Section 2.4 Introduction to Equations

1. (a) $x = 3$

$2(3) - 6 \stackrel{?}{=} 0$

$6 - 6 \stackrel{?}{=} 0$

$0 = 0$

3 *is* a solution.

(b) $x = 1$

$2(1) - 6 \stackrel{?}{=} 0$

$2 - 6 \stackrel{?}{=} 0$

$-4 \neq 0$

1 *is not* a solution.

3. (a) $x = 0$

$2(0) + 4 \stackrel{?}{=} 2$

$0 + 4 \stackrel{?}{=} 2$

$4 \neq 2$

0 *is not* a solution.

(b) $x = -1$

$2(-1) + 4 \stackrel{?}{=} 2$

$-2 + 4 \stackrel{?}{=} 2$

$2 = 2$

-1 *is* a solution.

5. (a) $x = -1$

$-1 + 5 \stackrel{?}{=} 2(-1)$

$4 \neq -2$

-1 *is not* a solution

(b) $x = 5$

$5 + 5 \stackrel{?}{=} 2(5)$

$10 = 10$

5 *is* a solution.

7. (a) $x = 11$

$11 + 3 \stackrel{?}{=} 2(11 - 4)$

$14 \stackrel{?}{=} 2(7)$

$14 = 14$

11 *is* a solution.

(b) $x = -5$

$-5 + 3 \stackrel{?}{=} 2(-5 - 4)$

$-2 \stackrel{?}{=} 2(-9)$

$-2 \neq -18$

-5 *is not* a solution.

9. (a) $x = \frac{3}{5}$

$2\left(\frac{3}{5}\right) + 10 \stackrel{?}{=} 7\left(\frac{3}{5} + 1\right)$

$\frac{6}{5} + \frac{10}{1} \stackrel{?}{=} 7\left(\frac{3}{5} + \frac{5}{5}\right)$

$\frac{6}{5} + \frac{50}{5} \stackrel{?}{=} \frac{7}{1}\left(\frac{8}{5}\right)$

$\frac{56}{5} = \frac{56}{5}$

$\frac{3}{5}$ *is* a solution.

(b) $x = \frac{2}{3}$

$2\left(\frac{2}{3}\right) + 10 \stackrel{?}{=} 7\left(\frac{2}{3} + 1\right)$

$\frac{4}{3} + \frac{10}{1} \stackrel{?}{=} 7\left(\frac{2}{3} + \frac{3}{3}\right)$

$\frac{4}{3} + \frac{30}{3} \stackrel{?}{=} \frac{7}{1}\left(\frac{5}{3}\right)$

$\frac{34}{3} \neq \frac{35}{3}$

$\frac{2}{3}$ *is not* a solution.

11. (a) $x = 3$

$3^2 - 4 \stackrel{?}{=} 3 + 2$

$9 - 4 \stackrel{?}{=} 3 + 2$

$5 = 5$

3 *is* a solution.

(b) $x = -2$

$(-2)^2 - 4 \stackrel{?}{=} -2 + 2$

$4 - 4 \stackrel{?}{=} 0$

$0 = 0$

-2 *is* a solution.

13. (a) $x = 3$

$\frac{2}{3} - \frac{1}{3} \stackrel{?}{=} 1$

$\frac{2 - 1}{3} \stackrel{?}{=} 1$

$\frac{1}{3} \neq 1$

3 *is not* a solution

(b) $x = \frac{1}{3}$

$\frac{2}{1/3} - \frac{1}{1/3} \stackrel{?}{=} 1$

$\left(2 \div \frac{1}{3}\right) - \left(1 \div \frac{1}{3}\right) \stackrel{?}{=} 1$

$\left(\frac{2}{1} \cdot \frac{3}{1}\right) - \left(\frac{1}{1} \cdot \frac{3}{1}\right) \stackrel{?}{=} 1$

$6 - 3 \stackrel{?}{=} 1$

$3 \neq 1$

$\frac{1}{3}$ *is not* a solution.

15. (a) $x = 3$

$\frac{5}{3 - 1} + \frac{1}{3} \stackrel{?}{=} 5$

$\frac{5}{2} + \frac{1}{3} \stackrel{?}{=} 5$

$\frac{15}{6} + \frac{2}{6} \stackrel{?}{=} 5$

$\frac{17}{6} \neq 5$

3 *is not* a solution.

