Section 2.2 Simplifying Algebraic Expressions

1. $u^2 \cdot u^4 = u^{2+4} = u^6$

3. $3x^3 \cdot x^4 = 3x^{3+4} = 3x^7$

5. $5x(x^6) = 5x^{1+6} = 5x^7$

7. $(-5z^3)(3z^2) = (-5 \cdot 3)(z^3 \cdot z^2)$
$= -15z^{3+2} = -15z^5$

9. $(-xz)(-2y^2z) = 2xy^2z^{1+1}$
$= 2xy^2z^2$

11. $2b^4(-ab)(3b^2) = (2)(-1)(3)(a)(b^4 \cdot b \cdot b^2)$
$= -6ab^{4+1+2} = -6ab^7$

13. $(t^2)^4 = t^{2(4)} = t^8$

15. $5(uv)^5 = 5u^5v^5$

17. $(-2s)^3 = (-2)^3s^3 = -8s^3$

19. $(a^2b)^3(ab^2)^4 = (a^2)^3 \cdot b^3 \cdot a^4(b^2)^4$
$= a^{2\cdot 3} \cdot a^4 \cdot b^3 \cdot b^{2\cdot 4}$
$= a^6 \cdot a^4 \cdot b^3 \cdot b^8$
$= a^{6+4}b^{3+8}$
$= a^{10}b^{11}$

21. $(u^2v^3)(-2uv^2)^4 = u^2v^3(-2)^4u^4v^{2\cdot 4}$
$= u^2v^3 \cdot 16u^4v^8$
$= 16u^{2+4}v^{3+8}$
$= 16u^6v^{11}$

23. $[(x-3)^4]^2 = (x-3)^{4\cdot 2} = (x-3)^8$

25. $(x-2y)^3(x-2y)^3 = (x-2y)^{3+3}$
$= (x-2y)^6$

27. $x^5 \cdot x^3 = x^{5+3} = x^8$
$x^5 \cdot x^3 \neq x^{15}$
No, they are *not* equal.

29. $-3x^3 = -3 \cdot x \cdot x \cdot x$
$-27x^3 = -27 \cdot x \cdot x \cdot x$
$-3x^3 \neq -27x^3$
No, they are *not* equal.

31. Commutative Property of Addition

33. Associative Property of Multiplication

35. Multiplicative Identity Property

37. Commutative Property of Multiplication

39. Additive Inverse Property

41. Multiplicative Inverse Property

43. Additive Inverse Property and Additive Identity Property

45. $(x+10) - (x+10) = 0$
Additive Inverse Property

47. $v(2) = 2v$
Commutative Property of Multiplication

49. $5(t-2) = 5(t) - 5(2)$
Distributive Property

51. $5x\left(\frac{1}{5x}\right) = 1$
Multiplicative Inverse Property

53. $12 + (8 - x) = (12 + 8) - x$
Associative Property of Addition

55. $-5(2x - y) = -5(2x) - (-5)(y)$
$= -10x + 5y$

57. $(x+2)(3) = x(3) + 2(3)$
$= 3x + 6$

59. $4(x + xy + y^2) = 4(x) + 4(xy) + 4(y^2)$
$= 4x + 4xy + 4y^2$

61. $3(x^2 + x) = 3(x^2) + 3(x)$
$= 3x^2 + 3x$

63. $-4y(3y - 4) = -4y(3y) - (-4y)(4)$
$= -12y^2 + 16y$

65. $-(u - v) = (-1)(u - v)$
$= (-1)(u) - (-1)(v)$
$= -u + v$

67. $x(3x - 4y) = x(3x) - x(4y)$
$= 3x^2 - 4xy$

69. The area of the rectangle on the left is ab. The area of the rectangle on the right is ac. The area of the entire rectangle can be written as $a(b + c)$ and as the sum of the two smaller rectangles $ab + ac$.

Using the Distributive Property, you can see that these expressions are equal. $a(b + c) = ab + ac$

71. The area of the rectangle on the left is $2a$. The area of the rectangle on the right is $2(b - a)$. The area of the entire rectangle can be written as $2b$ and as the sum of the two smaller rectangles $2a + 2(b - a)$.

