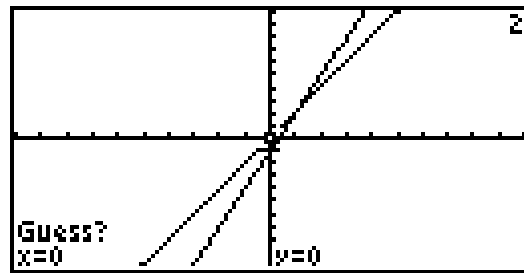


Figure II.14: Setting up to locate the intersection

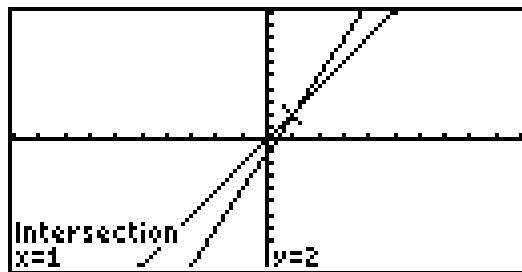


Figure II.15: Point of intersection

The TI-86 also has a simultaneous equation solver that you can use to solve a linear system. So press **2nd SIMULT 2 ENTER** for a system of two equations. Follow the prompts to enter the coefficients of the two equations (which are in the form $ax + by = c$). Finally, press **F5 [SOLVE]** for the solution.

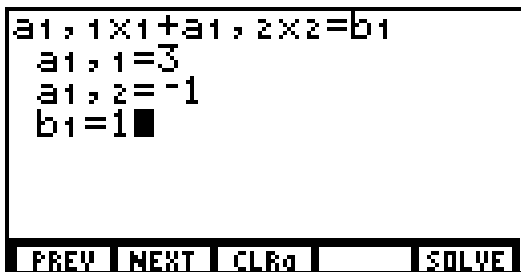


Figure II.16: Coefficients of $3x - y = 1$

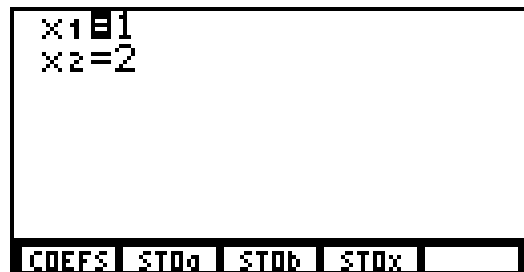


Figure II.17: Simultaneous equation solver

II.1.8 Making a Matrix: The TI-86 can display and use many different matrices, each with up to 255 rows

and up to 255 columns! Here's how to create this 3×4 matrix $\begin{bmatrix} 1 & -4 & 3 & 5 \\ -1 & 3 & -1 & -3 \\ 2 & 0 & -4 & 6 \end{bmatrix}$ in your calculator.

Press 2nd MATRX F2 [EDIT] to see the matrix edit menu (Figure II.18). You must first name the matrix; let's name this matrix A (the TI-86 is already set for alphabetic entry) and press ENTER to continue.

You may now change the dimensions of matrix A to 3×4 by pressing 3 ENTER 4 ENTER . Simply press ENTER or the down arrow key to accept an existing dimension. Next enter 1 in the first row and first column of the matrix, then press ENTER to move horizontally across this row to the second column. Continue to enter the top row of elements. Press ENTER after the last element of the first row has been entered to move to the second row. You may use the up and down arrow keys to move vertically through the columns of the matrix.

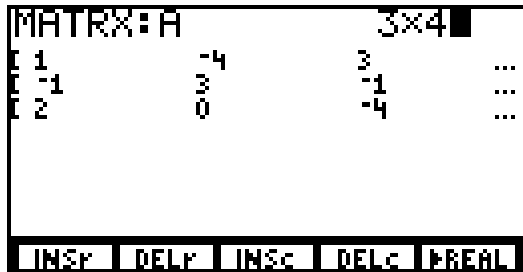


Figure II.18: MATRX EDIT menu

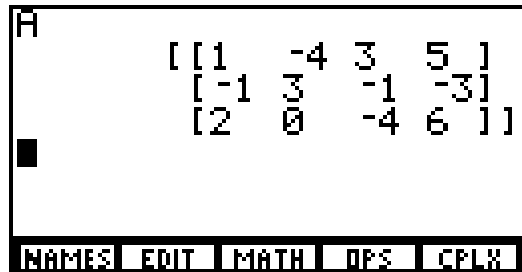


Figure II.19: Matrix A

Leave the matrix A editing screen by pressing EXIT or 2nd QUIT and return to the home screen.

See the matrix you just created by pressing ALPHA A ENTER (Figure II.19).

