PART I

CHAPTER P
Prerequisites

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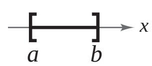
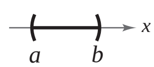
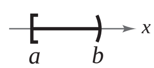
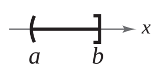
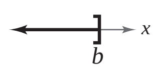
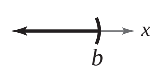
Section P.1 Review of Real Numbers and Their Properties

- You should know the following sets.
  - (a) The set of real numbers includes the rational numbers and the irrational numbers.
  - (b) The set of rational numbers includes all real numbers that can be written as the ratio p/q of two integers, where $q \neq 0$.
  - (c) The set of irrational numbers includes all real numbers which are not rational.
  - (d) The set of integers: $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$
  - (e) The set of whole numbers: $\{0, 1, 2, 3, 4, \ldots\}$
  - (f) The set of natural numbers: $\{1, 2, 3, 4, \ldots\}$
- The real number line is used to represent the real numbers.
- Know the inequality symbols.
  - (a) $a < b$ means a is less than b.
  - (b) $a \le b$ means a is less than or equal to b.
  - (c) $a > b$ means a is greater than b.
  - (d) $a \ge b$ means a is greater than or equal to b.
- Interval notation, inequality notation, graph and type:

Interval Notation	*Inequality Notation*	*Graph*	*Type*
$[a, b]$	$a \le x \le b$		Bounded and Closed
(a, b)	$a < x < b$		Bounded and Open
$[a, b)$	$a \le x < b$		Bounded
$(a, b]$	$a < x \le b$		Bounded
$[a, \infty)$	$x \ge a$		Unbounded
(a, ∞)	$x > a$		Unbounded
$(-\infty, b]$	$x \le b$		Unbounded
$(-\infty, b)$	$x < b$		Unbounded
$(-\infty, \infty)$	$-\infty < x < \infty$		Unbounded

- You should know that $|a| = \begin{cases} a, & \text{if } a \ge 0 \\ -a, & \text{if } a < 0 \end{cases}$.
- Know the properties of absolute value.
  - (a) $|a| \ge 0$
  - (b) $|-a| = |a|$
  - (c) $|ab| = |a|\,|b|$
  - (d) $\left|\dfrac{a}{b}\right| = \dfrac{|a|}{|b|}, b \neq 0$

—CONTINUED—

- The distance between a and b on the real line is $d(a, b) = |b - a| = |a - b|$.
- You should be able to identify the terms in an algebraic expression.
- You should know and be able to use the basic rules of algebra.
- Commutative Property
  (a) Addition: $a + b = b + a$ (b) Multiplication: $a \cdot b = b \cdot a$
- Associative Property
  (a) Addition: $(a + b) + c = a + (b + c)$ (b) Multiplication: $(ab)c = a(bc)$
- Identity Property
  (a) Addition: 0 is the identity; $a + 0 = 0 + a = a$. (b) Multiplication: 1 is the identity; $a \cdot 1 = 1 \cdot a = a$.
- Inverse Property
  (a) Addition: $-a$ is the additive inverse of a; $a + (-a) = -a + a = 0$.
  (b) Multiplication: $1/a$ is the multiplicative inverse of a, $a \neq 0$; $a(1/a) = (1/a)a = 1$.
- Distributive Property
  (a) $a(b + c) = ab + ac$ (b) $(a + b)c = ac + bc$
- Properties of Negation
  (a) $(-1)a = -a$ (b) $-(-a) = a$ (c) $(-a)b = a(-b) = -ab$
  (d) $(-a)(-b) = ab$ (e) $-(a + b) = (-a) + (-b) = -a - b$
- Properties of Equality
  (a) If $a = b$, then $a \pm c = b \pm c$. (b) If $a = b$, then $ac = bc$.
  (c) If $a \pm c = b \pm c$, then $a = b$. (d) If $ac = bc$ and $c \neq 0$, then $a = b$.
- Properties of Zero
  (a) $a \pm 0 = a$ (b) $a \cdot 0 = 0$ (c) $0 \div a = 0/a = 0, a \neq 0$
  (d) $a/0$ is undefined. (e) If $ab = 0$, then $a = 0$ or $b = 0$.
- Properties of Fractions $(b \neq 0, d \neq 0)$
  (a) Equivalent Fractions: $a/b = c/d$ if and only if $ad = bc$.
  (b) Rule of Signs: $-a/b = a/-b = -(a/b)$ and $-a/-b = a/b$
  (c) Equivalent Fractions: $a/b = ac/bc, c \neq 0$
  (d) Addition and Subtraction
  1. Like Denominators: $(a/b) \pm (c/b) = (a \pm c)/b$ 2. Unlike Denominators: $(a/b) \pm (c/d) = (ad \pm bc)/bd$
  (e) Multiplication: $(a/b) \cdot (c/d) = (ac)/(bd)$
  (f) Division: $(a/b) \div (c/d) = (a/b) \cdot (d/c) = (ad)/(bc)$ if $c \neq 0$.

Vocabulary Check

1. rational **2.** irrational **3.** absolute value

4. composite **5.** prime **6.** variables; constants

7. terms **8.** coefficient **9.** zero-factor property

1. $-9, -\frac{7}{2}, 5, \frac{2}{3}, \sqrt{2}, 0, 1, -4, 2, -11$

(a) Natural numbers: 5, 1, 2

(b) Whole numbers: 0, 5, 1, 2

(c) Integers: $-9, 5, 0, 1, -4, 2, -11$

(d) Rational numbers: $-9, -\frac{7}{2}, 5, \frac{2}{3}, 0, 1, -4, 2, -11$

(e) Irrational numbers: $\sqrt{2}$

3. $2.01, 0.666\ldots, -13, 0.010110111\ldots, 1, -6$

(a) Natural numbers: 1

(b) Whole numbers: 1

(c) Integers: $-13, 1, -6$

(d) Rational numbers: $2.01, 0.666\ldots, -13, 1, -6$

(e) Irrational numbers: $0.010110111\ldots$

5. $-\pi, -\frac{1}{3}, \frac{6}{3}, \frac{1}{2}\sqrt{2}, -7.5, -1, 8, -22$

(a) Natural numbers: $\frac{6}{3}$ (since it equals 2), 8

(b) Whole numbers: $\frac{6}{3}$, 8

(c) Integers: $\frac{6}{3}, -1, 8, -22$

(d) Rational numbers: $-\frac{1}{3}, \frac{6}{3}, -7.5, -1, 8, -22$

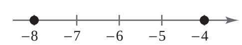
(e) Irrational numbers: $-\pi, \frac{1}{2}\sqrt{2}$

7. $\frac{5}{8} = 0.625$

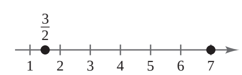
9. $\frac{41}{333} = 0.\overline{123}$

11. $-1 < 2.5$

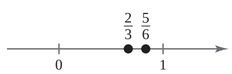
13. $-4 > -8$



15. $\frac{3}{2} < 7$

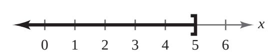


17. $\frac{5}{6} > \frac{2}{3}$



19. (a) The inequality $x \le 5$ denotes the set of all real numbers less than or equal to 5.

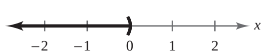
(b)



(c) The interval is unbounded.

21. (a) The inequality $x < 0$ denotes the set of all negative real numbers.

(b)



(c) The interval is unbounded.

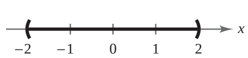
23. (a) The interval $[4, \infty)$ denotes the set of all real numbers greater than or equal to 4.

(b)

(c) The interval is unbounded.

25. (a) The inequality $-2 < x < 2$ denotes the set of all real numbers greater than -2 and less than 2.

(b)



(c) The interval is bounded.

27. (a) The inequality $-1 \le x < 0$ denotes the set of all negative real numbers greater than or equal to -1.

(b)

(c) The interval is bounded.

29. (a) The interval $[-2, 5)$ denotes the set of all real numbers greater than or equal to -2 and less than 5.

(b)

(c) The interval is bounded.

31. $-2 < x \le 4$

33. $y \ge 0$

35. $10 \le t \le 22$

37. $W > 65$

39. $|-10| = -(-10) = 10$

41. $|3 - 8| = |-5| = -(-5) = 5$

43. $|-1| - |-2| = 1 - 2 = -1$

45. $\dfrac{-5}{|-5|} = \dfrac{-5}{-(-5)} = \dfrac{-5}{5} = -1$

47. If $x < -2$, then $x + 2$ is negative.

Thus $\dfrac{|x + 2|}{x + 2} = \dfrac{-(x + 2)}{x + 2} = -1$.

49. $|-3| > -|-3|$ since $3 > -3$.

51. $-5 = -|5|$ since $-5 = -5$.

53. $-|-2| = -|2|$ since $-2 = -2$.

55. $d(126, 75) = |75 - 126| = 51$

57. $d\left(-\frac{5}{2}, 0\right) = \left|0 - \left(-\frac{5}{2}\right)\right| = \frac{5}{2}$

59. $d\left(\frac{16}{5}, \frac{112}{75}\right) = \left|\frac{112}{75} - \frac{16}{5}\right| = \frac{128}{75}$

61.

Budgeted Expense, b	*Actual Expense, a*	$\lvert a - b \rvert$	$0.05b$
\$112,700	\$113,356	\$656	$0.05(112{,}700) = \$5635$

Since \$656 < \$5635 but \$656 > \$500, the actual expense does not pass the "budget variance test."

63.

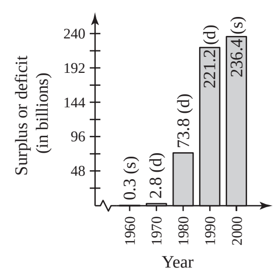
Budgeted Expense, b	*Actual Expense, a*	$\lvert a - b \rvert$	$0.05b$
\$37,640	\$37,335	\$305	$0.05(37{,}640) = \$1882$

Since \$305 < \$500 and \$305 < \$1882, the actual expense passes the "budget variance test."

65. (a)

Year	Expenditures (in billions)	Surplus or Deficit (in billions)
1960	\$92.2	$\lvert 92.5 - 92.2 \rvert = \$0.3$ surplus
1970	\$195.6	$\lvert 192.8 - 195.6 \rvert = \$2.8$ deficit
1980	\$590.9	$\lvert 517.1 - 590.9 \rvert = \$73.8$ deficit
1990	\$1253.2	$\lvert 1032.0 - 1253.2 \rvert = \$221.2$ deficit
2000	\$1788.8	$\lvert 2025.2 - 1788.8 \rvert = \$236.4$ surplus

(b)



67. $d(x, 5) = |x - 5|$ and $d(x, 5) \le 3$, thus $|x - 5| \le 3$.

69. $d(y, 0) = |y - 0| = |y|$ and $d(y, 0) \ge 6$, thus $|y| \ge 6$.

71. $d(326, 351) = |351 - 326| = 25$ miles

73. $7x + 4$

Terms: $7x, 4$

Coefficient: 7

75. $\sqrt{3}x^2 - 8x - 11$

Terms: $\sqrt{3}x^2, -8x, -11$

Coefficients: $\sqrt{3}, -8$

77. $4x^3 + \dfrac{x}{2} - 5$

Terms: $4x^3, \dfrac{x}{2}, -5$

Coefficients: $4, \dfrac{1}{2}$

79. $4x - 6$

(a) $4(-1) - 6 = -4 - 6 = -10$

(b) $4(0) - 6 = 0 - 6 = -6$

81. $x^2 - 3x + 4$

(a) $(-2)^2 - 3(-2) + 4 = 4 + 6 + 4 = 14$

(b) $(2)^2 - 3(2) + 4 = 4 - 6 + 4 = 2$

83. $\dfrac{x + 1}{x - 1}$

(a) $\dfrac{1 + 1}{1 - 1} = \dfrac{2}{0}$

Division by zero is undefined.

(b) $\dfrac{-1 + 1}{-1 - 1} = \dfrac{0}{-2} = 0$

85. $x + 9 = 9 + x$

Commutative Property of Addition

87. $\dfrac{1}{(h + 6)}(h + 6) = 1, h \neq -6$

Multiplicative Inverse Property

89. $2(x + 3) = 2x + 6$

Distributive Property

91. $1 \cdot (1 + x) = 1 + x$

Multiplicative Identity Property

93. $x + (y + 10) = (x + y) + 10$

Associative Property of Addition

95. $3(t - 4) = 3 \cdot t - 3 \cdot 4$

Distributive Property

97. $\frac{3}{16} + \frac{5}{16} = \frac{8}{16} = \frac{1}{2}$

99. $\frac{5}{8} - \frac{5}{12} + \frac{1}{6} = \frac{15}{24} - \frac{10}{24} + \frac{4}{24} = \frac{9}{24} = \frac{3}{8}$

101. $12 \div \dfrac{1}{4} = 12 \cdot \dfrac{4}{1} = 12 \cdot 4 = 48$

103. $\dfrac{2x}{3} - \dfrac{x}{4} = \dfrac{8x}{12} - \dfrac{3x}{12} = \dfrac{5x}{12}$

105. (a)

n	1	0.5	0.01	0.0001	0.000001
$5/n$	5	10	500	50,000	5,000,000

(b) The value of $5/n$ approaches infinity as n approaches 0.

107. False. If $a < b$, then $\dfrac{1}{a} > \dfrac{1}{b}$, where $a \neq b \neq 0$.

109. (a) $|u + v| \neq |u| + |v|$ if u is positive and v is negative or vice versa.

(b) $|u + v| \leq |u| + |v|$

They are equal when u and v have the same sign. If they differ in sign, $|u + v|$ is less than $|u| + |v|$.

111. The only even prime number is 2, because its factors are itself and 1.

113. (a) Since $A > 0$, $-A < 0$. The expression is negative.

(b) Since $B < A$, $B - A < 0$. The expression is negative.