(b) $x = \frac{1}{6}$

$\frac{5}{(1/6) - 1} + \frac{1}{1/6} \stackrel{?}{=} 5$

$\left[5 \div \left(\frac{1}{6} - 1\right)\right] + \left(1 \div \frac{1}{6}\right) \stackrel{?}{=} 5$

$\frac{5}{1} \div \left(-\frac{5}{6}\right) + \frac{1}{1} \div \frac{1}{6} \stackrel{?}{=} 5$

$\frac{5}{1} \cdot \frac{-6}{5} + \frac{1}{1} \cdot \frac{6}{1} \stackrel{?}{=} 5$

$\frac{-30}{5} + 6 \stackrel{?}{=} 5$

$-6 + 6 \stackrel{?}{=} 5$

$0 \neq 5$

$\frac{1}{6}$ *is not* a solution.

17. (a) $x = 1.2$

$1.2 + 3 \stackrel{?}{=} 3.5$

$4.2 \neq 3.5$

1.2 is *not* a solution.

(b) $x = 4.8$

$4.8 + 3 \stackrel{?}{=} 3.5$

$7.8 \neq 3.5$

4.8 is *not* a solution.

19. (a) $x = 12.25$

$40(12.25) - 490 \stackrel{?}{=} 0$

$0 = 0$

12.25 *is* a solution.

(b) $x = -12.25$

$40(-12.25) - 490 \stackrel{?}{=} 0$

$-980 \neq 0$

-12.25 is *not* a solution.

21. (a) $x = \frac{5}{2}$

$2\left(\frac{5}{2}\right)^2 - \frac{5}{2} - 10 \stackrel{?}{=} 0$

$0 = 0$

$\frac{5}{2}$ *is* a solution.

(b) $x = -1.09$

$2(-1.09)^2 - (-1.09) - 10 \stackrel{?}{=} 0$

$-6.5338 \neq 0$

-1.09 is *not* a solution.

23. (a) $x = 0$

$\frac{1}{0}$ is undefined

0 is *not* a solution

(b) $x = -2$

$\frac{1}{-2} - \frac{9}{-2-4} \stackrel{?}{=} 1$

$1 = 1$

-2 *is* a solution.

25. (a) $x = \frac{6}{5}$

$\left(\frac{6}{5}\right)^3 - 1.728 \stackrel{?}{=} 0$

$0 = 0$

$\frac{6}{5}$ *is* a solution.

(b) $x = -\frac{6}{5}$

$\left(-\frac{6}{5}\right)^3 - 1.728 \stackrel{?}{=} 0$

$-3.456 \neq 0$

$-\frac{6}{5}$ is *not* a solution.

27.

$5x + 12 = 22$	Given equation.
$5x + 12 - 12 = 22 - 12$	Subtract 12 from each side.
$5x = 10$	Combine like terms.
$\frac{5x}{5} = \frac{10}{5}$	Divide both sides by 5.
$x = 2$	Solution.

29.

$\frac{2}{3}x = 12$	Given equation
$\frac{3}{2}\left(\frac{2}{3}x\right) = \frac{3}{2}(12)$	Multiply both sides by $\frac{3}{2}$.
$x = 18$	Simplify. (solution)

31.

$2(x - 1) = x + 3$	Given equation.
$2x - 2 = x + 3$	Remove grouping symbols.
$-x + 2x - 2 = -x + x + 3$	Subtract x from each side.
$x - 2 = 3$	Combine like terms.
$x - 2 + 2 = 3 + 2$	Add 2 to both sides.
$x = 5$	Combine like terms. (solution)

33.

$x = -2(x + 3)$	Given equation.
$x = -2x - 6$	Distributive Property.
$2x + x = 2x - 2x - 6$	Add $2x$ to each side.
$3x = 0 - 6$	Combine like terms.
$3x = -6$	Additive identity property.
$\frac{3x}{3} = \frac{-6}{3}$	Divide each side by 3.
$x = -2$	Simplify. (solution)

35. $x + 4 = 6$

$x + 4 - 4 = 6 - 4$

$x = 2$

Check: $2 + 4 = 6$

37. $3x = 30$

$\frac{3x}{3} = \frac{30}{3}$

$x = 10$

Check: $3(10) = 30$

39. The sum of a number and 8 is 25.

or

A number is increased by 8, and the result is 25.