Technology Tip: The TI-86 enables you to create an identity matrix quickly. If you want to make the 3×3 identity matrix, for example, press 2nd MATRX F4 [OPS] F3 [ident] 3 ENTER (see Figure II.20). If you want to save the 4×4 identity matrix as matrix M, press 2nd MATRX F4 [OPS] F3 [ident] 4 STO ► M ENTER.

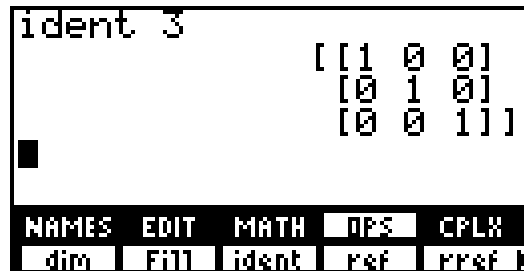


Figure II.20: Identity matrix

Technology Tip: The TI-86 also enables you to create a matrix of any size and fill it with random single-digit integers -9 to 9 . To create a 2×3 matrix filled with random integers, press 2nd MATRX F4 [OPS] MORE MORE F1 [randM] 2, 3) ENTER (see one result in Figure II.21; yours may well be different).

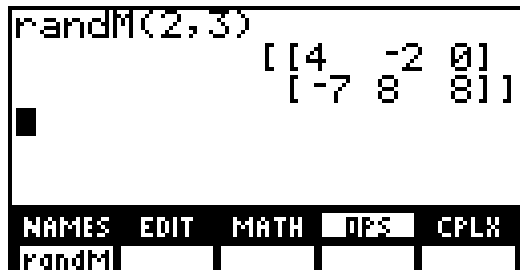


Figure II.21: Random matrix

II.1.9 Row Operations: Here are the keystrokes necessary to perform elementary row operations on a matrix. Your textbook provides more careful explanation of the elementary row operations and their uses.

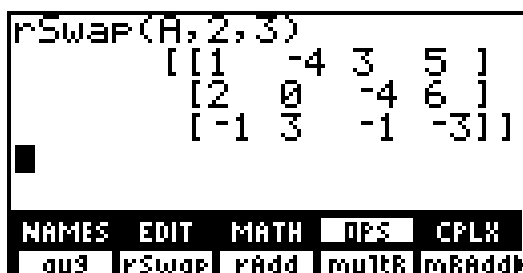


Figure II.22: Swap rows 2 and 3

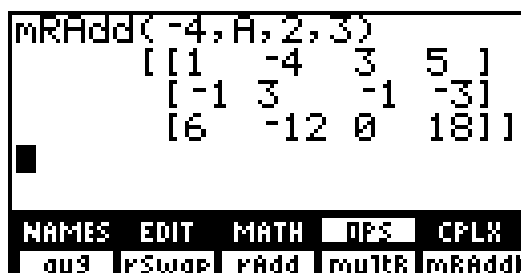


Figure II.23: Add -4 times row 2 to row 3

To interchange the second and third rows of the matrix A that was defined above, press these keys: 2nd MATRIX F4 [OPS] MORE F2 [rSwap] ALPHA A , 2 , 3) ENTER (see Figure II.22). The format of this command is rSwap (*matrix, row1, row2*).

To add row 2 and row 3 and store the results in row 3, press 2nd MATRIX F4 MORE F3 ALPHA A , 2 , 3) ENTER. The format of this command is rAdd (*matrix, row 1, row 2*).

To multiply row 2 by -4 and *store* the results in row 2, thereby replacing row 2 with new values, press 2nd MATRIX F4 MORE F4 (-) 4, ALPHA A , 2) ENTER. The format of this command is multR(*scalar, matrix, row*).

To multiply row 2 by -4 and *add* the results in row 3, thereby replacing row 3 with new values, press 2nd MATRIX F4 MORE F5 (-) 4, ALPHA A , 2 , 3) ENTER (see Figure II.23). The format of this command is mRAdd (*scalar, matrix, row1, row 2*).

Technology Tip: It is important to remember that your TI-86 does *not automatically* store a matrix obtained as the result of any row operations. So when you need to perform several row operations in succession, it is a good idea to store the result of each one in a temporary place.

For example, use elementary row operations to solve this system of linear equations:
$$\begin{cases} x - 2y + 3z = 9 \\ -x + 3y = -4 \\ 2x - 5y + 5z = 17 \end{cases}$$

First enter this *augmented matrix* as A in your TI-86:
$$\begin{bmatrix} 1 & -2 & 3 & 9 \\ -1 & 3 & 0 & -4 \\ 2 & -5 & 5 & 17 \end{bmatrix}$$
. Next store this matrix in C (press ALPHA A STO ► C ENTER) so you may keep the original in case you need to recall it.