115. Yes, if a is a negative number, then $-a$ is positive. Thus, $|a| = -a$ if a is negative.

Section P.2 Exponents and Radicals

- You should know the properties of exponents.
  - (a) $a^1 = a$
  - (b) $a^0 = 1, a \neq 0$
  - (c) $a^m a^n = a^{m+n}$
  - (d) $a^m/a^n = a^{m-n}, a \neq 0$
  - (e) $a^{-n} = 1/a^n = (1/a)^n, a \neq 0$
  - (f) $(a^m)^n = a^{mn}$
  - (g) $(ab)^n = a^n b^n$
  - (h) $(a/b)^n = a^n/b^n, b \neq 0$
  - (i) $(a/b)^{-n} = (b/a)^n, a \neq 0, b \neq 0$
  - (j) $|a^2| = |a|^2 = a^2$
- You should be able to write numbers in scientific notation, $c \times 10^n$, where $1 \le c < 10$ and n is an integer.
- You should be able to use your calculator to evaluate expressions involving exponents.
- You should know the properties of radicals.
  - (a) $\sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m, a > 0$
  - (b) $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$
  - (c) $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}}, b \neq 0$
  - (d) $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$
  - (e) $\left(\sqrt[n]{a}\right)^n = a$
  - (f) For n even, $\sqrt[n]{a^n} = |a|$. For n odd, $\sqrt[n]{a^n} = a$.
  - (g) $a^{1/n} = \sqrt[n]{a}$
  - (h) $a^{m/n} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}, a \ge 0$
- You should be able to simplify radicals.
  - (a) All possible factors have been removed from the radical sign.
  - (b) All fractions have radical-free denominators.
  - (c) The index for the radical has been reduced as far as possible.
- You should be able to use your calculator to evaluate radicals.

Vocabulary Check

1. exponent; base
2. scientific notation
3. square root
4. principal *n*th root
5. index; radicand
6. simplest form
7. conjugates
8. rationalizing
9. power; index

1. $8^5 = 8 \times 8 \times 8 \times 8 \times 8$

3. $(4.9)(4.9)(4.9)(4.9)(4.9)(4.9) = 4.9^6$

5. (a) $3^2 \cdot 3 = 3^3 = 27$

(b) $3 \cdot 3^3 = 3^4 = 81$

7. (a) $(3^3)^0 = 1$

(b) $-3^2 = -9$

9. (a) $\dfrac{3 \cdot 4^{-4}}{3^{-4} \cdot 4^{-1}} = 3^{1-(-4)} \cdot 4^{-4-(-1)} = 3^5 \cdot 4^{-3}$

$= \dfrac{3^5}{4^3} = \dfrac{243}{64}$

(b) $32(-2)^{-5} = \dfrac{32}{(-2)^5} = \dfrac{32}{-32} = -1$

11. (a) $2^{-1} + 3^{-1} = \dfrac{1}{2} + \dfrac{1}{3} = \dfrac{3}{6} + \dfrac{2}{6} = \dfrac{5}{6}$

(b) $(2^{-1})^{-2} = 2^{(-1)(-2)} = 2^2 = 4$

13. $(-4)^3(5^2) = (-64)(25) = -1600$

15. $\dfrac{3^6}{7^3} = \dfrac{729}{343} \approx 2.125$

17. When $x = 2$,
$-3x^3 = -3(2)^3 = -24.$

19. When $x = 10$,
$6x^0 = 6(10)^0 = 6(1) = 6.$

21. When $x = -3$,
$2x^3 = 2(-3)^3 = 2(-27) = -54.$

23. When $x = -\frac{1}{2}$,
$4x^2 = 4\left(-\frac{1}{2}\right)^2 = 4\left(\frac{1}{4}\right) = 1.$

25. (a) $(-5z)^3 = (-5)^3z^3 = -125z^3$
(b) $5x^4(x^2) = 5x^{4+2} = 5x^6$

27. (a) $6y^2(2y^0)^2 = 6y^2(2 \cdot 1)^2 = 6y^2(4) = 24y^2$
(b) $\dfrac{3x^5}{x^3} = 3x^{5-3} = 3x^2$

29. (a) $\dfrac{7x^2}{x^3} = 7x^{2-3} = 7x^{-1} = \dfrac{7}{x}$
(b) $\dfrac{12(x+y)^3}{9(x+y)} = \dfrac{4}{3}(x+y)^{3-1} = \dfrac{4}{3}(x+y)^2$

31. (a) $(x+5)^0 = 1,\ x \neq -5$
(b) $(2x^2)^{-2} = \dfrac{1}{(2x^2)^2} = \dfrac{1}{4x^4}$

33. (a) $(-2x^2)^3(4x^3)^{-1} = \dfrac{-8x^6}{4x^3} = -2x^3$
(b) $\left(\dfrac{x}{10}\right)^{-1} = \dfrac{10}{x}$

35. (a) $3^n \cdot 3^{2n} = 3^{n+2n} = 3^{3n}$
(b) $\left(\dfrac{a^{-2}}{b^{-2}}\right)\left(\dfrac{b}{a}\right)^3 = \left(\dfrac{b^2}{a^2}\right)\left(\dfrac{b^3}{a^3}\right) = \dfrac{b^5}{a^5}$

37. $57{,}300{,}000 = 5.73 \times 10^7$ square miles

39. $0.0000899 = 8.99 \times 10^{-5}$ gram per cubic centimeter

41. $4.568 \times 10^9 = 4{,}568{,}000{,}000$ ounces

43. $1.6022 \times 10^{-19} = 0.00000000000000000016022$ coulomb

45. (a) $\sqrt{25 \times 10^8} = 5 \times 10^4 = 50{,}000$
(b) $\sqrt[3]{8 \times 10^{15}} = 2 \times 10^5 = 200{,}000$

47. (a) $750\left(1 + \dfrac{0.11}{365}\right)^{800} \approx 954.448$
(b) $\dfrac{67{,}000{,}000 + 93{,}000{,}000}{0.0052} = 30{,}769{,}230{,}769.2$
$\approx 3.077 \times 10^{10}$

49. (a) $\sqrt{4.5 \times 10^9} \approx 67{,}082.039$
(b) $\sqrt[3]{6.3 \times 10^4} \approx 39.791$

51. (a) $\sqrt{9} = 3$
(b) $\sqrt[3]{\dfrac{27}{8}} = \dfrac{\sqrt[3]{27}}{\sqrt[3]{8}} = \dfrac{3}{2}$

53. (a) $32^{-3/5} = \dfrac{1}{32^{3/5}} = \dfrac{1}{(\sqrt[5]{32}\,)^3} = \dfrac{1}{(2)^3} = \dfrac{1}{8}$
(b) $\left(\dfrac{16}{81}\right)^{-3/4} = \left(\dfrac{81}{16}\right)^{3/4} = \left(\sqrt[4]{\dfrac{81}{16}}\right)^3 = \left(\dfrac{3}{2}\right)^3 = \dfrac{27}{8}$

55. (a) $\left(-\dfrac{1}{64}\right)^{-1/3} = (-64)^{1/3} = \sqrt[3]{-64} = -4$
(b) $\left(\dfrac{1}{\sqrt{32}}\right)^{-2/5} = \left(\sqrt{32}\right)^{2/5} = \sqrt[5]{\left(\sqrt{32}\right)^2} = \sqrt[5]{32} = 2$

57. (a) $\sqrt{57} \approx 7.550$
(b) $\sqrt[5]{-27^3} = (-27)^{3/5}$
≈ -7.225

59. (a) $(-12.4)^{-1.8} \approx -0.011$
(b) $\left(5\sqrt{3}\right)^{-2.5} \approx 0.005$

61. (a) $\left(\sqrt[3]{4}\right)^3 = 4^{3/3} = 4^1 = 4$
(b) $\sqrt[5]{96x^5} = \sqrt[5]{32x^5 \cdot 3}$
$= 2x\sqrt[5]{3}$
$= 2 \cdot 3^{1/5} \cdot x$

63. (a) $\sqrt{8} = \sqrt{4 \cdot 2} = \sqrt{4}\sqrt{2} = 2\sqrt{2}$

(b) $\sqrt[3]{24} = \sqrt[3]{8 \cdot 3} = \sqrt[3]{8}\,\sqrt[3]{3} = 2\sqrt[3]{3}$

65. (a) $\sqrt{72x^3} = \sqrt{36x^2 \cdot 2x} = 6x\sqrt{2x}$

(b) $\sqrt{\dfrac{18^2}{z^3}} = \dfrac{\sqrt{18^2}}{\sqrt{z^2 \cdot z}} = \dfrac{18}{z\sqrt{z}}$

67. (a) $\sqrt[3]{16x^5} = \sqrt[3]{8x^3 \cdot 2x^2} = 2x\sqrt[3]{2x^2}$

(b) $\sqrt{75x^2y^{-4}} = \sqrt{\dfrac{75x^2}{y^4}} = \dfrac{\sqrt{25x^2 \cdot 3}}{\sqrt{y^4}} = \dfrac{5|x|\sqrt{3}}{y^2}$

69. (a) $2\sqrt{50} + 12\sqrt{8} = 2\sqrt{25 \cdot 2} + 12\sqrt{4 \cdot 2} = 2(5\sqrt{2}) + 12(2\sqrt{2}) = 10\sqrt{2} + 24\sqrt{2} = 34\sqrt{2}$

(b) $10\sqrt{32} - 6\sqrt{18} = 10\sqrt{16 \cdot 2} - 6\sqrt{9 \cdot 2} = 10(4\sqrt{2}) - 6(3\sqrt{2}) = 40\sqrt{2} - 18\sqrt{2} = 22\sqrt{2}$

71. (a) $5\sqrt{x} - 3\sqrt{x} = 2\sqrt{x}$

(b) $-2\sqrt{9y} + 10\sqrt{y} = -2(3\sqrt{y}) + 10\sqrt{y} = -6\sqrt{y} + 10\sqrt{y} = 4\sqrt{y}$

73. (a) $3\sqrt{x+1} + 10\sqrt{x+1} = 13\sqrt{x+1}$

(b) $7\sqrt{80x} - 2\sqrt{125x} = 7\sqrt{16 \cdot 5x} - 2\sqrt{25 \cdot 5x} = 7(4\sqrt{5x}) - 2(5\sqrt{5x}) = 28\sqrt{5x} - 10\sqrt{5x} = 18\sqrt{5x}$

75. $\sqrt{5} + \sqrt{3} \approx 3.968$ and

$\sqrt{5+3} = \sqrt{8} \approx 2.828$

Thus, $\sqrt{5} + \sqrt{3} > \sqrt{5+3}$.

77. $\sqrt{3^2 + 2^2} = \sqrt{9+4} = \sqrt{13} \approx 3.606$

Thus, $5 > \sqrt{3^2 + 2^2}$.

79. $\dfrac{1}{\sqrt{3}} = \dfrac{1}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{\sqrt{3}}{3}$

81. $\dfrac{2}{5-\sqrt{3}} = \dfrac{2}{5-\sqrt{3}} \cdot \dfrac{5+\sqrt{3}}{5+\sqrt{3}} = \dfrac{2(5+\sqrt{3})}{5^2 - (\sqrt{3})^2} = \dfrac{2(5+\sqrt{3})}{25-3} = \dfrac{2(5+\sqrt{3})}{22} = \dfrac{5+\sqrt{3}}{11}$

83. $\dfrac{\sqrt{8}}{2} = \dfrac{\sqrt{4 \cdot 2}}{2} = \dfrac{2\sqrt{2}}{2} = \dfrac{\sqrt{2}}{1} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{2}{\sqrt{2}}$

85. $\dfrac{\sqrt{5}+\sqrt{3}}{3} = \dfrac{\sqrt{5}+\sqrt{3}}{3} \cdot \dfrac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}} = \dfrac{5-3}{3(\sqrt{5}-\sqrt{3})} = \dfrac{2}{3(\sqrt{5}-\sqrt{3})}$

	Radical Form	*Rational Exponent Form*
87.	$\sqrt{9} = 3$, Given	$9^{1/2} = 3$, Answer
89.	$\sqrt[5]{32} = 2$, Answer	$32^{1/5} = 2$, Given
91.	$\sqrt[3]{-216} = -6$, Given	$(-216)^{1/3} = -6$, Answer
93.	$\sqrt[4]{81^3} = 27$, Given	$81^{3/4} = 27$, Answer

95. $\dfrac{(2x^2)^{3/2}}{2^{1/2}x^4} = \dfrac{2^{3/2}(x^2)^{3/2}}{2^{1/2}x^4} = \dfrac{2^{3/2}x^3}{2^{1/2}x^4} = 2^{3/2-1/2}x^{3-4} = 2^1x^{-1} = \dfrac{2}{x}$

97. $\dfrac{x^{-3} \cdot x^{1/2}}{x^{3/2} \cdot x^{-1}} = \dfrac{x^{1/2} \cdot x^1}{x^{3/2} \cdot x^3} = x^{1/2+1-3/2-3} = x^{-3} = \dfrac{1}{x^3},\ x > 0$

99. (a) $\sqrt[4]{3^2} = 3^{2/4} = 3^{1/2} = \sqrt{3}$

(b) $\sqrt[6]{(x+1)^4} = (x+1)^{4/6} = (x+1)^{2/3} = \sqrt[3]{(x+1)^2}$

101. (a) $\sqrt{\sqrt{32}} = (32^{1/2})^{1/2}$

$= 32^{1/4} = \sqrt[4]{32} = \sqrt[4]{16 \cdot 2} = 2\sqrt[4]{2}$

(b) $\sqrt{\sqrt[4]{2x}} = ((2x)^{1/4})^{1/2} = (2x)^{1/8} = \sqrt[8]{2x}$

103. $T = 2\pi\sqrt{\dfrac{2}{32}}$

$= 2\pi\sqrt{\dfrac{1}{16}}$

$= 2\pi\left(\dfrac{1}{4}\right)$

$= \dfrac{\pi}{2} \approx 1.57$ seconds

105. $t = 0.03[12^{5/2} - (12 - h)^{5/2}],\ 0 \le h \le 12$

(a)

h (in centimeters)	t (in seconds)
0	0
1	2.93
2	5.48
3	7.67
4	9.53
5	11.08
6	12.32
7	13.29
8	14.00
9	14.50
10	14.80
11	14.93
12	14.96

(b) As h approaches 12, t approaches

$0.03(12^{5/2}) = 8.64\sqrt{3} \approx 14.96$ seconds.

107. True. When dividing variables, you subtract exponents.

109. $1 = \dfrac{a^m}{a^m} = a^{m-m} = a^0,\ a \ne 0$

111. When any positive integer is squared, the units digit is 0, 1, 4, 5, 6, or 9. Therefore, $\sqrt{5233}$ is not an integer.