41. The product of 10 and the difference of a number and 3 is 8 times the number.

43. The sum of a number and 1 is divided by 3, and the quotient is equal to 8.

45. *Verbal model:* [Original score] + [Additional points] = [Final score]

Labels: Original score = x (points)
Additional points = 6 (points)
Final score = 94 (points)

Equation: $x + 6 = 94$

Note: There are other equivalent ways to write this equation. Here are some other possibilities:

$$94 - x = 6$$
$$94 = x + 6$$
$$x = 94 - 6$$

There are also equivalent ways of writing equations for the *other* exercises in this section.

47. *Verbal model:* [Amount saved] + [Additional savings needed] = [Computer cost]

Labels: Amount saved = \$3650
Additional savings needed = x (dollars)
Computer cost = \$4532

Equation: $3650 + x = 4532$

Note: Remember that there are other equivalent ways of writing this equation.

49. *Verbal model:* [Unknown number] + [12] = [45]

Label: Unknown number = x

Equation: $x + 12 = 45$

51. *Verbal model:* [4] · [Sum of number and 6] = [100]

Label: Number = x

Equation: $4(x + 6) = 100$

Note: The parentheses are necessary to indicate that 4 is multiplied by the *sum* of x and 6. (If the equation were written as $4x + 6 = 100$, the 4 would be multiplied by the x only.)

53. *Verbal model:* $2 \cdot$ [Number] $- 14 =$ [Number] / 3

Labels: Number = x

Equation: $2x - 14 = \frac{x}{3}$

55. *Verbal model:* [Cost per mile] · [Number of miles] = [Cost for driv-

Label: Cost per mile = 0.32 (dollars per mile)
Number of miles = x
Cost for driving = 135.36 (dollars)

Equation: $0.32x = 135.36$

57. *Verbal model:* [2] · [Length of mirror] + [2] · [Width of mirror] = [Perimeter of mirror]

Label: Length of mirror = l (inches)
Width of mirror = $\frac{1}{3}l$ (inches)
Perimeter of mirror = 96 (inches)

Equation: $2l + 2\left(\frac{1}{3}l\right) = 96$

59. *Verbal model:*

$$\boxed{\text{Distance traveled}} + \boxed{\text{Remaining distance}} = \boxed{\text{Total distance}}$$

$$\boxed{\text{Travel rate}} \cdot \boxed{\text{Travel time}} + \boxed{\text{Remaining distance}} = \boxed{\text{Total distance}}$$

Labels: Travel rate $= x$ (miles per hour)
Travel time $= 3$ (hours)
Remaining distance $= 25$ (miles)
Total distance $= 160$ (miles)

Equation: $x \cdot 3 + 25 = 160$ or $3x + 25 = 160$

Note: Remember the formula $d = rt$; distance equals rate times time.

61. *Verbal model:*

$$\boxed{\text{Distance for first car}} = \boxed{\text{Distance for second car}}$$

$$\boxed{\text{Rate for first car}} \cdot \boxed{\text{Time for first car}} = \boxed{\text{Rate for second car}} \cdot \boxed{\text{Time for second car}}$$

Labels: Rate for first car $= 45$ (miles per hour)
Time for first car $= 3$ (hours)
Rate for second car $= x$ (miles per hour)
Time for second car $= 2.5$ (hours)

Equation: $45(3) = x(2.5)$ or $135 = 2.5x$

63. *Verbal model:*

$$\boxed{\text{Original price}} + \boxed{\text{Increase in price}} = \boxed{\text{Current price}}$$

Labels: Original price $= x$ (dollars)
Increase in price $= 45$ (dollars)
Current price $= 375$ (dollars)

Equation: $x + 45 = 375$

65. *Verbal model:*

$$\boxed{\text{Total depreciation}} = \boxed{3} \cdot \boxed{\text{Depreciation per year}} \text{ or}$$

$$\boxed{\text{Initial value}} - \boxed{\text{Final value}} = \boxed{3} \cdot \boxed{\text{Depreciation per year}}$$

Labels: Initial value $= \$750{,}000$
Final value $= \$75{,}000$
Depreciation per year $= x$ (dollars)

Equation: $750{,}000 - 75{,}000 = 3x$

Note: This equation could also be written as $\dfrac{750{,}000 - 75{,}000}{3} = x$ or as $750{,}000 - 3x = 75{,}000$.