Here are the row operations and their associated keystrokes. At each step, the result is stored in C and replaces the previous matrix C. The solution is shown in Figure II.24.

<i>Row Operation</i>	<i>Keystrokes</i>
rAdd(C, 1, 2)	2nd MATRX F4 MORE F3 ALPHA C , 1 , 2) STO ► C ENTER
mRAdd(-2, C, 1, 3)	F5 (-) 2, ALPHA C , 1 , 3) STO ► C ENTER
rAdd(C, 2, 3)	F3 ALPHA C , 2 , 3) STO ► C ENTER
multR(1/2, C, 3)	F4 1 ÷ 2, ALPHA C , 3) STO ► C ENTER

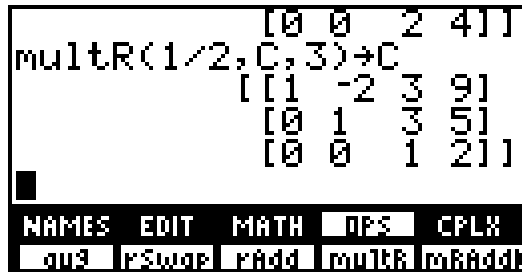


Figure II.24: Final matrix after row operations

Thus $z = 2$, so $y = -1$ and $x = 1$.

II.2 Matrices

II.2.1 Matrix Addition: From the home screen, you can perform many calculations with matrices. Add two matrices A and B by pressing ALPHA A + ALPHA B ENTER. Subtraction is similar.

II.2.2 Scalar Multiplication: Calculate the scalar multiplication $2A$ by pressing 2 ALPHA A ENTER. For example, use the matrix A shown here (Figure II.25).

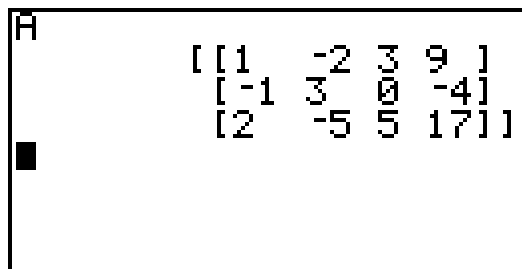


Figure II.25: Matrix A

To replace matrix B by $2A$, press 2 ALPHA STO ► B ENTER, or if you do this immediately after calculating $2A$, press only STO ► B ENTER (see Figure II.26). Press 2nd MATRX F2 B ENTER to verify that the dimensions of matrix B have been changed automatically to reflect this new value.

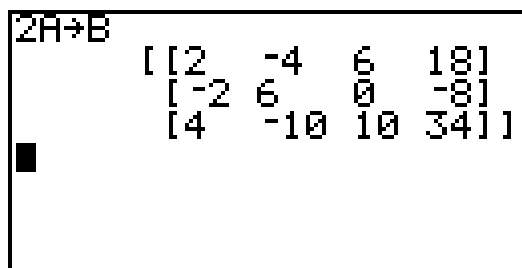


Figure II.26: Matrix B

II.2.3 Matrix Multiplication: Set the dimensions of matrix C to 2×3 and enter this matrix as C: $\begin{bmatrix} 2 & 0 & 3 \\ 1 & -5 & -1 \end{bmatrix}$. For matrix multiplication of C by A, press ALPHA C \times ALPHA A ENTER.

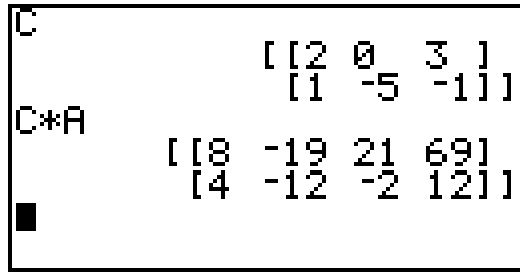


Figure II.27: Multiplication of two matrices

Technology Tip: If you tried to multiply A by C, your TI-86 would signal an error because the dimensions of the two matrices do not permit multiplication this way.

II.2.4 Inverse of a Matrix: Create the square matrix $\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ as matrix D in your calculator. The symbol for the inverse of D is D^{-1} . Then to calculate this inverse, press ALPHA D 2nd X^{-1} ENTER.

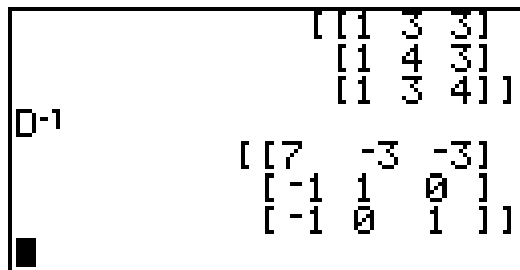


Figure II.28: Inverse

II.2.5 Transpose of a Matrix: The *transpose* of a matrix A is another matrix with the rows and columns interchanged. The symbol for the transpose of A is A^T . To calculate A^T , press ALPHA A 2nd **MATRIX** F3 **[MATH]** F2 **[T]** ENTER.

