

## Section 4.2 Logarithmic Functions and Their Graphs

**Objective:** In this lesson you learned how to recognize, evaluate, and graph logarithmic functions.

Course Number

Instructor

Date

### Important Vocabulary

Define each term or concept.

**Common logarithmic function**

**Natural logarithmic function**

### I. Logarithmic Functions (Pages 311–312)

The **logarithmic function with base  $a$**  is defined as

\_\_\_\_\_ , for  $x > 0$  and  $0 < a \neq 1$ , if and only if  $x = a^y$ .

The logarithmic function with base  $a$  is the \_\_\_\_\_ of the exponential function  $f(x) = a^x$ .

The equation  $x = a^y$  in exponential form is equivalent to the equation \_\_\_\_\_ in logarithmic form.

When evaluating logarithms, remember that a logarithm is a(n) \_\_\_\_\_. This means that  $\log_a x$  is the \_\_\_\_\_ to which  $a$  must be raised to obtain \_\_\_\_\_.

**Example 1:** Use the definition of logarithmic function to evaluate  $\log_5 125$ .

**Example 2:** Use a calculator to evaluate  $\log_{10} 300$ .

Complete the following properties of logarithms:

- 1)  $\log_a 1 =$  \_\_\_\_\_
- 2)  $\log_a a =$  \_\_\_\_\_
- 3)  $\log_a a^x =$  \_\_\_\_\_ and  $a^{\log_a x} =$  \_\_\_\_\_
- 4) If  $\log_a x = \log_a y$ , then \_\_\_\_\_.

### *What you should learn*

How to recognize and evaluate logarithmic functions with base  $a$

**Example 3:** Solve the equation  $\log_7 x = 1$  for  $x$ .

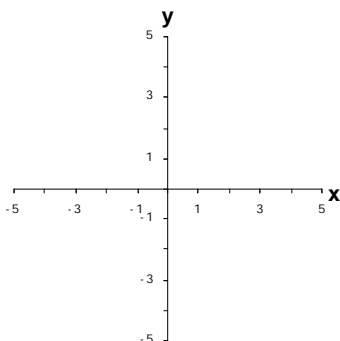
## II. Graphs of Logarithmic Functions (Pages 313–314)

For  $a > 1$ , is the graph of  $y = \log_a x$  increasing or decreasing over its domain? \_\_\_\_\_

For the graph of  $y = \log_a x$ ,  $a > 1$ , the domain is \_\_\_\_\_, the range is \_\_\_\_\_, and the intercept is \_\_\_\_\_.

Also, the graph has \_\_\_\_\_ as a vertical asymptote. The graph of  $y = \log_a x$  is a reflection of the graph of  $y = a^x$  about \_\_\_\_\_.

**Example 4:** Sketch the graph of the function  $f(x) = \log_3 x$ .



## III. The Natural Logarithmic Function (Pages 315–316)

Complete the following properties of natural logarithms:

- 1)  $\ln 1 =$  \_\_\_\_\_
- 2)  $\ln e =$  \_\_\_\_\_
- 3)  $\ln e^x =$  \_\_\_\_\_ and  $e^{\ln x} =$  \_\_\_\_\_
- 4) If  $\ln x = \ln y$ , then \_\_\_\_\_.