Section P.3 Polynomials and Special Products

- Given a polynomial in x, $a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0$, where $a_n \neq 0$, and n is a nonnegative integer, you should be able to identify the following.
  (a) Degree: n
  (b) Terms: $a_nx^n, a_{n-1}x^{n-1}, \ldots, a_1x, a_0$
  (c) Coefficients: $a_n, a_{n-1}, \ldots, a_1$
  (d) Leading coefficient: a_n
  (e) Constant term: a_0
- You should be able to add and subtract polynomials.
- You should be able to multiply polynomials by either
  (a) The Distributive Properties
  (b) The Vertical Method.
- You should know the special binomial products.
  (a) $(ax + b)(cx + d) = acx^2 + adx + bcx + bd$ FOIL
  $= acx^2 + (ad + bc)x + bd$
  (b) $(u + v)^2 = u^2 + 2uv + v^2$
  $(u - v)^2 = u^2 - 2uv + v^2$
  (c) $(u + v)(u - v) = u^2 - v^2$
  (d) $(u + v)^3 = u^3 + 3u^2v + 3uv^2 + v^3$
  $(u - v)^3 = u^3 - 3u^2v + 3uv^2 - v^3$

Vocabulary Check

1. n; a_n; a_0

2. descending

3. monomial; binomial; trinomial

4. like terms

5. First terms; Outer terms; Inner terms; Last terms

6. (c) the sum and difference of same terms

7. (a) a binomial sum squared

8. (b) a binomial difference squared

---

1. (d) 12 is a polynomial of degree zero.

3. (b) $1 - 2x^3 = -2x^3 + 1$ is a binomial with leading coefficient -2.

5. (f) $\frac{2}{3}x^4 + x^2 + 10$ is a trinomial with leading coefficient $\frac{2}{3}$.

7. $-2x^3$; $-2x^3 + 5$; $-2x^3 + 4x^2 - 3x + 20$, etc. (Answers will vary.)

9. $-15x^4 + 1$; $-3x^4 + 7x^2$; $-5x^4 - 6x$, etc. (Answers will vary.)

11. (a) Standard form: $-\frac{1}{2}x^5 + 14x$
(b) Degree: 5
Leading coefficient: $-\frac{1}{2}$
(c) Binomial

13. (a) Standard form: $-3x^4 + 2x^2 - 5$
(b) Degree: 4
Leading coefficient: -3
(c) Trinomial

15. (a) Standard form: $x^5 - 1$
(b) Degree: 5
Leading coefficient: 1
(c) Binomial

17. (a) Standard form: 3
(b) Degree: 0
Leading coefficient: 3
(c) Monomial

19. (a) Standard form: $-4x^5 + 6x^4 + 1$
(b) Degree: 5
Leading coefficient: -4
(c) Trinomial

21. (a) Standard form: $4x^3y$
(b) Degree: 4 (add the exponents on x and y)
Leading coefficient: 4
(c) Monomial

23. $2x - 3x^3 + 8$ *is* a polynomial.

Standard form: $-3x^3 + 2x + 8$

25. $\dfrac{3x + 4}{x} = 3 + \dfrac{4}{x} = 3 + 4x^{-1}$ is *not* a polynomial because it includes a term with a negative exponent.

27. $y^2 - y^4 + y^3$ *is* a polynomial.

Standard form: $-y^4 + y^3 + y^2$

29. $(6x + 5) - (8x + 15) = 6x + 5 - 8x - 15$

$= (6x - 8x) + (5 - 15)$

$= -2x - 10$

31. $-(x^3 - 2) + (4x^3 - 2x) = -x^3 + 2 + 4x^3 - 2x$

$= (4x^3 - x^3) - 2x + 2$

$= 3x^3 - 2x + 2$

33. $(15x^2 - 6) - (-8.3x^3 - 14.7x^2 - 17) = 15x^2 - 6 + 8.3x^3 + 14.7x^2 + 17$

$= 8.3x^3 + (15x^2 + 14.7x^2) + (-6 + 17)$

$= 8.3x^3 + 29.7x^2 + 11$

35. $5z - [3z - (10z + 8)] = 5z - (3z - 10z - 8)$

$= 5z - 3z + 10z + 8$

$= (5z - 3z + 10z) + 8$

$= 12z + 8$

37. $3x(x^2 - 2x + 1) = 3x(x^2) + 3x(-2x) + 3x(1)$

$= 3x^3 - 6x^2 + 3x$

39. $-5z(3z - 1) = -5z(3z) + (-5z)(-1)$

$= -15z^2 + 5z$

41. $(1 - x^3)(4x) = 1(4x) - x^3(4x)$

$= 4x - 4x^4$

$= -4x^4 + 4x$

43. $(2.5x^2 + 3)(3x) = (2.5x^2)(3x) + (3)(3x)$

$= 7.5x^3 + 9x$

45. $-4x\left(\frac{1}{8}x + 3\right) = (-4x)\left(\frac{1}{8}x\right) + (-4x)(3)$

$= -\frac{1}{2}x^2 - 12x$

47. $(7x^3 - 2x^2 + 8) + (-3x^3 - 4) = (7x^3 - 3x^3) + (-2x^2) + (8 - 4)$

$= 4x^3 - 2x^2 + 4$

49. $(5x^2 - 3x + 8) - (x - 3) = 5x^2 - 3x + 8 - x + 3$

$= 5x^2 + (-3x - x) + (8 + 3)$

$= 5x^2 - 4x + 11$

51. Multiply:

$$\begin{array}{r} -6x^2 + 15x - 4 \\ 5x + 3 \\ \hline -30x^3 + 75x^2 - 20x \\ -18x^2 + 45x - 12 \\ \hline -30x^3 + 57x^2 + 25x - 12 \end{array}$$

53. Multiply:

$$\begin{array}{r} x^2 - x - 4 \\ x^2 + 9 \\ \hline x^4 - x^3 - 4x^2 \qquad\qquad \\ 9x^2 - 9x - 36 \\ \hline x^4 - x^3 + 5x^2 - 9x - 36 \end{array}$$

55. $(x + 3)(x + 4) = x^2 + 4x + 3x + 12$ FOIL

$= x^2 + 7x + 12$

57. $(3x - 5)(2x + 1) = 6x^2 + 3x - 10x - 5$ FOIL

$= 6x^2 - 7x - 5$

59. Multiply:

$$\begin{array}{r} x^2 - x + 1 \\ x^2 + x + 1 \\ \hline x^4 - x^3 + x^2 \qquad\qquad \\ x^3 - x^2 + x \qquad \\ x^2 - x + 1 \\ \hline x^4 - 0x^3 + x^2 + 0x + 1 \end{array} = x^4 + x^2 + 1$$

61. $(x + 10)(x - 10) = x^2 - 10^2 = x^2 - 100$

63. $(x + 2y)(x - 2y) = x^2 - (2y)^2 = x^2 - 4y^2$

65. $(2x + 3)^2 = (2x)^2 + 2(2x)(3) + 3^2$
$= 4x^2 + 12x + 9$

67. $(2x - 5y)^2 = (2x)^2 - 2(2x)(5y) + (5y)^2$
$= 4x^2 - 20xy + 25y^2$

69. $(x + 1)^3 = x^3 + 3x^2(1) + 3x(1^2) + 1^3$
$= x^3 + 3x^2 + 3x + 1$

71. $(2x - y)^3 = (2x)^3 - 3(2x)^2y + 3(2x)y^2 - y^3$
$= 8x^3 - 12x^2y + 6xy^2 - y^3$

73. $(4x^3 - 3)^2 = (4x^3)^2 - 2(4x^3)(3) + (3)^2$
$= 16x^6 - 24x^3 + 9$

75. $[(m - 3) + n][(m - 3) - n] = (m - 3)^2 - n^2$
$= m^2 - 6m + 9 - n^2$
$= m^2 - n^2 - 6m + 9$

77. $[(x - 3) + y]^2 = (x - 3)^2 + 2y(x - 3) + y^2$
$= x^2 - 6x + 9 + 2xy - 6y + y^2$
$= x^2 + 2xy + y^2 - 6x - 6y + 9$

79. $(2r^2 - 5)(2r^2 + 5) = (2r^2)^2 - 5^2 = 4r^4 - 25$

81. $\left(\frac{1}{2}x - 3\right)^2 = \left(\frac{1}{2}x\right)^2 - 2\left(\frac{1}{2}x\right)(3) + 3^2$
$= \frac{1}{4}x^2 - 3x + 9$

83. $\left(\frac{1}{3}x - 2\right)\left(\frac{1}{3}x + 2\right) = \left(\frac{1}{3}x\right)^2 - (2)^2$
$= \frac{1}{9}x^2 - 4$

85. $(1.2x + 3)^2 = (1.2x)^2 + 2(1.2x)(3) + 3^2$
$= 1.44x^2 + 7.2x + 9$

87. $(1.5x - 4)(1.5x + 4) = (1.5x)^2 - 4^2$
$= 2.25x^2 - 16$

89. $5x(x + 1) - 3x(x + 1) = 2x(x + 1)$
$= 2x^2 + 2x$

91. $(u + 2)(u - 2)(u^2 + 4) = (u^2 - 4)(u^2 + 4)$
$= u^4 - 16$

93. $\left(\sqrt{x} + \sqrt{y}\right)\left(\sqrt{x} - \sqrt{y}\right) = \left(\sqrt{x}\right)^2 - \left(\sqrt{y}\right)^2$
$= x - y$

95. $\left(x - \sqrt{5}\right)^2 = x^2 - 2(x)\left(\sqrt{5}\right) + \left(\sqrt{5}\right)^2$
$= x^2 - 2\sqrt{5}x + 5$

97. (a) Profit = Revenue − Cost
Profit $= 95x - (73x + 25{,}000)$
$= 95x - 73x - 25{,}000 = 22x - 25{,}000$

(b) For $x = 5000$:
Profit $= 22(5000) - 25{,}000$
$= 110{,}000 - 25{,}000 = \$85{,}000$

99. (a) $500(1 + r)^2 = 500(r + 1)^2 = 500(r^2 + 2r + 1)$

$= 500r^2 + 1000r + 500$

(b)

r	$2\frac{1}{2}\%$	3%	4%	$4\frac{1}{2}\%$	5%
$500(1 + r)^2$	\$525.31	\$530.45	\$540.80	\$546.01	\$551.25

(c) As r increases, the amount increases.

101. (a) $V = l \cdot w \cdot h = (26 - 2x)(18 - 2x)(x)$

$= 2(13 - x)(2)(9 - x)(x)$

$= 4x(-1)(x - 13)(-1)(x - 9)$

$= 4x(x - 13)(x - 9)$

$= 4x^3 - 88x^2 + 468x$

(b)

x (cm)	1	2	3
V (cm^3)	384	616	720

103. (a) Area of shaded region = Area of outer rectangle − Area of inner rectangle

$A = 2x(2x + 6) - x(x + 4)$

$= 4x^2 + 12x - x^2 - 4x$

$= 3x^2 + 8x$

(b) Area of shaded region = Area of outer triangle − Area of inner triangle

$A = \frac{1}{2}(9x)(12x) - \frac{1}{2}(6x)(8x)$

$= 54x^2 - 24x^2$

$= 30x^2$

(c) Area of shaded region = Area of outer triangle − Area of inner triangle

$A = \frac{1}{2}(x + 2)(5x) - \frac{1}{2}(x + 1)(3x)$

$= \frac{5}{2}x^2 + 5x - \frac{3}{2}x^2 - \frac{3}{2}x$

$= x^2 + \frac{7}{2}x$

OR

Area of shaded region = Area of a trapezoid

$A = \frac{1}{2}(2x)[(x + 2) + (x + 1)]$

$= x(2x + 3)$

$= 2x^2 + 3x$

Note: $x = \frac{1}{2}$ is the only x-value yielding a triangle with the given dimensions.

(d) Area of shaded region = Area of rectangle − Area of triangle

$A = (3x + 10)(x + 6) - \frac{1}{2}(3x + 10)(x + 6)$

$= \frac{1}{2}(3x + 10)(x + 6)$

$= \frac{1}{2}(3x^2 + 28x + 60)$

$= \frac{3}{2}x^2 + 14x + 30$

105. Area = length × width

$= (2x + 14)(22)$

$= (2x)(22) + (14)(22)$

$= 44x + 308$

107. (a) Estimates will vary. Actual safe loads for $x = 12$:

$S_6 = (0.06(12)^2 - 2.42(12) + 38.71)^2 = 335.2561$ (using a calculator)

$S_8 = (0.08(12)^2 - 3.30(12) + 51.93)^2 = 568.8225$ (using a calculator)

Difference in safe loads $= 568.8225 - 335.2561 = 233.5664$ pounds

(b) The difference in safe loads decreases in magnitude as the span increases.

109. $(x + 1)(x + 4) = x^2 + 4x + x + 4$

This illustrates the Distributive Property.

111. False.

$(4x^2 + 1)(3x + 1) = 12x^3 + 4x^2 + 3x + 1$

113. Since $x^m x^n = x^{m+n}$, the degree of the product is $m + n$.

115. $(x - 3)^2 \neq x^2 + 9$

The student did not remember the middle term when squaring the binomial.
The correct method for squaring this binomial is:

$(x - 3)^2 = (x)^2 - 2(x)(3) + (3)^2 = x^2 - 6x + 9$

117. No; $(x^2 + 1) + (-x^2 + 3) = 4$,
which is not a second-degree polynomial.

119. $(x + y)^2 \neq x^2 + y^2$

Let $x = 3$ and $y = 4$.

$(3 + 4)^2 = (7)^2 = 49$

$3^2 + 4^2 = 9 + 16 = 25$

Not Equal

If either x or y is zero, then $(x + y)^2$ would equal $x^2 + y^2$.

Section P.4 Factoring Polynomials

- You should be able to factor out all common factors, the first step in factoring.
- You should be able to factor the following special polynomial forms.

  (a) $u^2 - v^2 = (u + v)(u - v)$

  (b) $u^2 + 2uv + v^2 = (u + v)^2$
  $u^2 - 2uv + v^2 = (u - v)^2$

  (c) $u^3 + v^3 = (u + v)(u^2 - uv + v^2)$
  $u^3 - v^3 = (u - v)(u^2 + uv + v^2)$
- You should be able to factor trinomials with binomial factors.
- You should be able to factor by grouping.
- You should be able to factor some trinomials by grouping.