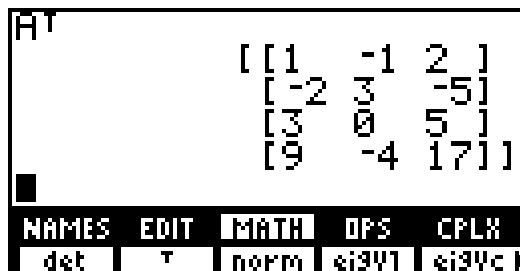


Figure II.29: Transpose

II.2.6 LU-Factorization: Use this 3×3 square matrix for A: $\begin{bmatrix} 1 & -2 & 3 \\ -1 & 3 & 0 \\ 2 & -5 & 5 \end{bmatrix}$. To calculate its LU-factorization, press 2nd **MATRIX** F3 **[MATH]** MORE F3 **[LU]** ALPHA A, ALPHA L, ALPHA U, ALPHA P) ENTER. The format of this command is LU(matrix, lower triangular matrix, upper triangular matrix, permutation matrix). In the current example, see the lower triangular matrix L (Figure II.30), the upper triangular matrix U, and the permutation matrix P.

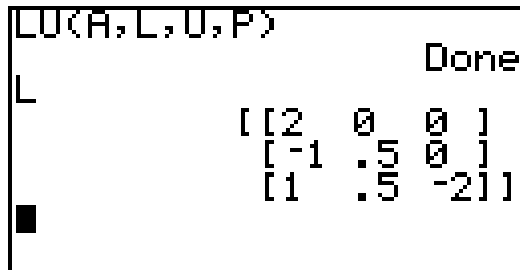


Figure II.30: LU-factorization and the lower triangular matrix

II.3 Determinants

II.3.1 Determinant of a Matrix: Enter this 3×3 square matrix as A: $\begin{bmatrix} 1 & -2 & 3 \\ -1 & 3 & 0 \\ 2 & -5 & 5 \end{bmatrix}$. To calculate its determinate, press 2nd MATRX F3 [MATH] F1 [det] ALPHA A ENTER. You should find that $|A| = 2$.

II.3.2 Eigenvalues: Enter this 3×3 square matrix as A: $\begin{bmatrix} 1 & -2 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$. To calculate the eigenvalues, press 2nd MATRX F3 [MATH] F4 [eigV] ALPHA A ENTER. Your TI-86 returns a list of the eigenvalues of a real or complex square matrix.

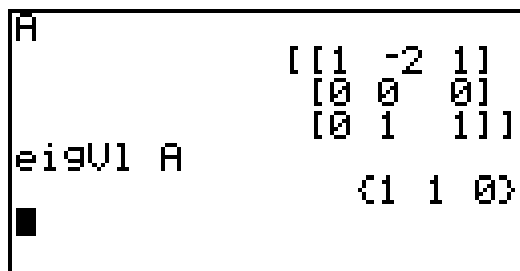


Figure II.31: Eigenvalues of matrix A

II.3.3 Eigenvectors: Calculate eigenvectors of matrix $A = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ by pressing 2nd MATRX F3 [MATH] F5 [eigVc] ALPHA A ENTER. The calculator returns a matrix, each column of which is an eigenvector corresponding to an eigenvalue.

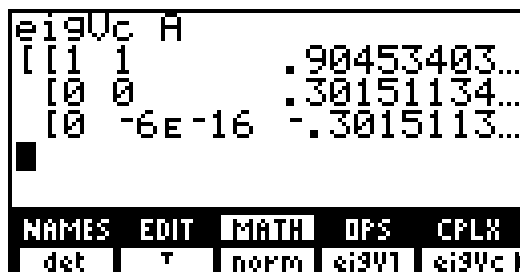


Figure II.32: Eigenvector of matrix A

Technology Tip: The entry $-6E - 16$ in Figure II.32 (third row, second column of the eigenvector matrix) should be taken as essentially 0.

II.4 Additional Topics

Length and Dot Product in R^n : Create a vector in the TI-86 with square brackets. For example, make the vector $V = (0, -2, 1, 4, -2)$ by pressing $2\text{nd } [0, (-) 2, 1, 4, (-) 2 2\text{nd }] \text{STO} \blacktriangleright V \text{ENTER}$.