Using the Distributive Property, you can see that these expressions are equal.
$2a + 2(b - a) = 2a + 2b - 2a = 2b$

73.

Term	*Coefficient*
$6x^2$	6
$-3xy$	-3
y^2	1

75. In this expression, $16t^3$ and $3t^3$ are like terms, and the constants 4 and -5 are like terms.

77. In this expression, $6x^2y$ and $-4x^2y$ are like terms.

79. $\frac{1}{2}x^2y$ and $-\frac{5}{2}xy^2$ are not like terms because their variable factors are not alike.

Note: $x^2y = x \cdot x \cdot y$ and $xy^2 = x \cdot y \cdot y$.

81. $3y - 5y = (3 - 5)y = -2y$

83. $x + 5 - 3x = x - 3x + 5$
$= (1 - 3)x + 5$
$= -2x + 5$

85. $2x + 9x + 4 = (2 + 9)x + 4$
$= 11x + 4$

87. $5r + 6 - 2r + 1 = 5r - 2r + 6 + 1$
$= (5 - 2)r + (6 + 1)$
$= 3r + 7$

89. $x^2 - 2xy + 4 + xy = x^2 - 2xy + xy + 4$
$= x^2 + (-2 + 1)xy + 4$
$= x^2 + (-1)xy + 4$
$= x^2 - xy + 4$

91. $5z - 5 + 10z + 2z + 16 = 5z + 10z + 2z - 5 + 16$
$= (5 + 10 + 2)z + (-5 + 16)$
$= 17z + 11$

93. $z^3 + 2z^2 + z + z^2 + 2z + 1 = z^3 + 2z^2 + z^2 + z + 2z + 1$
$= z^3 + (2 + 1)z^2 + (1 + 2)z + 1$
$= z^3 + 3z^2 + 3z + 1$

95. $2x^2y + 5xy^2 - 3x^2y + 4xy + 7xy^2 = (2x^2y - 3x^2y) + (5xy^2 + 7xy^2) + 4xy$
$= (2 - 3)x^2y + (5 + 7)xy^2 + 4xy$
$= -x^2y + 12xy^2 + 4xy$

97. $3\left(\frac{1}{x}\right) - \frac{1}{x} + 8 = (3 - 1)\left(\frac{1}{x}\right) + 8$
$= 2\left(\frac{1}{x}\right) + 8$ or $\frac{2}{x} + 8$

99. $5\left(\frac{1}{t}\right) + 6\left(\frac{1}{t}\right) - 2t = (5 + 6)\left(\frac{1}{t}\right) - 2t$
$= 11\left(\frac{1}{t}\right) - 2t$ or $\frac{11}{t} - 2t$

101. False

$3(x - 4) = 3 \cdot x - 3 \cdot 4$

$= 3x - 12$

Therefore, $3(x - 4) \neq 3x - 4$.

103. True

$6x - 4x = (6 - 4)x = 2x$

Therefore, $6x - 4x = 2x$.

105. $8(52) = 8(50 + 2)$

$= 8(50) + 8(2)$

$= 400 + 16$

$= 416$

107. $5(7.98) = 5(8 - 0.02)$

$= 5(8) - 5(0.02)$

$= 40 - 0.10$

$= 39.9$

109. $2(6x) = (2 \cdot 6)x = 12x$

111. $-(-4x) = [-1(-4)]x = 4x$

113. $(-2x)(-3x) = (-2)(-3)(x \cdot x)$

$= 6x^2$

115. $(-5z)(2z^2) = (-5)(2)(z \cdot z^2)$

$= -10z^3$

117. $\frac{18a}{5} \cdot \frac{15}{6} = \frac{18a(15)}{5(6)}$

$= \frac{9\cancel{(2)}(a)\cancel{(5)}\cancel{(3)}}{\cancel{5}\cancel{(3)}\cancel{(2)}}$