Vocabulary Check

1. factoring

2. irreducible

3. completely factored

4. factoring by grouping

5. (a) **ii** Difference of two squares

(b) **iii** Difference of two cubes

(c) **i** Perfect square trinomial

1. $90 = 2 \cdot 3 \cdot 3 \cdot 5$

$300 = 2 \cdot 2 \cdot 3 \cdot 5 \cdot 5$

Greatest common factor: $2 \cdot 3 \cdot 5 = 30$

3. $12x^2y^3 = 2 \cdot 2 \cdot 3 \cdot x \cdot x \cdot y \cdot y \cdot y$

$18x^2y = 2 \cdot 3 \cdot 3 \cdot x \cdot x \cdot y$

$24x^3y^2 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot x \cdot x \cdot x \cdot y \cdot y$

Greatest common factor: $2 \cdot 3 \cdot x \cdot x \cdot y = 6x^2y$

5. $3x + 6 = 3(x + 2)$

7. $2x^3 - 6x = 2x(x^2 - 3)$

9. $x(x - 1) + 6(x - 1) = (x - 1)(x + 6)$

11. $(x + 3)^2 - 4(x + 3) = (x + 3)[(x + 3) - 4]$
$= (x + 3)(x - 1)$

13. $\frac{1}{2}x + 4 = \frac{1}{2}x + \frac{8}{2}$
$= \frac{1}{2}(x + 8)$

15. $\frac{1}{2}x^3 + 2x^2 - 5x = \frac{1}{2}x^3 + \frac{4}{2}x^2 - \frac{10}{2}x$
$= \frac{1}{2}x(x^2 + 4x - 10)$

17. $\frac{2}{3}x(x - 3) - 4(x - 3) = \frac{2}{3}x(x - 3) - \frac{12}{3}(x - 3)$
$= \frac{2}{3}(x - 3)(x - 6)$

19. $x^2 - 81 = x^2 - 9^2$
$= (x + 9)(x - 9)$

21. $32y^2 - 18 = 2(16y^2 - 9)$
$= 2[(4y)^2 - 3^2]$
$= 2(4y + 3)(4y - 3)$

23. $16x^2 - \frac{1}{9} = (4x)^2 - \left(\frac{1}{3}\right)^2$
$= \left(4x + \frac{1}{3}\right)\left(4x - \frac{1}{3}\right)$

25. $(x - 1)^2 - 4 = (x - 1)^2 - (2)^2$
$= [(x - 1) + 2][(x - 1) - 2]$
$= (x + 1)(x - 3)$

27. $9u^2 - 4v^2 = (3u)^2 - (2v)^2$
$= (3u + 2v)(3u - 2v)$

29. $x^2 - 4x + 4 = x^2 - 2(2)x + 2^2$
$= (x - 2)^2$

31. $4t^2 + 4t + 1 = (2t)^2 + 2(2t)(1) + 1^2$
$= (2t + 1)^2$

33. $25y^2 - 10y + 1 = (5y)^2 - 2(5y)(1) + 1^2$
$= (5y - 1)^2$

35. $9u^2 + 24uv + 16v^2 = (3u)^2 + 2(3u)(4v) + (4v)^2$
$= (3u + 4v)^2$

37. $x^2 - \frac{4}{3}x + \frac{4}{9} = x^2 - 2(x)\left(\frac{2}{3}\right) + \left(\frac{2}{3}\right)^2$
$= \left(x - \frac{2}{3}\right)^2$

39. $x^3 - 8 = x^3 - 2^3$
$= (x - 2)(x^2 + 2x + 4)$

41. $y^3 + 64 = y^3 + 4^3$
$= (y + 4)(y^2 - 4y + 16)$

43. $8t^3 - 1 = (2t)^3 - 1^3$
$= (2t - 1)(4t^2 + 2t + 1)$

45. $u^3 + 27v^3 = u^3 + (3v)^3$
$= (u + 3v)(u^2 - 3uv + 9v^2)$

47. $25 - 5x^2 = -5(-5 + x^2)$
$= -5(x^2 - 5)$

49. $-2t^3 + 4t + 6 = -2(t^3 - 2t - 3)$

51. $x^2 + x - 2 = (x + 2)(x - 1)$

53. $s^2 - 5s + 6 = (s - 3)(s - 2)$

55. $20 - y - y^2 = -(y^2 + y - 20)$
$= -(y + 5)(y - 4)$

57. $x^2 - 30x + 200 = (x - 20)(x - 10)$

59. $3x^2 - 5x + 2 = (3x - 2)(x - 1)$

61. $5x^2 + 26x + 5 = (5x + 1)(x + 5)$

63. $-9z^2 + 3z + 2 = -(9z^2 - 3z - 2)$
$= -(3z - 2)(3z + 1)$

65. $x^3 - x^2 + 2x - 2 = x^2(x - 1) + 2(x - 1)$
$= (x - 1)(x^2 + 2)$

67. $2x^3 - x^2 - 6x + 3 = x^2(2x - 1) - 3(2x - 1)$
$= (2x - 1)(x^2 - 3)$

69. $6 + 2x - 3x^3 - x^4 = 2(3 + x) - x^3(3 + x)$
$= (3 + x)(2 - x^3)$

71. $6x^3 - 2x + 3x^2 - 1 = 2x(3x^2 - 1) + 1(3x^2 - 1)$
$= (3x^2 - 1)(2x + 1)$

73. $a \cdot c = (3)(8) = 24$. Rewrite the middle term, $10x = 6x + 4x$, since $(6)(4) = 24$ and $6 + 4 = 10$.

$3x^2 + 10x + 8 = 3x^2 + 6x + 4x + 8$
$= 3x(x + 2) + 4(x + 2)$
$= (x + 2)(3x + 4)$

75. $a \cdot c = (6)(-2) = -12$. Rewrite the middle term, $x = 4x - 3x$, since $4(-3) = -12$ and $4 + (-3) = 1$.

$6x^2 + x - 2 = 6x^2 + 4x - 3x - 2$
$= 2x(3x + 2) - 1(3x + 2)$
$= (2x - 1)(3x + 2)$

77. $a \cdot c = (15)(2) = 30$. Rewrite the middle term, $-11x = -6x - 5x$, since $(-6)(-5) = 30$ and $(-6) + (-5) = -11$.

$15x^2 - 11x + 2 = 15x^2 - 6x - 5x + 2$
$= 3x(5x - 2) - 1(5x - 2)$
$= (3x - 1)(5x - 2)$

79. $6x^2 - 54 = 6(x^2 - 9)$
$= 6(x + 3)(x - 3)$

81. $x^3 - 4x^2 = x^2(x - 4)$

83. $x^2 - 2x + 1 = (x - 1)^2$

85. $1 - 4x + 4x^2 = (1 - 2x)^2$

87. $2x^2 + 4x - 2x^3 = -2x(-x - 2 + x^2)$
$= -2x(x^2 - x - 2)$
$= -2x(x + 1)(x - 2)$

89. $9x^2 + 10x + 1 = (9x + 1)(x + 1)$

91. $\frac{1}{81}x^2 + \frac{2}{9}x - 8 = \frac{1}{81}x^2 + \frac{18}{81}x - \frac{648}{81}$
$= \frac{1}{81}(x^2 + 18x - 648)$
$= \frac{1}{81}(x + 36)(x - 18)$

93. $3x^3 + x^2 + 15x + 5 = x^2(3x + 1) + 5(3x + 1)$
$= (3x + 1)(x^2 + 5)$

95. $x^4 - 4x^3 + x^2 - 4x = x(x^3 - 4x^2 + x - 4)$
$= x[x^2(x - 4) + (x - 4)]$
$= x(x - 4)(x^2 + 1)$

97. $\frac{1}{4}x^3 + 3x^2 + \frac{3}{4}x + 9 = \frac{1}{4}x^3 + \frac{12}{4}x^2 + \frac{3}{4}x + \frac{36}{4}$
$= \frac{1}{4}(x^3 + 12x^2 + 3x + 36)$
$= \frac{1}{4}[x^2(x + 12) + 3(x + 12)]$
$= \frac{1}{4}(x + 12)(x^2 + 3)$

99. $(t - 1)^2 - 49 = (t - 1)^2 - (7)^2$
$= [(t - 1) + 7][(t - 1) - 7]$
$= (t + 6)(t - 8)$

101. $(x^2 + 8)^2 - 36x^2 = (x^2 + 8)^2 - (6x)^2$
$= [(x^2 + 8) - 6x][(x^2 + 8) + 6x]$
$= (x^2 - 6x + 8)(x^2 + 6x + 8)$
$= (x - 4)(x - 2)(x + 4)(x + 2)$

103. $5x^3 + 40 = 5(x^3 + 8)$

$= 5(x^3 + 2^3)$

$= 5(x + 2)(x^2 - 2x + 4)$

105. $5(3 - 4x)^2 - 8(3 - 4x)(5x - 1) = (3 - 4x)[5(3 - 4x) - 8(5x - 1)]$

$= (3 - 4x)[15 - 20x - 40x + 8]$

$= (3 - 4x)(23 - 60x)$

107. $7(3x + 2)^2(1 - x)^2 + (3x + 2)(1 - x)^3 = (3x + 2)(1 - x)^2[7(3x + 2) + (1 - x)]$

$= (3x + 2)(1 - x)^2(21x + 14 + 1 - x)$

$= (3x + 2)(1 - x)^2(20x + 15)$

$= 5(3x + 2)(1 - x)^2(4x + 3)$

109. $3(x - 2)^2(x + 1)^4 + (x - 2)^3(4)(x + 1)^3 = (x - 2)^2(x + 1)^3[3(x + 1) + 4(x - 2)]$

$= (x - 2)^2(x + 1)^3(3x + 3 + 4x - 8)$

$= (x - 2)^2(x + 1)^3(7x - 5)$

111. $5(x^6 + 1)^4(6x^5)(3x + 2)^3 + 3(3x + 2)^2(3)(x^6 + 1)^5 = 3(x^6 + 1)^4(3x + 2)^2[10x^5(3x + 2) + 3(x^6 + 1)]$

$= 3(x^6 + 1)^4(3x + 2)^2(30x^6 + 20x^5 + 3x^6 + 3)$

$= 3(x^6 + 1)^4(3x + 2)^2(33x^6 + 20x^5 + 3)$

$= 3[(x^2)^3 + 1]^4(3x + 2)^2(33x^6 + 20x^5 + 3)$

$= 3[(x^2 + 1)(x^4 - x^2 + 1)]^4(3x + 2)^2(33x^6 + 20x^5 + 3)$

$= 3(x^2 + 1)^4(x^4 - x^2 + 1)^4(3x + 2)^2(33x^6 + 20x^5 + 3)$

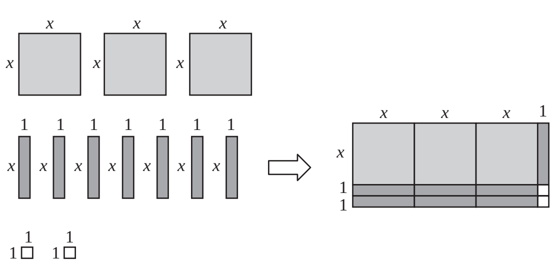
113. $a^2 - b^2 = (a + b)(a - b)$

Matches model (b).

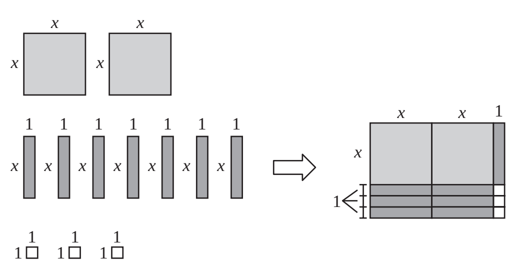
115. $a^2 + 2a + 1 = (a + 1)^2$

Matches model (a).

117. $3x^2 + 7x + 2 = (3x + 1)(x + 2)$



119. $2x^2 + 7x + 3 = (2x + 1)(x + 3)$



121. $A = \pi(r + 2)^2 - \pi r^2$

$= \pi[(r + 2)^2 - r^2]$

$= \pi[r^2 + 4r + 4 - r^2]$

$= \pi(4r + 4)$

$= 4\pi(r + 1)$

123. $A = 8(18) - 4x^2$

$= 4(36 - x^2)$

$= 4(6 - x)(6 + x)$

125. For $x^2 + bx - 15$ to be factorable, b must equal $m + n$ where $mn = -15$.

Factors of -15	Sum of factors
$(15)(-1)$	$15 + (-1) = 14$
$(-15)(1)$	$-15 + 1 = -14$
$(3)(-5)$	$3 + (-5) = -2$
$(-3)(5)$	$-3 + 5 = 2$

The possible b-values are 14, -14, -2, or 2.

127. For $x^2 + bx - 12$ to be factorable, b must equal $m + n$ where $mn = -12$.

Factors of -12	Sum of factors
$(12)(-1)$	$12 + (-1) = 11$
$(-12)(1)$	$-12 + 1 = -11$
$(2)(-6)$	$2 + (-6) = -4$
$(-2)(6)$	$-2 + 6 = 4$
$(3)(-4)$	$3 + (-4) = -1$
$(-3)(4)$	$-3 + 4 = 1$

The possible b-values are 11, -11, -4, 4, -1, 1.

129. For $2x^2 + 5x + c$ to be factorable, the factors of $2c$ must add up to 5.

Possible c-values	$2c$	Factors of $2c$ that add up to 5
2	4	$(1)(4) = 4$ and $1 + 4 = 5$
3	6	$(2)(3) = 6$ and $2 + 3 = 5$
-3	-6	$(6)(-1) = -6$ and $6 + (-1) = 5$
-7	-14	$(7)(-2) = -14$ and $7 + (-2) = 5$
-12	-24	$(8)(-3) = -24$ and $8 + (-3) = 5$

These are a few possible c-values. There are *many* correct answers.

If $c = 2: 2x^2 + 5x + 2 = (2x + 1)(x + 2)$

If $c = 3: 2x^2 + 5x + 3 = (2x + 3)(x + 1)$

If $c = -3: 2x^2 + 5x - 3 = (2x - 1)(x + 3)$

If $c = -7: 2x^2 + 5x - 7 = (2x + 7)(x - 1)$

If $c = -12: 2x^2 + 5x - 12 = (2x - 3)(x + 4)$

131. For $3x^2 - x + c$ to be factorable, the factors of $3c$ must add up to -1.

Possible c-values	$3c$	Factors of $3c$ must add up to -1
-2	-6	$(2)(-3) = -6$ and $2 + (-3) = -1$
-4	-12	$(3)(-4) = -12$ and $3 + (-4) = -1$
-10	-30	$(5)(-6) = -30$ and $5 + (-6) = -1$

These are a few possible c-values. There are *many* correct answers.

If $c = -2: 3x^2 - x - 2 = (3x + 2)(x - 1)$

If $c = -4: 3x^2 - x - 4 = (3x - 4)(x + 1)$

If $c = -10: 3x^2 - x - 10 = (3x + 5)(x - 2)$

133. $9x^2 - 9x - 54 = 9(x^2 - x - 6) = 9(x + 2)(x - 3)$

The error in the problem in the book was that 3 was factored out of the first binomial but not out of the second binomial.