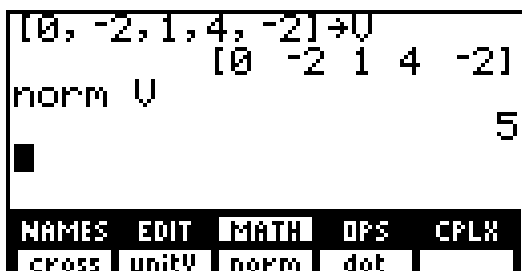


Figure II.33: Vector and its length

To calculate the length (norm) of this vector press $2\text{nd } \text{VECTR } F3 [MATH] F3 [norm] \text{ALPHA } V \text{ENTER}$ (see Figure II.33).

Now define the two vectors $U = (1, 2, 0, -3)$ and $V = (3, -2, 4, 2)$. Calculate the dot product $U \cdot V$ of these vectors by pressing $2\text{nd } \text{VECTR } F3 [MATH] F4 [dot] \text{ALPHA } U, \text{ALPHA } V) \text{ENTER}$

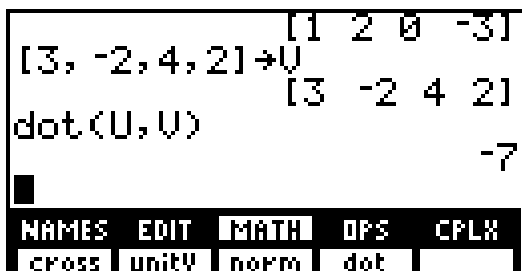


Figure II.34: Vector dot product

Cross Product: Evaluate the cross product of two vectors $\mathbf{u} = \mathbf{i} - 2\mathbf{j} + \mathbf{k}$ and $\mathbf{v} = 3\mathbf{i} + \mathbf{j} - 2\mathbf{k}$, by pressing $2\text{nd } \text{VECTR } F3 [MATH] F1 [cross] 2\text{nd } [1, (-)2, 1 2\text{nd }], 2\text{nd } [3, 1, (-) 2 2\text{nd }]) \text{ENTER}$. Their cross product is $\mathbf{u} \times \mathbf{v} = 3\mathbf{i} + 5\mathbf{j} + 7\mathbf{k}$.

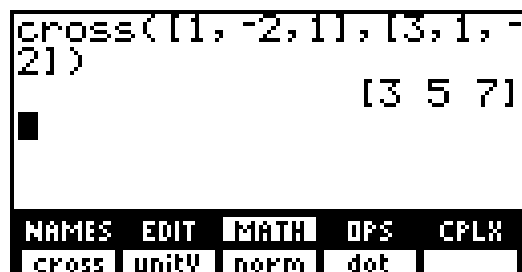


Figure II.35: Vector cross product

Complex Numbers: On the TI-86, the complex number $a + bi$ is entered as the ordered pair (a, b) . In particular, the complex unit i is represented as $(0, 1)$.

Press $2\text{nd } \text{CPLX}$ to display the menu of special complex number operations (Figure II.36).

Expression	Keystrokes	Answer
$(2 + 3i) + (5 - 7i)$	(2 , 3) + (5 , (-) 7) ENTER	$7 - 4i$
$(2 + 3i)(5 - 7i)$	(2 , 3) x (5 , (-) 7) ENTER	$31 + i$
$\frac{31 + i}{5 - 7i}$	(31 , 1) ÷ (5 , (-) 7) ENTER	$2 + 3i$
$ 5 - 12i $	F4 [abs] (5 , (-) 12) ENTER	13
$\overline{5 - 12i}$	F1 [conj] (5 , (-) 12) ENTER	$5 + 12i$

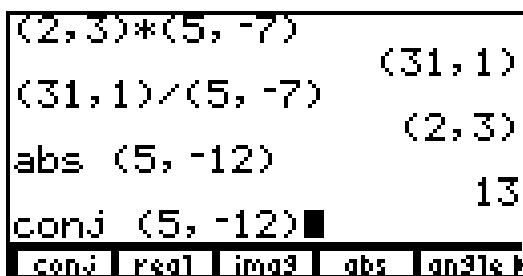


Figure II.36: Complex number arithmetic

Rectangular-Polar Conversion: The CPLX menu (Figure II.37) also provides functions for converting between rectangular and polar coordinate systems.

Given rectangular coordinates $(x, y) = (4, -3)$ convert *from* these rectangular coordinates *to* polar coordinates (r, θ) by pressing 2nd CPLX MORE (4 , (-) 3) F2 ENTER. We see that $r = 5$ and $\theta = -.6435$. The measure of angle θ is displayed in radians, because that is the current default angle measure chosen in the MODE menu.

Suppose $(r, \theta) = (3, \pi)$. To convert *from* these polar coordinates *to* rectangular coordinates (x, y) press 2nd CPLX MORE (3 2ND \angle 2nd π) F1 ENTER. Then $x = -3$ and $y = 0$.