**Example 5:** Use a calculator to evaluate  $\ln 10$ .

**What you should learn**  
How to graph logarithmic functions

**What you should learn**  
How to recognize, evaluate, and graph natural logarithmic functions

**Example 6:** Find the domain of the function  $f(x) = \ln(x + 3)$ .

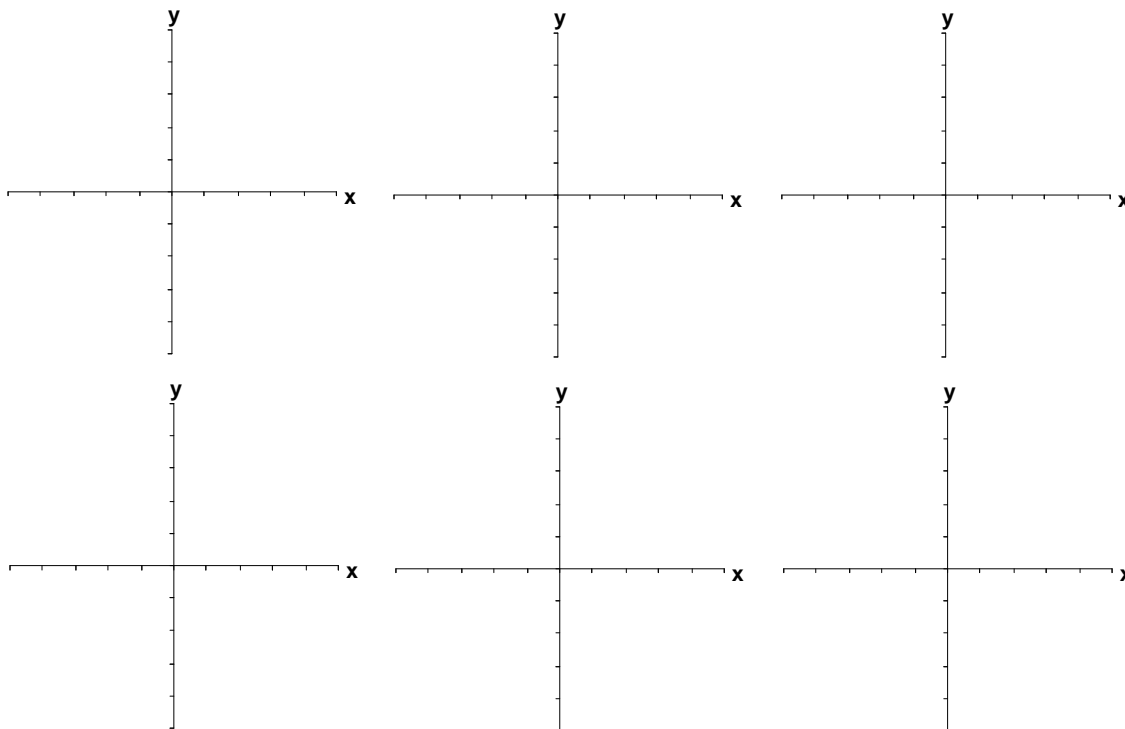
#### IV. Applications of Logarithmic Functions (Page 317)

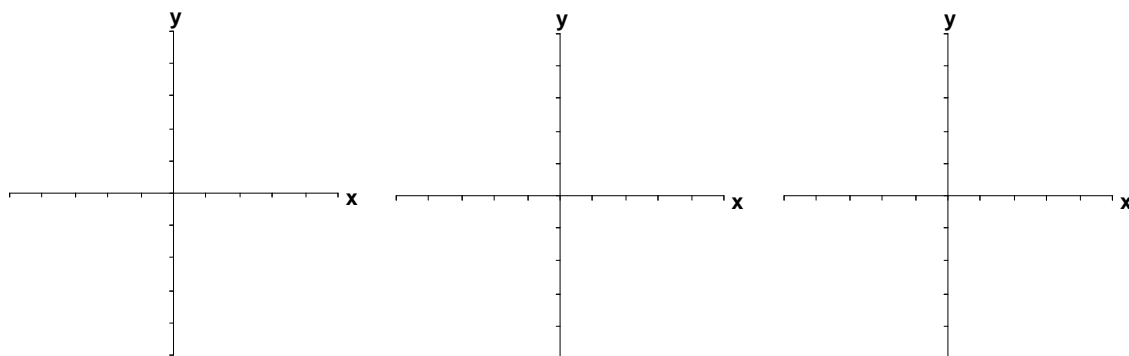
Describe a real-life situation in which logarithms are used.

***What you should learn***

How to use logarithmic functions to model and solve real-life problems

**Example 7:** A principal  $P$ , invested at 6% interest and compounded continuously, increases to an amount  $K$  times the original principal after  $t$  years, where  $t$  is given by  $t = \frac{\ln K}{0.06}$ . How long will it take the original investment to double in value? To triple in value?



**Additional notes****Homework Assignment**

Page(s)

Exercises