$(3x + 6)(3x - 9) = 3(x + 2)(3)(x - 3) = 9(x + 2)(x - 3)$

135. $kQx - kx^2 = kx(Q - x)$

137. True. $a^2 - b^2 = (a + b)(a - b)$

139. $x^{2n} - y^{2n} = (x^n)^2 - (y^n)^2$
$= (x^n + y^n)(x^n - y^n)$

141. $x^{3n} - y^{2n} = (x^n)^3 - (y^n)^2 = x^{3n} - y^{2n}$ is completely factored.

143. Answers will vary. Some examples:
$x^2 - 3;\ x^2 + x + 1;\ x^2 + 16$

Section P.5 Rational Expressions

- You should be able to find the domain of a rational expression.
- You should know that a rational expression is the quotient of two polynomials.
- You should be able to simplify rational expressions by reducing them to lowest terms. This may involve factoring both the numerator and the denominator.
- You should be able to add, subtract, multiply, and divide rational expressions.
- You should be able to simplify complex fractions.
- You should be able to simplify expressions with negative or fraction exponents.

Vocabulary Check

1. domain **2.** rational expression **3.** complex
4. smaller **5.** equivalent **6.** difference quotient

1. The domain of the polynomial $3x^2 - 4x + 7$ is the set of all real numbers.

3. The domain of the polynomial $4x^3 + 3$, $x \geq 0$ is the set of non-negative real numbers, since the polynomial is restricted to that set.

5. The domain of $1/(x - 2)$ is the set of all real numbers x such that $x \neq 2$.

7. The domain of $\sqrt{x + 1}$ is the set of all real numbers x such that $x \geq -1$.

9. $\dfrac{5}{2x} = \dfrac{5(3x)}{(2x)(3x)} = \dfrac{5(3x)}{6x^2},\quad x \neq 0$

The missing factor is $3x$, $x \neq 0$.

11. $\dfrac{15x^2}{10x} = \dfrac{5x(3x)}{5x(2)} = \dfrac{3x}{2},\quad x \neq 0$

13. $\dfrac{3xy}{xy + x} = \dfrac{x(3y)}{x(y + 1)} = \dfrac{3y}{y + 1},\ x \neq 0$

15. $\dfrac{4y - 8y^2}{10y - 5} = \dfrac{-4y(2y - 1)}{5(2y - 1)}$
$= -\dfrac{4y}{5},\ y \neq \dfrac{1}{2}$

17. $\dfrac{x - 5}{10 - 2x} = \dfrac{x - 5}{-2(x - 5)}$
$= -\dfrac{1}{2},\ x \neq 5$

19. $\dfrac{y^2 - 16}{y + 4} = \dfrac{(y + 4)(y - 4)}{y + 4}$
$= y - 4,\ y \neq -4$

21. $\dfrac{x^3 + 5x^2 + 6x}{x^2 - 4} = \dfrac{x(x + 2)(x + 3)}{(x + 2)(x - 2)} = \dfrac{x(x + 3)}{x - 2},\quad x \neq -2$

23. $\dfrac{y^2 - 7y + 12}{y^2 + 3y - 18} = \dfrac{(y - 3)(y - 4)}{(y + 6)(y - 3)} = \dfrac{y - 4}{y + 6},\quad y \neq 3$

25. $\dfrac{2 - x + 2x^2 - x^3}{x^2 - 4} = \dfrac{(2 - x) + x^2(2 - x)}{(x + 2)(x - 2)}$

$= \dfrac{(2 - x)(1 + x^2)}{(x + 2)(x - 2)}$

$= \dfrac{-(x - 2)(x^2 + 1)}{(x + 2)(x - 2)}$

$= -\dfrac{x^2 + 1}{x + 2}, \quad x \neq 2$

27. $\dfrac{z^3 - 8}{z^2 + 2z + 4} = \dfrac{(z - 2)(z^2 + 2z + 4)}{z^2 + 2z + 4} = z - 2$

29.

x	0	1	2	3	4	5	6
$\dfrac{x^2 - 2x - 3}{x - 3}$	1	2	3	Undef.	5	6	7
$x + 1$	1	2	3	4	5	6	7

The expressions are equivalent except at $x = 3$.

31. $\dfrac{5x^3}{2x^3 + 4} = \dfrac{5x^3}{2(x^3 + 2)}$

There are no common factors so this expression cannot be simplified. In this case factors of terms were incorrectly cancelled.

33. $\dfrac{\pi r^2}{(2r)^2} = \dfrac{\pi r^2}{4r^2} = \dfrac{\pi}{4}, \ r \neq 0$

35. $\dfrac{5}{x - 1} \cdot \dfrac{x - 1}{25(x - 2)} = \dfrac{1}{5(x - 2)}, \ x \neq 1$

37. $\dfrac{r}{r - 1} \cdot \dfrac{r^2 - 1}{r^2} = \dfrac{r(r + 1)(r - 1)}{r^2(r - 1)} = \dfrac{r + 1}{r}, \ r \neq 1, r \neq 0$

39. $\dfrac{t^2 - t - 6}{t^2 + 6t + 9} \cdot \dfrac{t + 3}{t^2 - 4} = \dfrac{(t - 3)(t + 2)(t + 3)}{(t + 3)^2(t + 2)(t - 2)} = \dfrac{t - 3}{(t + 3)(t - 2)}, \quad t \neq -2$

41. $\dfrac{x^2 - 36}{x} \div \dfrac{x^3 - 6x^2}{x^2 + x} = \dfrac{x^2 - 36}{x} \cdot \dfrac{x^2 + x}{x^3 - 6x^2}$

$= \dfrac{(x + 6)(x - 6)}{x} \cdot \dfrac{x(x + 1)}{x^2(x - 6)}$

$= \dfrac{(x + 6)(x + 1)}{x^2}, \quad x \neq 6$

43. $\dfrac{5}{x - 1} + \dfrac{x}{x - 1} = \dfrac{5 + x}{x - 1} = \dfrac{x + 5}{x - 1}$

45. $6 - \dfrac{5}{x + 3} = \dfrac{6(x + 3)}{(x + 3)} - \dfrac{5}{x + 3}$

$= \dfrac{6(x + 3) - 5}{x + 3}$

$= \dfrac{6x + 18 - 5}{x + 3}$

$= \dfrac{6x + 13}{x + 3}$

47. $\dfrac{3}{x - 2} + \dfrac{5}{2 - x} = \dfrac{3}{x - 2} - \dfrac{5}{x - 2} = -\dfrac{2}{x - 2}$

49. $\dfrac{1}{x^2 - x - 2} - \dfrac{x}{x^2 - 5x + 6} = \dfrac{1}{(x - 2)(x + 1)} - \dfrac{x}{(x - 2)(x - 3)}$

$= \dfrac{(x - 3) - x(x + 1)}{(x + 1)(x - 2)(x - 3)} = \dfrac{x - 3 - x^2 - x}{(x + 1)(x - 2)(x - 3)}$

$= \dfrac{-x^2 - 3}{(x + 1)(x - 2)(x - 3)} = -\dfrac{x^2 + 3}{(x + 1)(x - 2)(x - 3)}$

51. $$-\frac{1}{x}+\frac{2}{x^2+1}+\frac{1}{x^3+x}=\frac{-(x^2+1)}{x(x^2+1)}+\frac{2x}{x(x^2+1)}+\frac{1}{x(x^2+1)}$$
$$=\frac{-x^2-1+2x+1}{x(x^2+1)}=\frac{-x^2+2x}{x(x^2+1)}=\frac{-x(x-2)}{x(x^2+1)}$$
$$=-\frac{x-2}{x^2+1}=\frac{2-x}{x^2+1},\quad x\neq 0$$

53. $$\frac{x+4}{x+2}-\frac{3x-8}{x+2}=\frac{(x+4)-(3x-8)}{x+2}$$
$$=\frac{x+4-3x+8}{x+2}=\frac{-2x+12}{x+2}=\frac{-2(x-6)}{x+2}$$

The error was incorrect subtraction in the numerator.

55. $$\frac{\left(\frac{x}{2}-1\right)}{(x-2)}=\frac{\left(\frac{x}{2}-\frac{2}{2}\right)}{\left(\frac{x-2}{1}\right)}$$
$$=\frac{x-2}{2}\cdot\frac{1}{x-2}$$
$$=\frac{1}{2},\quad x\neq 2$$

57. $$\frac{\left[\frac{x^2}{(x+1)^2}\right]}{\left[\frac{x}{(x+1)^3}\right]}=\frac{x^2}{(x+1)^2}\cdot\frac{(x+1)^3}{x}$$
$$=x(x+1),\quad x\neq -1, 0$$

59. $$\frac{\left(\sqrt{x}-\frac{1}{2\sqrt{x}}\right)}{\sqrt{x}}=\frac{\left(\sqrt{x}-\frac{1}{2\sqrt{x}}\right)}{\sqrt{x}}\cdot\frac{2\sqrt{x}}{2\sqrt{x}}=\frac{2x-1}{2x},\quad x>0$$

61. $$x^5-2x^{-2}=x^{-2}(x^7-2)=\frac{x^7-2}{x^2}$$

63. $$x^2(x^2+1)^{-5}-(x^2+1)^{-4}=(x^2+1)^{-5}\left[x^2-(x^2+1)\right]=-\frac{1}{(x^2+1)^5}$$

65. $$2x^2(x-1)^{1/2}-5(x-1)^{-1/2}=(x-1)^{-1/2}\left[2x^2(x-1)^1-5\right]=\frac{2x^3-2x^2-5}{(x-1)^{1/2}}$$

67. $$\frac{3x^{1/3}-x^{-2/3}}{3x^{-2/3}}=\frac{3x^{1/3}-x^{-2/3}}{3x^{-2/3}}\cdot\frac{x^{2/3}}{x^{2/3}}=\frac{3x^1-x^0}{3x^0}=\frac{3x-1}{3},\ x\neq 0$$

69. $$\frac{\left(\frac{1}{x+h}-\frac{1}{x}\right)}{h}=\frac{\left(\frac{1}{x+h}-\frac{1}{x}\right)}{h}\cdot\frac{x(x+h)}{x(x+h)}$$
$$=\frac{x-(x+h)}{hx(x+h)}$$
$$=\frac{-h}{hx(x+h)}$$
$$=-\frac{1}{x(x+h)},\quad h\neq 0$$

71. $$\frac{\left(\frac{1}{x+h-4}-\frac{1}{x-4}\right)}{h} = \frac{\left(\frac{1}{x+h-4}-\frac{1}{x-4}\right)}{h}\cdot\frac{(x-4)(x+h-4)}{(x-4)(x+h-4)}$$
$$= \frac{(x-4)-(x+h-4)}{h(x-4)(x+h-4)}$$
$$= \frac{-h}{h(x-4)(x+h-4)}$$
$$= -\frac{1}{(x-4)(x+h-4)}, \quad h \neq 0$$

73. $$\frac{\sqrt{x+2}-\sqrt{x}}{2} = \frac{\sqrt{x+2}-\sqrt{x}}{2}\cdot\frac{\sqrt{x+2}+\sqrt{x}}{\sqrt{x+2}+\sqrt{x}}$$
$$= \frac{(x+2)-x}{2\left(\sqrt{x+2}+\sqrt{x}\right)}$$
$$= \frac{2}{2\left(\sqrt{x+2}+\sqrt{x}\right)}$$
$$= \frac{1}{\sqrt{x+2}+\sqrt{x}}$$

75. $$\frac{\sqrt{x+h+1}-\sqrt{x+1}}{h} = \frac{\sqrt{x+h+1}-\sqrt{x+1}}{h}\cdot\frac{\sqrt{x+h+1}+\sqrt{x+1}}{\sqrt{x+h+1}+\sqrt{x+1}}$$
$$= \frac{(x+h+1)-(x+1)}{h\left(\sqrt{x+h+1}+\sqrt{x+1}\right)}$$
$$= \frac{h}{h\left(\sqrt{x+h+1}+\sqrt{x+1}\right)}$$
$$= \frac{1}{\sqrt{x+h+1}+\sqrt{x+1}}, \quad h \neq 0$$

77. Probability $= \dfrac{\text{Shaded area}}{\text{Total area}} = \dfrac{x(x/2)}{x(2x+1)} = \dfrac{x/2}{2x+1}\cdot\dfrac{2}{2} = \dfrac{x}{2(2x+1)}$

79. (a) $\dfrac{1}{16}$ minute

(b) $x\left(\dfrac{1}{16}\right) = \dfrac{x}{16}$ minutes

(c) $\dfrac{60}{16} = \dfrac{15}{4}$ minutes

81. (a) $$r = \frac{\left(\frac{24[48(400)-16{,}000]}{48}\right)}{\left[16{,}000+\frac{48(400)}{12}\right]} \approx 0.0909 = 9.09\%$$

(b) $$r = \frac{\left[\frac{24(NM-P)}{N}\right]}{\left(P+\frac{NM}{12}\right)} = \frac{24(NM-P)}{N}\cdot\frac{12}{12P+NM} = \frac{288(NM-P)}{N(12P+NM)}$$

$$r = \frac{288[48(400)-16{,}000]}{48[12(16{,}000)+48(400)]} \approx 0.0909 = 9.09\%$$

83. $T = 10\left(\frac{4t^2 + 16t + 75}{t^2 + 4t + 10}\right)$

(a)

t	0	2	4	6	8	10	12	14	16	18	20	22
T	75°	55.9°	48.3°	45°	43.3°	42.3°	41.7°	41.3°	41.1°	40.9°	40.7°	40.6°

(b) T is approaching 40°.

85. False. In order for the simplified expression to be equivalent to the original expression, the domain of the simplified expression needs to be restricted. If n is even, $x \neq \pm 1$. If n is odd, $x \neq 1$.

87. Completely factor the numerator and the denominator. A rational expression is in **simplest** form if there are no common factors in the numerator and the denominator other than ± 1.

Section P.6 Errors and the Algebra of Calculus

- You should be able to recognize and avoid the common algebraic errors involving parentheses, fractions, exponents, radicals, and cancellation.
- You should be able to "unsimplify" algebraic expressions by the following methods.
  - (a) Unusual Factoring
  - (b) Rewriting with Negative Exponents
  - (c) Writing a Fraction as a Sum of Terms
  - (d) Inserting Factors or Terms

Vocabulary Check

1. numerator

2. reciprocal

1. $2x - (3y + 4) \neq 2x - 3y + 4$

Change all signs when distributing the minus sign.

$2x - (3y + 4) = 2x - 3y - 4$

3. $\frac{4}{16x - (2x + 1)} \neq \frac{4}{14x + 1}$

Change all signs when distributing the minus sign.