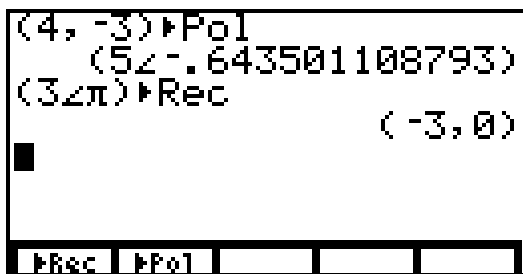


Figure II.37: Coordinate conversions

II.5 Program: Visualizing Row Operations

II.5.1 Entering the Program: The TI-86 is a programmable calculator that can store sequences of commands for later replay. Here's a useful program that demonstrates how elementary matrix row operations used in Gauss-Jordan elimination may be interpreted graphically.

Press PRGM to access the programming menu. The TI-86 has space for many programs, each identified by a name that is up to eight characters long. The names of all your programs are listed alphabetically in the PRGM NAMES menu.

To create a new program, press PRGM F2 [EDIT] and enter its name. The cursor is now a blinking A, indicating the calculator is set to receive upper case alphabetic characters. Call this program ROWOPS and press ENTER when you have finished.

Within the program itself, each line begins with a colon : supplied automatically by the calculator after you press ENTER. Any command you could enter directly in the TI-86's home screen can be entered as a line in a program. There are also special programming commands.

Note that the TI-86 calculator checks for program errors as it runs a program, not while you enter or edit it.

```
PROGRAM:ROWOPS
:Disp "ENTER A 2x3 MA
TRIX"
:Disp "A B C"
:Disp "D E F"
:Promt A,B,C,D,E,F
:[ [A,B,C] [D,E,F] ]→M
PAGE|PAGE| I/O | CTL | INSc
```

Figure II.38: Part of program ROWOPS

Input the program ROWOPS by pressing the keystrokes given in the listing below.

<i>Program Line</i>	<i>Keystrokes</i>
:Disp " ENTER A 2 BY 3 MATRIX "	F3 [I/O] F3 [Disp] MORE MORE F1 ["] ALPHA ALPHA E N T E R ␣ A ␣ ALPHA 2 ALPHA ALPHA ␣ B Y ␣ ALPHA 3 ALPHA ALPHA ␣ M A T R I X F1 ["] ENTER
:Disp "A B C"	MORE F3 [Disp] MORE MORE F1 ALPHA ALPHA A ␣ B ␣ C F1 ["] ENTER
:Disp "D E F"	MORE F3 [Disp] MORE MORE F1 ALPHA ALPHA D ␣ E ␣ F F1 ["] ENTER
:Prompt A, B, C, D, E, F	MORE F2 [Promp] ALPHA A , ALPHA B , ALPHA C , ALPHA D , ALPHA E , ALPHA F ENTER
: [[A, B, C][D E F]→M	2nd [2nd [ALPHA A , ALPHA B , ALPHA C 2nd] 2nd [ALPHA D , ALPHA E , ALPHA F 2nd] 2nd] STO ► M ENTER
:CLLCD	MORE F5 [CLLCD] ENTER
:Disp "ORIGINAL MATRIX"	MORE MORE F3 [Disp] MORE MORE F1 ["] ALPHA ALPHA O R I G I N A L ␣ M A T R I X F1 ["] ENTER
:Pause M ► Frac	2nd M4 [CTL] MORE MORE F3 [Pause] ALPHA M 2nd MATH F5 [Misc] MORE F1 ► Frac] ENTER
:y2=B ⁻¹ (C-A x)	2nd alpha Y 2 ALPHA = ALPHA B 2nd x ⁻¹ (ALPHA C – ALPHA A x-VAR) ENTER
:y1=E ⁻¹ (F-D x)	2nd alpha Y 1 ALPHA = ALPHA E 2nd x ⁻¹ (ALPHA F – ALPHA D x-VAR) ENTER
:Zstd:Pause :CLLCD	GRAPH F3 [ZOOM] F4 [ZStd] 2nd: EXIT F3 [Pause] 2nd: 2nd M3 [I/O] MORE F5 [CLLCD] ENTER

:Disp "OBTAIN LEADING 1"
 MORE MORE F3 [Disp] MORE MORE F1 ["] ALPHA
 ALPHA O B T A I N ◀ L E A D I N G ◀ ALPHA 1 F1 ["]
 ENTER

:Disp "IN ROW 1"
 MORE F3 [Disp] MORE MORE F1 ["] ALPHA ALPHA I N
 ◀ R O W ◀ ALPHA 1 F1 ["] ENTER

:multR(A⁻¹, M, 1)→M
 2nd MATRX F3 [OPS] MORE F4 [multR] ALPHA A 2nd x⁻¹,
 ALPHA M , 1) STO ► M ENTER