67. *Verbal model:*

$$\boxed{\text{Profit per box}} \cdot \boxed{\text{Number of boxes}} = \boxed{\text{Amount earned}}$$

Labels: Profit per box $= 1.75$ (dollars)
Number of boxes $= x$
Amount earned $= 2000$ (dollars)

Equation: $1.75x = 2000$

69. $\frac{3 \text{ dollars}}{\cancel{\text{unit}}} \cdot (5 \cancel{\text{units}}) = 15 \text{ dollars}$

71. $\frac{3 \text{ dollars}}{\cancel{\text{pound}}} \cdot (5 \cancel{\text{pounds}}) = 15 \text{ dollars}$

73. $\frac{5 \text{ feet}}{\cancel{\text{second}}} \cdot \frac{60 \cancel{\text{seconds}}}{\cancel{\text{minute}}} \cdot (20 \cancel{\text{minutes}}) = 6000 \text{ feet}$

75. A conditional equation has a solution set that is not the entire set of real numbers, but the solution set for any identity is all real numbers.

77. An expression doesn't contain an equal sign. Simplifying an expression is writing the expression in an equivalent form. Simplifying can involve combining like terms or removing symbols of grouping.

This expression can be simplified: $5x + 2(x + 6)$

$$5x + 2(x + 6) = 5x + 2x + 12 = 7x + 12$$

An equation contains an equal sign, and it can be solved by finding all the values of the variable for which the equation is true.

This equation can be solved: $5x = 2(x + 6)$

$$\begin{aligned} 5x &= 2x + 12 \\ 3x &= 12 \\ x &= 4 \end{aligned}$$

79. To transform an equation into an equivalent equation,

(a) Simplify either side by removing symbols of grouping, combining like terms, or reducing fractions on one or both sides.

(b) Add (or subtract) the same quantity to (from) both sides of the equation.

(c) Multiply (or divide) both sides of the equation by the same nonzero quantity.

(d) Interchange the sides of the equation.

Review Exercises for Chapter 2

1. Terms: $4, -\frac{1}{2}x^3$
Coefficient: $-\frac{1}{2}$

3. Terms: $y^2, -10yz, \frac{2}{3}z^2$
Coefficients: $1, -10, \frac{2}{3}$

5. $5z \cdot 5z \cdot 5z = (5z)^3$ or 5^3z^3

7. $a(b - c) \cdot a(b - c) = [a(b - c)]^2$ or $a^2(b - c)^2$

9. (a) When $x = 0$, the value of $x^2 - 2x + 5$ is $0^2 - 2 \cdot 0 + 5 = 0 - 0 + 5 = 5$.

(b) When $x = 2$, the value of $x^2 - 2x + 5$ is $2^2 - 2 \cdot 2 + 5 = 4 - 4 + 5 = 5$.

11. When $x = 2$ and $y = -1$, the value of $x^2 - x(y + 1) = 2^2 - 2(-1 + 1) = 4 - 2(0) = 4 - 0 = 4$

When $x = 1$ and $y = 2$, the value of $x^2 - x(y + 1) = 1^2 - 1[2 + 1] = 1 - 1(3) = 1 - 3 = -2$

13. $x^2 \cdot x \cdot x = x^{2+1+4} = x^7$

15. $(x^3)^2 = x^{3 \cdot 2} = x^6$

17. $t^4(-2t^2) = -2t^{4+2} = -2t^6$

19. $(xy)(-5x^2y^3) = -5x^{1+2}y^{1+3}$
$= -5x^3y^4$

21. $(-2y^2)^3(8y) = (-2)^3(y^2)^3(8y)$
$= -8y^{2 \cdot 3}(8y)$
$= -8 \cdot 8 \cdot y^6 \cdot y$
$= -64y^{6+1}$
$= -64y^7$

23. Multiplicative Inverse Property