$\frac{4}{16x - (2x + 1)} = \frac{4}{16x - 2x - 1} = \frac{4}{14x - 1}$

5. $(5z)(6z) \neq 30z$

z occurs twice as a factor.

$(5z)(6z) = 30z^2$

7. $a\left(\frac{x}{y}\right) \neq \frac{ax}{ay}$

The fraction as a whole is multiplied by a, not the numerator and denominator separately.

$a\left(\frac{x}{y}\right) = \frac{a}{1} \cdot \frac{x}{y} = \frac{ax}{y}$

9. $\sqrt{x + 9} \neq \sqrt{x} + 3$

Do not apply the radical to the terms.

$\sqrt{x + 9}$ does not simplify.

11. $\frac{2x^2 + 1}{5x} \neq \frac{2x + 1}{5}$

Divide out common factors not common terms.

$\frac{2x^2 + 1}{5x}$ cannot be simplified.

13. $\dfrac{1}{a^{-1} + b^{-1}} \neq \left(\dfrac{1}{a + b}\right)^{-1}$

To get rid of negative exponents:

$$\frac{1}{a^{-1} + b^{-1}} = \frac{1}{a^{-1} + b^{-1}} \cdot \frac{ab}{ab} = \frac{ab}{b + a}$$

15. $(x^2 + 5x)^{1/2} \neq x(x + 5)^{1/2}$

Factor within grouping symbols before applying the exponent to each factor.

$$(x^2 + 5x)^{1/2} = [x(x + 5)]^{1/2} = x^{1/2}(x + 5)^{1/2}$$

17. $\dfrac{3}{x} + \dfrac{4}{y} = \dfrac{3}{x} \cdot \dfrac{y}{y} + \dfrac{4}{y} \cdot \dfrac{x}{x} = \dfrac{3y + 4x}{xy}$

To add fractions, they must have a common denominator.

19. $\dfrac{3x + 2}{5} = \dfrac{1}{5}(3x + 2)$

The required factor is $3x + 2$.

21. $\dfrac{2}{3}x^2 + \dfrac{1}{3}x + 5 = \dfrac{2}{3}x^2 + \dfrac{1}{3}x + \dfrac{15}{3} = \dfrac{1}{3}(2x^2 + x + 15)$

The required factor is $2x^2 + x + 15$.

23. $x^2(x^3 - 1)^4 = \dfrac{1}{3}(x^3 - 1)^4(3x^2)$

The required factor is $\frac{1}{3}$.

25. $\dfrac{4x + 6}{(x^2 + 3x + 7)^3} = \dfrac{2(2x + 3)}{(x^2 + 3x + 7)^3} = \dfrac{2}{1} \cdot \dfrac{(2x + 3)}{1} \cdot \dfrac{1}{(x^2 + 3x + 7)^3} = (2)\dfrac{1}{(x^2 + 3x + 7)^3}(2x + 3)$

The required factor is 2.

27.
$$\frac{3}{x} + \frac{5}{2x^2} - \frac{3}{2}x = \frac{6x}{2x^2} + \frac{5}{2x^2} - \frac{3x^3}{2x^2}$$
$$= \left(\frac{1}{2x^2}\right)(6x + 5 - 3x^3)$$

The required factor is $\dfrac{1}{2x^2}$.

29.
$$\frac{9x^2}{25} + \frac{16y^2}{49} = \frac{9}{25} \cdot \frac{x^2}{1} + \frac{16}{49} \cdot \frac{y^2}{1}$$
$$= \frac{1}{25/9} \cdot \frac{x^2}{1} + \frac{1}{49/16} \cdot \frac{y^2}{1}$$
$$= \frac{x^2}{(25/9)} + \frac{y^2}{(49/16)}$$

The required factors are $\frac{25}{9}$ and $\frac{49}{16}$.

31. $\dfrac{x^2}{1/12} - \dfrac{y^2}{2/3} = x^2\left(\dfrac{12}{1}\right) - y^2\left(\dfrac{3}{2}\right) = \dfrac{12x^2}{1} - \dfrac{3y^2}{2}$

The required factors are 1 and 2.

33. $x^{1/3} - 5x^{4/3} = x^{1/3}(1 - 5x^{3/3}) = x^{1/3}(1 - 5x)$

The required factor is $1 - 5x$.

35.
$$(1 - 3x)^{4/3} - 4x(1 - 3x)^{1/3} = (1 - 3x)^{1/3}[(1 - 3x)^1 - 4x]$$
$$= (1 - 3x)^{1/3}(1 - 7x)$$

The required factor is $1 - 7x$.

37.
$$\tfrac{1}{10}(2x + 1)^{5/2} - \tfrac{1}{6}(2x + 1)^{3/2} = \tfrac{3}{30}(2x + 1)^{3/2}(2x + 1)^1 - \tfrac{5}{30}(2x + 1)^{3/2}$$
$$= \tfrac{1}{30}(2x + 1)^{3/2}[3(2x + 1) - 5]$$
$$= \tfrac{1}{30}(2x + 1)^{3/2}(6x - 2)$$
$$= \tfrac{1}{30}(2x + 1)^{3/2}2(3x - 1)$$
$$= \tfrac{1}{15}(2x + 1)^{3/2}(3x - 1)$$

The required factor is $3x - 1$.

39. $\dfrac{3x^2}{(2x - 1)^3} = 3x^2(2x - 1)^{-3}$

41. $\dfrac{4}{3x} + \dfrac{4}{x^4} - \dfrac{7x}{\sqrt[3]{2x}} = 4(3x)^{-1} + 4x^{-4} - 7x(2x)^{-1/3}$

43. $\dfrac{16 - 5x - x^2}{x} = \dfrac{16}{x} - \dfrac{5x}{x} - \dfrac{x^2}{x} = \dfrac{16}{x} - 5 - x$

45. $\dfrac{4x^3 - 7x^2 + 1}{x^{1/3}} = \dfrac{4x^3}{x^{1/3}} - \dfrac{7x^2}{x^{1/3}} + \dfrac{1}{x^{1/3}}$

$= 4x^{3-1/3} - 7x^{2-1/3} + \dfrac{1}{x^{1/3}}$

$= 4x^{8/3} - 7x^{5/3} + \dfrac{1}{x^{1/3}}$

47. $\dfrac{3 - 5x^2 - x^4}{\sqrt{x}} = \dfrac{3}{\sqrt{x}} - \dfrac{5x^2}{\sqrt{x}} - \dfrac{x^4}{\sqrt{x}}$

$= \dfrac{3}{\sqrt{x}} - 5x^{2-1/2} - x^{4-1/2}$

$= \dfrac{3}{x^{1/2}} - 5x^{3/2} - x^{7/2}$

49. $\dfrac{-2(x^2 - 3)^{-3}(2x)(x + 1)^3 - 3(x + 1)^2(x^2 - 3)^{-2}}{[(x + 1)^3]^2} = \dfrac{(x^2 - 3)^{-3}(x + 1)^2[-4x(x + 1) - 3(x^2 - 3)]}{(x + 1)^6}$

$= \dfrac{-4x^2 - 4x - 3x^2 + 9}{(x^2 - 3)^3(x + 1)^4}$

$= \dfrac{-7x^2 - 4x + 9}{(x^2 - 3)^3(x + 1)^4}$

51. $\dfrac{(6x + 1)^3(27x^2 + 2) - (9x^3 + 2x)(3)(6x + 1)^2(6)}{[(6x + 1)^3]^2} = \dfrac{(6x + 1)^2[(6x + 1)(27x^2 + 2) - 18(9x^3 + 2x)]}{(6x + 1)^6}$

$= \dfrac{162x^3 + 12x + 27x^2 + 2 - 162x^3 - 36x}{(6x + 1)^4}$

$= \dfrac{27x^2 - 24x + 2}{(6x + 1)^4}$

53. $\dfrac{(x + 2)^{3/4}(x + 3)^{-2/3} - (x + 3)^{1/3}(x + 2)^{-1/4}}{[(x + 2)^{3/4}]^2} = \dfrac{(x + 2)^{-1/4}(x + 3)^{-2/3}[(x + 2) - (x + 3)]}{(x + 2)^{6/4}}$

$= \dfrac{x + 2 - x - 3}{(x + 2)^{1/4}(x + 3)^{2/3}(x + 2)^{6/4}}$

$= -\dfrac{1}{(x + 3)^{2/3}(x + 2)^{7/4}}$

55. $\dfrac{2(3x - 1)^{1/3} - (2x + 1)(1/3)(3x - 1)^{-2/3}(3)}{(3x - 1)^{2/3}} = \dfrac{(3x - 1)^{-2/3}[2(3x - 1) - (2x + 1)]}{(3x - 1)^{2/3}}$

$= \dfrac{6x - 2 - 2x - 1}{(3x - 1)^{2/3}(3x - 1)^{2/3}}$

$= \dfrac{4x - 3}{(3x - 1)^{4/3}}$

57. $\dfrac{1}{(x^2 + 4)^{1/2}} \cdot \dfrac{1}{2}(x^2 + 4)^{-1/2}(2x) = \dfrac{1}{(x^2 + 4)^{1/2}} \cdot \dfrac{1}{(x^2 + 4)^{1/2}} \cdot \dfrac{1}{2}(2x)$

$= \dfrac{1}{(x^2 + 4)^1}(x)$

$= \dfrac{x}{x^2 + 4}$

59. $(x^2+5)^{1/2}\left(\frac{3}{2}\right)(3x-2)^{1/2}(3)+(3x-2)^{3/2}\left(\frac{1}{2}\right)(x^2+5)^{-1/2}(2x)=\frac{9}{2}(x^2+5)^{1/2}(3x-2)^{1/2}+x(x^2+5)^{-1/2}(3x-2)^{3/2}$

$$=\frac{9}{2}(x^2+5)^{1/2}(3x-2)^{1/2}+\frac{2}{2}x(x^2+5)^{-1/2}(3x-2)^{3/2}$$

$$=\frac{1}{2}(x^2+5)^{-1/2}(3x-2)^{1/2}[9(x^2+5)^1+2x(3x-2)^1]$$

$$=\frac{1}{2}(x^2+5)^{-1/2}(3x-2)^{1/2}(9x^2+45+6x^2-4x)$$

$$=\frac{(3x-2)^{1/2}(15x^2-4x+45)}{2(x^2+5)^{1/2}}$$

61. $t=\frac{\sqrt{x^2+4}}{2}+\frac{\sqrt{(4-x)^2+4}}{6}$

(a)

x	t
0.5	1.70
1.0	1.72
1.5	1.78
2.0	1.89
2.5	2.02
3.0	2.18
3.5	2.36
4.0	2.57

(b) She should swim to a point about $\frac{1}{2}$ mile down the coast to minimize the time required to reach the finish line.

(c) $\frac{1}{2}x(x^2+4)^{-1/2}+\frac{1}{6}(x-4)(x^2-8x+20)^{-1/2}=\frac{3}{6}x(x^2+4)^{-1/2}+\frac{1}{6}(x-4)(x^2-8x+20)^{-1/2}$

$$=\frac{1}{6}\left[3x(x^2+4)^{-1/2}+(x-4)(x^2-8x+20)^{-1/2}\right]$$

$$=\frac{1}{6}\left[\frac{3x}{(x^2+4)^{1/2}}+\frac{x-4}{(x^2-8x+20)^{1/2}}\right]$$

$$=\frac{3x\sqrt{x^2-8x+20}+(x-4)\sqrt{x^2+4}}{6\sqrt{x^2+4}\sqrt{x^2-8x+20}}$$

63. True.

$$x^{-1}+y^{-2}=\frac{1}{x}+\frac{1}{y^2}=\frac{y^2+x}{xy^2}$$

65. True.

$$\frac{1}{\sqrt{x}+4}=\frac{1}{\sqrt{x}+4}\cdot\frac{\sqrt{x}-4}{\sqrt{x}-4}=\frac{\sqrt{x}-4}{x-16}$$

67. $x^n\cdot x^{3n}\neq x^{3n^2}$

Add exponents when multiplying powers with like bases.

$x^n\cdot x^{3n}=x^{4n}$

69. $x^{2n}+y^{2n}\neq(x^n+y^n)^2$

When squaring binomials, there is also a middle term.

$(x^n+y^n)^2=x^{2n}+2x^ny^n+y^{2n}$

71. The two answers are equivalent and can be obtained by factoring.

$$\tfrac{1}{10}(2x-1)^{5/2}+\tfrac{1}{6}(2x-1)^{3/2}=\tfrac{1}{60}(2x-1)^{3/2}[6(2x-1)+10]$$

$$=\tfrac{1}{60}(2x-1)^{3/2}(12x+4)$$

$$=\tfrac{4}{60}(2x-1)^{3/2}(3x+1)$$

$$=\tfrac{1}{15}(2x-1)^{3/2}(3x+1)$$

(a) $\tfrac{2}{3}x(2x-3)^{3/2}-\tfrac{2}{15}(2x-3)^{5/2}=\tfrac{2}{15}(2x-3)^{3/2}[5x-(2x-3)]$

$$=\tfrac{2}{15}(2x-3)^{3/2}(3x+3)$$

$$=\tfrac{2}{15}(2x-3)^{3/2}3(x+1)$$

$$=\tfrac{2}{5}(2x-3)^{3/2}(x+1)$$

—CONTINUED—

71. —CONTINUED—

(b) $\frac{2}{3}x(4 + x)^{3/2} - \frac{2}{15}(4 + x)^{5/2} = \frac{2}{15}(4 + x)^{3/2}[5x - (4 + x)]$

$$= \frac{2}{15}(4 + x)^{3/2}(4x - 4) = \frac{2}{15}(4 + x)^{3/2}4(x - 1) = \frac{8}{15}(4 + x)^{3/2}(x - 1)$$

Section P.7 The Rectangular Coordinate System and Graphs

- You should be able to plot points and sketch scatter plots.
- You should know that the distance between (x_1, y_1) and (x_2, y_2) in the plane is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
- You should know that the midpoint of the line segment joining (x_1, y_1) and (x_2, y_2) is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

Vocabulary Check

1. (a) **v** horizontal real number line (b) **vi** vertical real number line

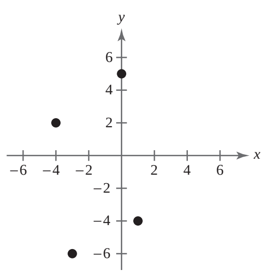
(c) **i** point of intersection of vertical axis and horizontal axis (d) **iv** four regions of the coordinate plane

(e) **iii** directed distance from the y-axis (f) **ii** directed distance from the x-axis

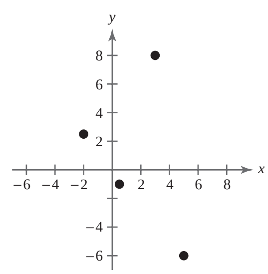
2. Cartesian **3.** Distance Formula **4.** Midpoint Formula

1. A: (2, 6), B: (−6, −2), C: (4, −4), D: (−3, 2)

3.