:Pause M ► Frac:CL Drw
 EXIT 2nd M4 [CTL] MORE MORE F3 [Pause] ALPHA M
 2nd MATH F5 [Misc] MORE F1 ► Frac] 2nd:
 GRAPH MORE F2 [DRAW] MORE F5 [CIDrw] ENTER

:y2=(A/B)(C/A-x)
 2nd alpha Y2 ALPHA = (ALPHA A ÷ ALPHA B) (ALPHA
 C ÷ ALPHA A – x-Var) ENTER

:DispG:Pause :CLLCD
 GRAPH F5 [DispG] 2nd: EXIT F3 [Pause] 2nd: 2nd M3
 [/O] MORE F5 [CLLCD] ENTER

:Disp "OBTAIN Ø BELOW"
 MORE MORE F3 [Disp] MORE MORE F1 ["] ALPHA
 ALPHA O B T A I N ◀ ALPHA 0 ALPHA ALPHA ◀
 B E L O W F1 ["] ENTER

:Disp "LEADING 1 IN"
 MORE F3 [Disp] MORE MORE F1 ["] ALPHA ALPHA
 L E A D I N G ◀ ALPHA 1 ALPHA ALPHA ◀ I N F1 ["]
 ENTER

:Disp "COLUMN 1"
 MORE F3 [Disp] MORE MORE F1 ["] ALPHA ALPHA
 C O L U M N ◀ ALPHA 1 F1 ["] ENTER

:mRAdd (-D, M, 1, 2)→M
 2nd MATRX F3 [OPS] MORE F5 [mRAdd](-)ALPHA D,
 ALPHA M , 1 , 2) STO ► M ENTER

:Pause M ► Frac:CLDrw
 EXIT 2nd M4 [CTL] MORE MORE F3 [Pause] ALPHA M
 2nd MATH F5 [Misc] MORE F1 ► Frac] 2nd:
 GRAPH MORE F2 [DRAW] MORE F5 [CIDrw] ENTER

:y1=(E-(B*D/A))⁻¹(F-(D*C/A))
 2nd alpha Y 1 ALPHA = (ALPHA E – (ALPHA B × ALPHA
 D ÷ ALPHA A)) 2nd x⁻¹(ALPHA F – (ALPHA D × ALPHA
 C ÷ ALPHA A)) ENTER

:DispG:Pause :CLLCD
 GRAPH F5 [DispG] 2nd : EXIT 2nd M4 [CTL] MORE
 MORE F3 2nd : 2nd M3 [/O] MORE F5 [CLLCD] ENTER

:M(2, 2)→G
 ALPHA M (2 , 2) STO ► G ENTER

:If G≠0
 2nd M4 [CTL] F1 [If] ALPHA G 2nd TEST MORE F1 [≠] 0
 ENTER

:Then
 2nd M2 [Then] ENTER

:multR(G⁻¹, M, 2)→M
 2nd MATRX F3 [OPS] MORE F4 [multR] ALPHA G 2nd x⁻¹,
 ALPHA M , 2) STO ► M ENTER

:Disp "OBTAIN LEADING" EXIT 2nd M3 [I/O] F3 MORE MORE F1 [""] ALPHA ALPHA
O B T A I N \leftarrow L E A D I N G \leftarrow F1 [""] ENTER

:Disp "1 IN ROW 2" MORE F3 [Disp] MORE MORE F1 [""] 1 ALPHA ALPHA \leftarrow
I N \leftarrow R O W \leftarrow ALPHA 2 F1 [""] ENTER

:Pause M \blacktriangleright Frac:CLDrw 2nd M4 [CTL] MORE MORE F3 [Pause] ALPHA M
2nd MATH F5 [Misc] MORE F1 \blacktriangleright Frac] 2nd:
GRAPH MORE F2 [DRAW] MORE F5 [CLDrw] ENTER

:DispG:Pause :CLLCD GRAPH F5 [DispG] 2nd: EXIT 2nd M4 [CTL] MORE MORE
F3 [Pause] 2nd: EXIT 2nd M3 MORE F5 [CLLCD] ENTER

:Disp "OBTAIN \emptyset ABOVE" EXIT 2nd M3 [I/O] F3 [Disp] MORE MORE F1 [""] ALPHA
ALPHA O B T A I N \leftarrow ALPHA 0 ALPHA ALPHA \leftarrow A B O
V E F1 [""] ENTER

:Disp "LEADING 1 IN" MORE F3 [Disp] MORE MORE F1 [""] ALPHA ALPHA
L E A D I N G \leftarrow ALPHA 1 ALPHA ALPHA \leftarrow I N F1 [""]
ENTER

