Chapter 6 Matrices and Determinants

Section 6.1

Matrix – (Informal) A rectangular array of real numbers

(Formal) If m and n are positive integers, an $m \times n$ matrix is a rectangular array

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} \dots & a_{2n} \\ a_{31} & a_{32} & a_{33} \dots & a_{3n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & a_{m3} \dots & a_{mn} \end{bmatrix}$$

in which each entry, a_{ij}, of the matrix is a number.

Entry of a matrix – One of the real numbers that make up a matrix

Order of a matrix – Indicates the number of rows and columns of a matrix. A matrix having m rows and n columns is said to be of order $m \times n$.

Square matrix – A matrix in which the number of rows and the number of columns is equal

Main diagonal – For a square matrix, all entries, a_{ij}, in which $i = j$

Row matrix – A matrix that has only one row

Column matrix – A matrix that has only one column

Augmented matrix – A matrix derived from a system of linear equations (each written in standard form with the constant term on the right)

Coefficient matrix – A matrix derived from the coefficients of a system of linear equations (but not including the constant terms)

Elementary row operations – A set of operations that can be performed on an augmented matrix of a given system of linear equations that produce a new augmented matrix corresponding to a new (but equivalent) system of linear equations

Row-equivalent matrices – Two matrices are row-equivalent if one can be obtained from the other by a sequence of elementary row operations

Row-echelon form – A matrix in row-echelon form has the following properties:

1. All rows consisting entirely of zeros occur at the bottom of the matrix
2. For each row that does not consist entirely of zeros, the first nonzero entry is 1 (called a leading 1)
3. For two successive (nonzero) rows, the leading 1 in the higher row is farther to the left than the leading 1 in the lower row

Reduced row-echelon form – A matrix in row-echelon form that has zeros in every position above and below its leading 1

Gauss-Jordan elimination – The process of reducing a matrix to reduced row-echelon form

Section 6.2

Scalar multiple – If $A = [a_{ij}]$ is an $m \times n$ matrix and c is a scalar, the scalar multiple of A by c is the $m \times n$ matrix given by $cA = [ca_{ij}]$

Zero matrix – A matrix consisting entirely of zeros

additive identity – An $m \times n$ zero matrix O that does not alter the $m \times n$ matrix A when added to A. That is, $A + O = A$.

Matrix multiplication – If $A = [a_{ij}]$ is an $m \times n$ matrix and $B = [b_{ij}]$ is an $n \times p$ matrix, the product AB is an $m \times p$ matrix $AB = [c_{ij}]$ where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots + a_{in}b_{nj}$

Identity matrix of order *n* – The $n \times n$ matrix that consists of 1's on its main diagonal and 0's elsewhere

Section 6.3

Inverse of a matrix – Let A be an $n \times n$ matrix and let I_n be the $n \times n$ identity matrix. If there exists matrix A^{-1} such that $AA^{-1} = I_n = A^{-1}A$ then A^{-1} is called the inverse of A

Invertible or nonsingular matrix – A matrix that has an inverse

Singular matrix – A matrix that does not have an inverse

Section 6.4

Determinant – If A is a square matrix (of order 2×2 or greater), the determinant of A is the sum of the entries in any row (or column) of A multiplied by their respective cofactors. The determinant is a real number.

Minors – If A is a square matrix, the minor M_{ij} of the entry a_{ij} is the determinant of the matrix obtained by deleting the ith row and jth column of A

Cofactors – The cofactor C_{ij} of the entry a_{ij} is $C_{ij} = (-1)^{i+j}M_{ij}$

Expanding by cofactors – The process of applying the definition of a determinant to find the determinant of a matrix

Triangular matrix – A square matrix with all zeros either above or below its main diagonal entries

Upper triangular matrix – A square matrix that has all zero entries below its main diagonal entries

Lower triangular matrix – A square matrix that has all zero entries above its main diagonal entries

Diagonal matrix – A square matrix that is both upper and lower triangular

Section 6.5

Collinear points – Points that lie on the same line

Cramer's Rule – If a system of n linear equations in n variables has a coefficient matrix A with a nonzero determinant $|A|$, the solution of the system is

$$x_1 = \frac{|A_1|}{|A|}, x_2 = \frac{|A_2|}{|A|}, \ldots, x_n = \frac{|A_n|}{|A|}$$

where the ith column of A_i is the column of constants in the system of equations

Uncoded row matrices – Matrices that represent a decoded message

Coded row matrices – Matrices that represent an encoded message