5.



7. (−3, 4)

9. (−5, −5)

11. $x > 0$ and $y < 0$ in Quadrant IV.

13. $x = -4$ and $y > 0$ in Quadrant II.

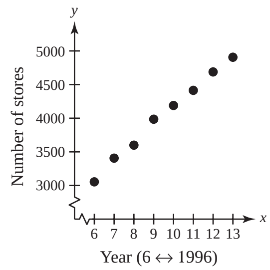
15. $y < -5$ in Quadrants III and IV.

17. $(x, -y)$ is in the second Quadrant means that (x, y) is in Quadrant III.

19. (x, y), $xy > 0$ means x and y have the same signs. This occurs in Quadrants I and III.

21.

Year, x	Number of stores, y
1996	3054
1997	3406
1998	3599
1999	3985
2000	4189
2001	4414
2002	4688
2003	4906



23. $d = |5 - (-3)| = 8$

25. $d = |2 - (-3)| = 5$

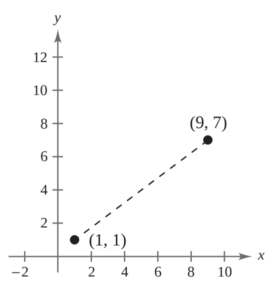
27. (a) The distance between $(0, 2)$ and $(4, 2)$ is 4.
The distance between $(4, 2)$ and $(4, 5)$ is 3.
The distance between $(0, 2)$ and $(4, 5)$ is
$\sqrt{(4-0)^2 + (5-2)^2} = \sqrt{16+9} = \sqrt{25} = 5.$

(b) $4^2 + 3^2 = 16 + 9 = 25 = 5^2$

29. (a) The distance between $(-1, 1)$ and $(9, 1)$ is 10.
The distance between $(9, 1)$ and $(9, 4)$ is 3.
The distance between $(-1, 1)$ and $(9, 4)$ is
$\sqrt{(9-(-1))^2 + (4-1)^2} = \sqrt{100+9} = \sqrt{109}.$

(b) $10^2 + 3^2 = 109 = \left(\sqrt{109}\right)^2$

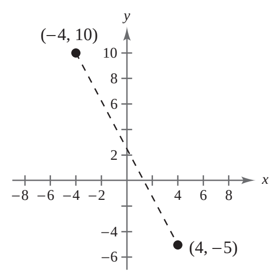
31. (a)



(b) $d = \sqrt{(9-1)^2 + (7-1)^2}$
$= \sqrt{64 + 36} = 10$

(c) $\left(\frac{9+1}{2}, \frac{7+1}{2}\right) = (5, 4)$

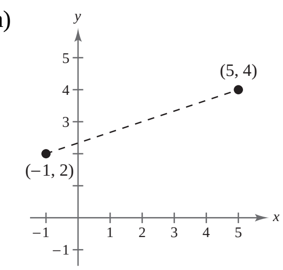
33. (a)



(b) $d = \sqrt{(4+4)^2 + (-5-10)^2}$
$= \sqrt{64 + 225} = 17$

(c) $\left(\frac{4-4}{2}, \frac{-5+10}{2}\right) = \left(0, \frac{5}{2}\right)$

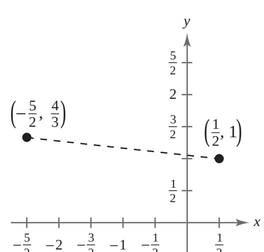
35. (a)



(b) $d = \sqrt{(5+1)^2 + (4-2)^2}$
$= \sqrt{36 + 4} = 2\sqrt{10}$

(c) $\left(\frac{-1+5}{2}, \frac{2+4}{2}\right) = (2, 3)$

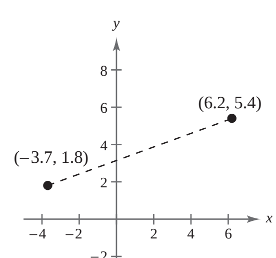
37. (a)



(b) $d = \sqrt{\left(\frac{1}{2} + \frac{5}{2}\right)^2 + \left(1 - \frac{4}{3}\right)^2}$
$= \sqrt{9 + \frac{1}{9}} = \frac{\sqrt{82}}{3}$

(c) $\left(\frac{-(5/2) + (1/2)}{2}, \frac{(4/3) + 1}{2}\right) = \left(-1, \frac{7}{6}\right)$

39. (a)



(b) $d = \sqrt{(6.2 + 3.7)^2 + (5.4 - 1.8)^2}$
$= \sqrt{98.01 + 12.96} = \sqrt{110.97}$

(c) $\left(\frac{6.2 - 3.7}{2}, \frac{5.4 + 1.8}{2}\right) = (1.25, 3.6)$

41. $d_1 = \sqrt{(4-2)^2 + (0-1)^2} = \sqrt{5}$

$d_2 = \sqrt{(4+1)^2 + (0+5)^2} = \sqrt{50}$

$d_3 = \sqrt{(2+1)^2 + (1+5)^2} = \sqrt{45}$

$(\sqrt{5})^2 + (\sqrt{45})^2 = (\sqrt{50})^2$

43. Since $x_m = \dfrac{x_1 + x_2}{2}$ and $y_m = \dfrac{y_1 + y_2}{2}$ we have:

$2x_m = x_1 + x_2$ $\qquad 2y_m = y_1 + y_2$

$2x_m - x_1 = x_2$ $\qquad 2y_m - y_1 = y_2$

Thus, $(x_2, y_2) = (2x_m - x_1, 2y_m - y_1)$.

45. The midpoint of the given line segment is $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$.

The midpoint between (x_1, y_1) and $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$ is $\left(\dfrac{x_1 + \frac{x_1 + x_2}{2}}{2}, \dfrac{y_1 + \frac{y_1 + y_2}{2}}{2}\right) = \left(\dfrac{3x_1 + x_2}{4}, \dfrac{3y_1 + y_2}{4}\right)$.

The midpoint between $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$ and (x_2, y_2) is $\left(\dfrac{\frac{x_1 + x_2}{2} + x_2}{2}, \dfrac{\frac{y_1 + y_2}{2} + y_2}{2}\right) = \left(\dfrac{x_1 + 3x_2}{4}, \dfrac{y_1 + 3y_2}{4}\right)$.

Thus, the three points are

$\left(\dfrac{3x_1 + x_2}{4}, \dfrac{3y_1 + y_2}{4}\right), \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$, and $\left(\dfrac{x_1 + 3x_2}{4}, \dfrac{y_1 + 3y_2}{4}\right)$.

47. $d = \sqrt{(42-18)^2 + (50-12)^2}$

$= \sqrt{24^2 + 38^2}$

$= \sqrt{2020}$

$= 2\sqrt{505}$

≈ 45 yards

49. $\left(\dfrac{2001 + 2003}{2}, \dfrac{3433 + 4174}{2}\right) = (2002, 3803.5)$

In 2002, the sales for Big Lots was approximately \$3803.5 million.

51. $(-2 + 2, -4 + 5) = (0, 1)$

$(2 + 2, -3 + 5) = (4, 2)$

$(-1 + 2, -1 + 5) = (1, 4)$

53. $(-7 + 4, -2 + 8) = (-3, 6)$

$(-2 + 4, 2 + 8) = (2, 10)$

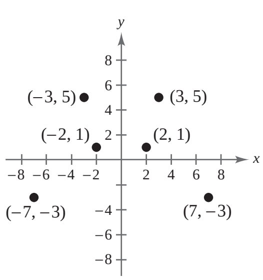
$(-2 + 4, -4 + 8) = (2, 4)$

$(-7 + 4, -4 + 8) = (-3, 4)$

55. The highest price of butter is approximately \$3.31 per pound. This occurred in 2001.

57. $\left[\dfrac{2400 - 700}{700}\right](100) \approx 242.9\%$ increase

59.



(a) The point is reflected through the y-axis.

(b) The point is reflected through the x-axis.

(c) The point is reflected through the origin.

61. (a) The minimum wage had the greatest increase in the 1990s.

(b) Minimum wage in 1990: \$3.80

Minimum wage in 1995: \$4.25

Percent increase: $\left(\dfrac{\$4.25 - \$3.80}{\$3.80}\right)(100) \approx 11.8\%$

Minimum wage in 1995: \$4.25

Minimum wage in 2000: \$5.15

Percent increase: $\left(\dfrac{\$5.15 - \$4.25}{\$4.25}\right)(100) \approx 21.2\%$

(c) $\$5.15 + 0.212(\$5.15) \approx \$6.24$

(d) The political nature of the minimum wage makes it difficult to predict, but this does seem like a reasonable value.

63. (1996, 18,546), (2004, 21,900)

By Exercise 45 we have the following:

$$\left(\frac{3(1996) + 2004}{4}, \frac{3(18{,}546) + 21{,}900}{4}\right) = (1998, 19{,}384.5)$$

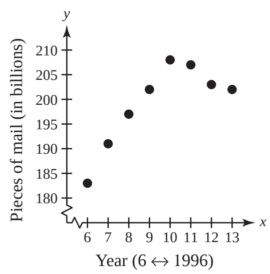
$$\left(\frac{1996 + 2004}{2}, \frac{18{,}546 + 21{,}900}{2}\right) = (2000, 20{,}223)$$

$$\left(\frac{1996 + 3(2004)}{4}, \frac{18{,}546 + 3(21{,}900)}{4}\right) = (2002, 21{,}061.5)$$

Year	Sales for Pepsi Bottling Group, Inc.
1998	\$19,384.5 million
2000	\$20,223 million
2002	\$21,061.5 million

65. (a)

Year, x	Pieces of mail, y (in billions)
1996	183
1997	191
1998	197
1999	202
2000	208
2001	207
2002	203
2003	202

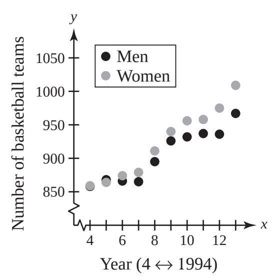


(b) The greatest decrease occurred in 2002.

(c) Answers will vary. Technology now enables us to transport information in ways other than by mail. The Internet is one example.

67. (a)

Year, x	Men's teams, M	Women's teams, W
1994	858	859
1995	868	864
1996	866	874
1997	865	879
1998	895	911
1999	926	940
2000	932	956
2001	937	958
2002	936	975
2003	967	1009



(b) In 1994, the number of men's and women's teams were nearly equal.

(c) In 2003, the difference between the number of teams was greatest: $1009 - 967 = 42$ teams

69. False, you would have to use the Midpoint Formula 15 times.

71. On the x-axis: $y = 0$; on the y-axis: $x = 0$

73. Since (x_0, y_0) lies in Quadrant II, $(x_0, -y_0)$ must lie in Quadrant III. Matches (b).

75. Since (x_0, y_0) lies in Quadrant II, $\left(x_0, \frac{1}{2}y_0\right)$ must lie in Quadrant II. Matches (d).

77. Use the Midpoint Formula to prove the diagonals of the parallelogram bisect each other.

$$\left(\frac{b+a}{2}, \frac{c+0}{2}\right) = \left(\frac{a+b}{2}, \frac{c}{2}\right)$$

$$\left(\frac{a+b+0}{2}, \frac{c+0}{2}\right) = \left(\frac{a+b}{2}, \frac{c}{2}\right)$$

Review Exercises for Chapter P

1. $\{11, -14, -\frac{8}{9}, \frac{5}{2}, \sqrt{6}, 0.4\}$

(a) Natural numbers: 11

(b) Whole numbers: 0, 11

(c) Integers: 11, -14

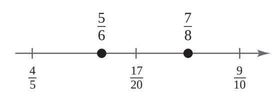
(d) Rational numbers: 11, -14, $-\frac{8}{9}, \frac{5}{2}$, 0.4

(e) Irrational numbers: $\sqrt{6}$

3. (a) $\frac{5}{6} = 0.8\overline{3}$

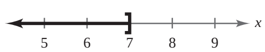
(b) $\frac{7}{8} = 0.875$

$\frac{5}{6} < \frac{7}{8}$



5. $x \leq 7$

The set consists of all real numbers less than or equal to 7.



7. $d(-92, 63) = |63 - (-92)|$
$= 155$

9. $d(x, 7) = |x - 7|$ and $d(x, 7) \geq 4$, thus $|x - 7| \geq 4$.

11. $d(y, -30) = |y - (-30)|$
$= |y + 30|$

and $d(y, -30) < 5$, thus $|y + 30| < 5$.

13. $12x - 7$

(a) $12(0) - 7 = -7$

(b) $12(-1) - 7 = -19$

15. $-x^2 + x - 1$

(a) $-(1)^2 + 1 - 1 = -1$

(b) $-(-1)^2 + (-1) - 1 = -3$

17. $2x + (3x - 10) = (2x + 3x) - 10$

Illustrates the Associative Property of Addition

19. $0 + (a - 5) = a - 5$

Illustrates the Additive Identity Property

21. $|-3| + 4(-2) - 6 = 3 - 8 - 6$
$= -11$

23. $\frac{5}{18} \div \frac{10}{3} = \frac{\cancel{5}}{\cancel{18}_6} \cdot \frac{\cancel{3}}{\cancel{10}_2} = \frac{1}{12}$

25. $6[4 - 2(6 + 8)] = 6[4 - 2(14)]$
$= 6[4 - 28]$
$= 6(-24)$
$= -144$

27. (a) $3x^2(4x^3)^3 = 3x^2(64x^9) = 192x^{11}$

(b) $\frac{5y^6}{10y} = \frac{y^{6-1}}{2} = \frac{y^5}{2}, \; y \neq 0$

29. (a) $\frac{6^2u^3v^{-3}}{12u^{-2}v} = \frac{36u^{3-(-2)}v^{-3-1}}{12} = 3u^5v^{-4} = \frac{3u^5}{v^4}$

(b) $\frac{3^{-4}m^{-1}n^{-3}}{9^{-2}mn^{-3}} = \frac{9^2n^3}{3^4mmn^3} = \frac{81}{81m^2} = \frac{1}{m^2} = m^{-2}$

31. $2{,}134{,}300{,}000 = 2.1343 \times 10^9$

33. $4.837 \times 10^8 = 483{,}700{,}000$

35. (a) $\sqrt[3]{27^2} = \left(\sqrt[3]{27}\right)^2 = (3)^2 = 9$

(b) $\sqrt{49^3} = \left(\sqrt{49}\right)^3 = (7)^3 = 343$

37. (a) $\left(\sqrt[3]{216}\right)^3 = \left(\sqrt[3]{6^3}\right)^3 = (6)^3 = 216$

(b) $\sqrt[4]{32^4} = \left(\sqrt[4]{32}\right)^4 = 32$

39. (a) $\sqrt{50} - \sqrt{18} = \sqrt{25 \cdot 2} - \sqrt{9 \cdot 2} = 5\sqrt{2} - 3\sqrt{2} = 2\sqrt{2}$

(b) $2\sqrt{32} + 3\sqrt{72} = 2\sqrt{16 \cdot 2} + 3\sqrt{36 \cdot 2} = 2(4\sqrt{2}) + 3(6\sqrt{2}) = 8\sqrt{2} + 18\sqrt{2} = 26\sqrt{2}$

41. These are not like terms. Radicals cannot be combined by addition or subtraction unless the index and the radicand are the same.