$= 9a$

119. $\left(-\frac{3x^2}{2}\right)(4x^3) = -\frac{3}{2} \cdot 4(x^2 \cdot x^3)$

$= -6x^5$

121. $(12xy^2)(-2x^3y^2) = 12(-2)(x \cdot x^3)(y^2 \cdot y^2)$

$= -24x^4y^4$

123. $2(x - 2) + 4 = 2x - 4 + 4 = 2x$

125. $6(2s - 1) + s + 4 = 12s - 6 + s + 4$

$= 12s + s - 6 + 4$

$= 13s - 2$

127. $m - 3(m - 5) = m - 3m + 15$

$= -2m + 15$

129. $-6(1 - 2x) + 10(5 - x) = -6 + 12x + 50 - 10x$

$= -6 + 50 + 12x - 10x$

$= 44 + 2x$

131. $\frac{2}{3}(12x + 15) + 16 = 8x + 10 + 16$

$= 8x + 26$

133. $3 - 2[6 + (4 - x)] = 3 - 2[6 + 4 - x]$

$= 3 - 2[10 - x]$

$= 3 - 20 + 2x$

$= 2x - 17$ or $17 + 2x$

Note: This expression may also be simplified as follows:

$3 - 2[6 + (4 - x)] = 3 - 2[6 + 4 - x]$

$= 3 - 12 - 8 + 2x$

$= 2x - 17$

135. $7x(2 - x) - 4x = 14x - 7x^2 - 4x$

$= -7x^2 + 10x$

137. $4x^2 + x(5 - x) = 4x^2 + 5x - x^2$

$= 4x^2 - x^2 + 5x$

$= 3x^2 + 5x$

139. $-3t(4 - t) + t(t + 1) = -12t + 3t^2 + t^2 + t$

$= 4t^2 - 11t$

141. $3t[4 - (t - 3)] + t(t + 5) = 3t[4 - t + 3] + t^2 + 5t$

$= 12t - 3t^2 + 9t + t^2 + 5t$

$= -2t^2 + 26t$

143. $\frac{2x}{3} - \frac{x}{3} = \frac{2}{3}x - \frac{1}{3}x$

$= \left(\frac{2}{3} - \frac{1}{3}\right)x$

$= \frac{1}{3}x \quad \text{or} \quad \frac{x}{3}$

145. $\frac{4z}{5} + \frac{3z}{5} = \frac{4}{5}z + \frac{3}{5}z$

$= \left(\frac{4}{5} + \frac{3}{5}\right)z$

$= \frac{7}{5}z \quad \text{or} \quad \frac{7z}{5}$

147. $\frac{x}{3} - \frac{5x}{4} = \frac{1}{3}x - \frac{5}{4}x$

$= \left(\frac{1}{3} - \frac{5}{4}\right)x$

$= \left(\frac{1(4)}{3(4)} - \frac{5(3)}{4(3)}\right)x$

$= \left(\frac{4}{12} - \frac{15}{12}\right)x$

$= -\frac{11}{12}x \quad \text{or} \quad -\frac{11x}{12}$

149. $\frac{3x}{10} - \frac{x}{10} + \frac{4x}{5} = \frac{3}{10}x - \frac{1}{10}x + \frac{4}{5}x$

$= \left(\frac{3}{10} - \frac{1}{10} + \frac{4}{5}\right)x$

$= \left(\frac{3}{10} - \frac{1}{10} + \frac{4(2)}{5(2)}\right)x$

$= \left(\frac{3}{10} - \frac{1}{10} + \frac{8}{10}\right)x$

$= 1 \cdot x$

$= x$

151. If $P = 10{,}000$, $r = 0.08$, and $t = 10$, the value of $P(1 + r)^t$ is $10{,}000(1 + 0.08)^{10} \approx 21{,}589.25$.

153. Area = (length)(width)

$= x \cdot x$

$= x^2$

Volume = (length)(width)(height)

$= x \cdot x \cdot x$

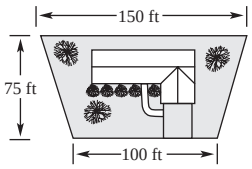
$= x^3$

155. $(x - 2) + (x + 11) + (2x + 3) = x - 2 + x + 11 + 2x + 3$

$= (1 + 1 + 2)x + (-2 + 11 + 3)$

$= 4x + 12$

157.