:Disp "COLUMN 2" MORE F3 [Disp] MORE MORE F1 [""] ALPHA ALPHA
C O L U M N \leftarrow ALPHA 2 F1 [""] ENTER

:M(1, 2) \rightarrow H ALPHA M (1 , 2) STO \blacktriangleright H ENTER

:mRAdd(-H,M,2,1) \rightarrow M 2nd MATRX F3 [OPS] MORE F5 [mRAdd](-)ALPHA H,
ALPHA M , 2 , 1) STO \blacktriangleright M ENTER

:Pause M \blacktriangleright Frac:CLDrw:FnOff 2
EXIT 2nd M4 [CTL] MORE MORE F3 [Pause] ALPHA M
2nd MATH F5 [Misc] MORE F1 \blacktriangleright Frac] 2nd:
GRAPH MORE F2 [DRAW] MORE F5 [CLDrw] 2nd:
GRAPH F1 [VARS] MORE MORE F2 [FnOff] 2 ENTER

:M(1,3) \rightarrow J ALPHA M (1 , 3) STO \blacktriangleright J ENTER

:Vert J GRAPH MORE F2 [DRAW] F3 [Vert] ALPHA J ENTER

:DispG:Pause :CLLCD GRAPH F5 [DispG] 2nd : EXIT F3 [Pause] 2nd : 2nd M3
[I/O] MORE F5 [CLLCD] ENTER

:Disp "THE POINT OF" MORE MORE F3 [Disp] MORE MORE F1 [""] ALPHA
ALPHA T H E \leftarrow P O I N T \leftarrow O F F1 [""] ENTER

:Disp "INTERSECTION IS" MORE F3 [Disp] MORE MORE F1 [""] ALPHA ALPHA
I N T E R S E C T I O N \leftarrow I S F1 [""] ENTER

:Disp "x=", M(1,3) \blacktriangleright Frac, "y=", M(2,3) \blacktriangleright Frac
MORE F3 [Disp] MORE MORE F1 [""] 2nd alpha X ALPHA =
F1 [""], ALPHA M (1 , 3) 2nd MATH F5 [Misc] MORE F1
 \blacktriangleright Frac], 2nd M1 [""] 2nd alpha Y ALPHA = 2nd M1 [""],
ALPHA M (2 , 3) F1 \blacktriangleright Frac] ENTER

:Stop EXIT 2nd M4 [CTL] MORE MORE F5 [Stop] ENTER

:Else MORE MORE F3 [Else] ENTER

:If M(2,3)= \emptyset F1 ALPHA M (2 , 3) 2nd TEST F1 0 ENTER

:Then EXIT F2 [Then] ENTER

```

:Disp "INFINITELY MANY"      2nd M3 [I/O] F3 [Disp] MORE MORE F1[""] ALPHA ALPHA
                              I N F I N I T E L Y  M A N Y F1 [""] ENTER
:Disp "SOLUTIONS"           MORE F3 [Disp] MORE MORE F1 [""] ALPHA ALPHA
                              S O L U T I O N S F1 [""] ENTER
:Else                        2nd M4 [CTL] F3 [Else] ENTER
:Disp "INCONSISTENT"       2nd M3 [I/O] F3 [Disp] MORE MORE F1[""] ALPHA ALPHA
                              I N C O N S I S T E N T F1 [""] ENTER
:Disp "SYSTEM"             MORE F3 [Disp] MORE MORE F1[""] ALPHA ALPHA
                              S Y S T E M F1 [""] ENTER
:End                        2nd M4 [CTL] F5 [End]

```

When you have finished, press 2nd QUIT to leave the program editor.

You may remove a program from memory by pressing 2nd MEM F2 [DELET] MORE F5 [PRGM]. Then move the cursor to the program's name and press ENTER to delete the entire program.

II.5.2 Running the Program: To run the program just entered, press PRGM F1 [NAMES] and look for ROWOPS. The names of programs are listed alphabetically; press MORE to advance through the listing. Press the function key below [ROWOP] to select this program, then press ENTER.

The program has been written to prompt you for values of the coefficients a , b , c , d , e and f in two linear equations, $ax + by = c$ and $dx + ey = f$. Input each value, then press ENTER to continue the program. This demonstration is most effective for equations that do not correspond to vertical or horizontal lines, and whose y -intercepts are between -10 and 10 .

While the demonstration is running, note that each elementary row operation creates an equivalent system. This equivalence is reinforced graphically by the fact that while a row operation may change the *slope* of the lines, their *point of intersection* remains the same.

When the program comes to a pause, press ENTER to continue the program. If you need to interrupt a program during execution, press ON.

The instruction manual for your TI-86 gives detailed information about programming. Refer to it to learn more about programming and how to use other features of your calculator.