When $b_1 = 150$, $b_2 = 100$, and $h = 75$, the value of $\frac{1}{2}h(b_1 + b_2)$ is

$\frac{1}{2}(75)(150 + 100) = \frac{1}{2}(75)(250) = 9375$.

The area of the lot is 9375 square feet.

159. $(6x)^4 = 6^4x^4 = 1296x^4$

Thus, $(6x)^4 \neq 6x^4$.

161. (a) $12x^8$ is already in simplified form.

(b) $12(x^3)^5 = 12x^{15}$

(c) $12x^3x^5 = 12x^8$

(d) $3 \cdot 2(x^2)^4 = 3 \cdot 4x^8 = 12x^8$

(e) $3 \cdot 5x^8 = 15x^8$

Thus, (a), (c), and (d) are equivalent.

163. To combine like terms, add their coefficients and attach the common variable factor.

Examples:

$5a + 7b - 2a + b = 3a + 8b$

$2x^2 - 7x + 10x = 2x^2 + 3x$

165. To remove nested symbols of grouping, remove the innermost grouping symbols first and combine like terms.

167. $[x - (3 \cdot 4)] \div 5 = \dfrac{x - 12}{5}$

$[x - 3 \cdot 4] \div 5 = \dfrac{x - 12}{5}$

If the parentheses are removed, the expression is unchanged because multiplication is a higher-order operation than subtraction.

$x - (3 \cdot 4) \div 5 = x - \dfrac{12}{5}$

If the brackets are removed, the expression is changed because division is a higher-order operation than subtraction.

Mid-Chapter Quiz for Chapter 2

1. (a) If $x = 3$, the value of $x^2 - 3x$ is $3^2 - 3(3) = 9 - 9 = 0$.

(b) If $x = -2$, the value of $x^2 - 3x$ is $(-2)^2 - 3(-2) = 4 + 6 = 10$.

(c) If $x = 0$, the value of $x^2 - 3x$ is $0^2 - 3(0) = 0 - 0 = 0$.

2. (a) If $x = 2$ and $y = 4$, the value of $\dfrac{x}{y - 3}$ is $\dfrac{2}{4 - 3} = \dfrac{2}{1} = 2$.

(b) If $x = 0$ and $y = -1$, the value of $\dfrac{x}{y - 3}$ is $\dfrac{0}{-1 - 3} = \dfrac{0}{-4} = 0$.

(c) If $x = 5$ and $y = 3$, the value of $\dfrac{x}{y - 3}$ is undefined because $\dfrac{5}{3 - 3} = \dfrac{5}{0}$ and division by zero is undefined.

3. (a) The coefficient is -5.

(b) The coefficient is $\frac{5}{16}$.

4. (a) $3y \cdot 3y \cdot 3y \cdot 3y = (3y)^4$

(b) $2 \cdot (x - 3)(x - 3)2 \cdot 2 = 2^3(x - 3)^2$

5. $x^4 \cdot x^3 = x^{4+3} = x^7$

6. $(v^2)^5 = v^{2(5)}v^{10}$

7. $(-3y)^2y^3 = (-3)^2y^2y^3$
$= 9y^{2+3}$
$= 9y^5$

8. $8(x - 4)^2(x - 4)^4 = 8(x - 4)^{2+4}$
$= 8(x - 4)^6$

9. $\dfrac{2z^2}{3y} \cdot \dfrac{5z}{7y^3} = \dfrac{2z^2(5z)}{3y(7y^3)} = \dfrac{10z^3}{21y^4}$

10. $\left(\dfrac{x}{y}\right)^2\left(\dfrac{x}{y}\right)^5 = \left(\dfrac{x}{y}\right)^{2+5} = \left(\dfrac{x}{y}\right)^7$

11. Associative Property Multiplication

12. Distributive Property

13. Multiplicative Inverse Property

14. Commutative Property of Addition