43. $$\frac{1}{2-\sqrt{3}} = \frac{1}{2-\sqrt{3}} \cdot \frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{2+\sqrt{3}}{2^2 - (\sqrt{3})^2} = \frac{2+\sqrt{3}}{4-3} = \frac{2+\sqrt{3}}{1} = 2 + \sqrt{3}$$

45. $$\frac{\sqrt{7}+1}{2} = \frac{\sqrt{7}+1}{2} \cdot \frac{\sqrt{7}-1}{\sqrt{7}-1} = \frac{(\sqrt{7})^2 - 1^2}{2(\sqrt{7}-1)} = \frac{7-1}{2(\sqrt{7}-1)} = \frac{6}{2(\sqrt{7}-1)} = \frac{3}{\sqrt{7}-1}$$

47. $(16)^{3/2} = \sqrt{16^3} = (\sqrt{16})^3 = (4)^3 = 64$

49. $(3x^{2/5})(2x^{1/2}) = 6x^{2/5+1/2} = 6x^{9/10}$

51. Standard form: $-11x^2 + 3$

Degree: 2

Leading coefficient: -11

53. Standard form: $-12x^2 - 4$

Degree: 2

Leading coefficient: -12

55. $$-(3x^2 + 2x) + (1 - 5x) = -3x^2 - 2x + 1 - 5x = -3x^2 - 7x + 1$$

57. $$2x(x^2 - 5x + 6) = (2x)(x^2) + (2x)(-5x) + (2x)(6) = 2x^3 - 10x^2 + 12x$$

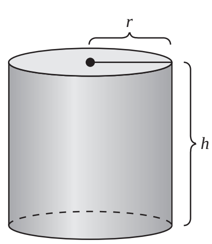
59. $$(3x - 6)(5x + 1) = 15x^2 + 3x - 30x - 6 = 15x^2 - 27x - 6$$

61. $$(2x - 3)^2 = (2x)^2 - 2(2x)(3) + 3^2 = 4x^2 - 12x + 9$$

63. $$(3\sqrt{5} + 2)(3\sqrt{5} - 2) = (3\sqrt{5})^2 - 2^2 = 9(5) - 4 = 41$$

65. (a) The surface is the sum of the area of the side, $2\pi rh$, and the areas of the top and bottom which are each πr^2.

$S = 2\pi rh + \pi r^2 + \pi r^2 = 2\pi rh + 2\pi r^2$



(b) $S = 2\pi(6)(8) + 2\pi(6)^2$

$= 96\pi + 72\pi$

$= 168\pi \approx 527.79 \text{ in}^2$

67. $x^3 - x = x(x^2 - 1) = x(x + 1)(x - 1)$

69. $25x^2 - 49 = (5x)^2 - 7^2 = (5x + 7)(5x - 7)$

71. $x^3 - 64 = x^3 - 4^3 = (x - 4)(x^2 + 4x + 16)$

73. $2x^2 + 21x + 10 = (2x + 1)(x + 10)$

75. $$x^3 - x^2 + 2x - 2 = x^2(x - 1) + 2(x - 1) = (x - 1)(x^2 + 2)$$

77. The domain of $\dfrac{1}{x+6}$ is the set of all real numbers except $x = -6$.

79. $\dfrac{x^2-64}{5(3x+24)} = \dfrac{(x+8)(x-8)}{5\cdot 3(x+8)} = \dfrac{x-8}{15},\quad x \neq -8$

81. $\dfrac{x^2-4}{x^4-2x^2-8}\cdot\dfrac{x^2+2}{x^2} = \dfrac{(x^2-4)(x^2+2)}{(x^2-4)(x^2+2)x^2}$

$$= \frac{1}{x^2},\quad x \neq \pm 2$$

83. $\dfrac{1}{x-1} + \dfrac{1-x}{x^2+x+1} = \dfrac{x^2+x+1+(1-x)(x-1)}{(x-1)(x^2+x+1)}$

$$= \frac{x^2+x+1+x-1-x^2+x}{(x-1)(x^2+x+1)}$$

$$= \frac{3x}{(x-1)(x^2+x+1)}$$

85. $\dfrac{\left(\dfrac{3a}{\dfrac{a^2}{x}-1}\right)}{\left(\dfrac{a}{x}-1\right)} = \dfrac{\left(\dfrac{3a}{\dfrac{a^2-x}{x}}\right)}{\left(\dfrac{a-x}{x}\right)}$

$$= \frac{3a}{1}\cdot\frac{x}{a^2-x}\cdot\frac{x}{a-x}$$

$$= \frac{3ax^2}{(a^2-x)(a-x)}$$

87. $\dfrac{\dfrac{1}{2(x+h)}-\dfrac{1}{2x}}{h} = \dfrac{\dfrac{x-(x+h)}{2x(x+h)}}{h}$

$$= \frac{-h}{2x(x+h)}\cdot\frac{1}{h}$$

$$= \frac{-1}{2x(x+h)},\quad h \neq 0$$

89. $10(4\cdot 7) = 10(28) = 280$

The multiplication in parentheses comes first.

91. $(3^4)^4 = 3^{4\cdot 4} = 3^{16}$

Multiply exponents when raising a power to a power.

93. $(5+8)^2 = 13^2 \neq 5^2 + 8^2$

Add the numbers in parentheses before squaring.

95. $\dfrac{2}{3}x^4 - \dfrac{3}{8}x^3 + \dfrac{5}{6}x^2 = \dfrac{16}{24}x^4 - \dfrac{9}{24}x^3 + \dfrac{20}{24}x^2$

$$= \frac{1}{24}x^2(16x^2 - 9x + 20)$$

The missing factor is $16x^2 - 9x + 20$.

97. $2x(x^2-3)^{1/3} - 5(x^2-3)^{4/3} = (x^2-3)^{1/3}[2x - 5(x^2-3)]$

$$= (x^2-3)^{1/3}(-5x^2 + 2x + 15)$$

The missing factor is $-5x^2 + 2x + 15$.

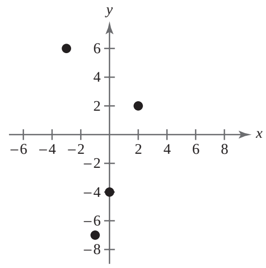
99. $\dfrac{x^3+5x^2+7}{x} = \dfrac{x^3}{x} + \dfrac{5x^2}{x} + \dfrac{7}{x} = x^2 + 5x + \dfrac{7}{x}$

101. $\dfrac{x(x+2)^{-1/2} + (x+2)^{1/2}}{(x+2)^{3/2}} = \dfrac{(x+2)^{-1/2}[x + (x+2)]}{(x+2)^{3/2}}$

$$= \frac{2x+2}{(x+2)^{1/2}(x+2)^{3/2}}$$

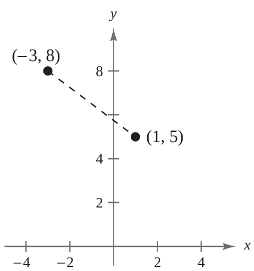
$$= \frac{2(x+1)}{(x+2)^2}$$

103.



105. $x > 0$ and $y = -2$ in Quadrant IV.

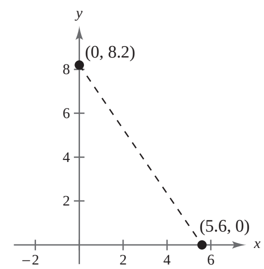
107. (a)



(b) $d = \sqrt{(-3-1)^2 + (8-5)^2} = \sqrt{16+9} = 5$

(c) Midpoint: $\left(\frac{-3+1}{2}, \frac{8+5}{2}\right) = \left(-1, \frac{13}{2}\right)$

109. (a)



(b) $d = \sqrt{(5.6-0)^2 + (0-8.2)^2}$

$= \sqrt{31.36 + 67.24} = \sqrt{98.6} \approx 9.9$

(c) Midpoint: $\left(\frac{0+5.6}{2}, \frac{8.2+0}{2}\right) = (2.8, 4.1)$

111. $(4-2, 8-3) = (2, 5)$

$(6-2, 8-3) = (4, 5)$

$(4-2, 3-3) = (2, 0)$

$(6-2, 3-3) = (4, 0)$

113. (2001, 539.1), (2003, 773.8)

$\left(\frac{2001+2003}{2}, \frac{539.1+773.8}{2}\right) = (2002, 656.45)$

In 2002, the sales were approximately \$656.45 million.

115. False, $(a+b)^2 = a^2 + 2ab + b^2 \neq a^2 + b^2$

There is also a cross-product term when a binomial sum is squared.

117. $\frac{ax-b}{b-ax} = -1$ for all real nonzero numbers a and b except for $x = \frac{b}{a}$. This makes the expression undefined.

Problem Solving for Chapter P

1. (a) Men's

Minimum Volume: $V = \frac{4}{3}\pi(55)^3 \approx 696{,}910 \text{ mm}^3$

Maximum Volume: $V = \frac{4}{3}\pi(65)^3 \approx 1{,}150{,}347 \text{ mm}^3$

Women's

Minimum Volume: $V = \frac{4}{3}\pi\left(\frac{95}{2}\right)^3 \approx 448{,}921 \text{ mm}^3$

Maximum Volume: $V = \frac{4}{3}\pi(55)^3 \approx 696{,}910 \text{ mm}^3$

(b) Men's

Minimum density: $\frac{7.26}{1{,}150{,}347} \approx 6.31 \times 10^{-6} \text{ kg/mm}^3$

Maximum density: $\frac{7.26}{696{,}910} \approx 1.04 \times 10^{-5} \text{ kg/mm}^3$

Women's

Minimum density: $\frac{4.00}{696{,}910} \approx 5.74 \times 10^{-6} \text{ kg/mm}^3$

Maximum density: $\frac{4.00}{448{,}921} \approx 8.91 \times 10^{-6} \text{ kg/mm}^3$

(c) No. The weight would be different. Cork is much lighter than iron so it would have a much smaller density.

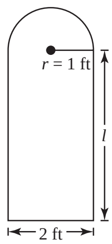
3. One golf ball: $\dfrac{1.6 \times 10^7}{1.58 \times 10^8} \approx 0.101$ pound

$0.101(16) = 1.62$ ounces

5. To say that a number has n significant digits means that the number has n digits with the left most non-zero digit and ending with the right most non-zero digit. For example; 28,000, 1.400, 0.00079 each have two significant digits.

7. $r = 1 - \left(\dfrac{3225}{12{,}000}\right)^{1/4} \approx 0.280$ or 28%

9.



Perimeter: $P = 2l + w + \pi r$

$13.14 = 2l + 2 + \pi(1)$

$l = \dfrac{13.14 - 2 - \pi}{2} \approx 4$ feet

Amount of glass = Area of window

$A = lw + \dfrac{1}{2}\pi r^2$

$= (4)(2) + \dfrac{1}{2}\pi(1)^2$

≈ 9.57 square feet

11. $y_1 = 2x\sqrt{1 - x^2} - \dfrac{x^3}{\sqrt{1 - x^2}}$ $\qquad y_2 = \dfrac{2 - 3x^2}{\sqrt{1 - x^2}}$

When $x = 0$, $y_1 = 0$. When $x = 0$, $y_2 = 2$.

Thus, $y_1 \neq y_2$.

$$y_1 = \frac{2x\sqrt{1 - x^2}}{1} - \frac{x^3}{\sqrt{1 - x^2}}$$

$$= \frac{2x\sqrt{1 - x^2}}{1} \cdot \frac{\sqrt{1 - x^2}}{\sqrt{1 - x^2}} - \frac{x^3}{\sqrt{1 - x^2}}$$

$$= \frac{2x(1 - x^2) - x^3}{\sqrt{1 - x^2}} = \frac{2x - 2x^3 - x^3}{\sqrt{1 - x^2}}$$

$$= \frac{2x - 3x^3}{\sqrt{1 - x^2}}$$

$$= \frac{x(2 - 3x^2)}{\sqrt{1 - x^2}}$$

Let $y_2 = \dfrac{x(2 - 3x^2)}{\sqrt{1 - x^2}}$. Then $y_1 = y_2$.

13. (a) $(1, -2)$ and $(4, 1)$

The points of trisection are:

$$\left(\frac{2(1) + 4}{3}, \frac{2(-2) + 1}{3}\right) = (2, -1)$$

$$\left(\frac{1 + 2(4)}{3}, \frac{-2 + 2(1)}{3}\right) = (3, 0)$$

(b) $(-2, -3)$ and $(0, 0)$

The points of trisection are:

$$\left(\frac{2(-2) + 0}{3}, \frac{2(-3) + 0}{3}\right) = \left(-\frac{4}{3}, -2\right)$$

$$\left(\frac{-2 + 2(0)}{3}, \frac{-3 + 2(0)}{3}\right) = \left(-\frac{2}{3}, -1\right)$$

Practice Test for Chapter P

1. Evaluate $\dfrac{|-42| - 20}{15 - |-4|}$.

2. Simplify $\dfrac{x}{z} - \dfrac{z}{y}$.

3. The distance between x and 7 is no more than 4. Use absolute value notation to describe this expression.

4. Evaluate $10(-x)^3$ for $x = 5$.

5. Simplify $(-4x^3)(-2x^{-5})\left(\frac{1}{16}x\right)$.

6. Change 0.0000412 to scientific notation.

7. Evaluate $125^{2/3}$.

8. Simplify $\sqrt[4]{64x^7y^9}$.

9. Rationalize the denominator and simplify $\dfrac{6}{\sqrt{12}}$.

10. Simplify $3\sqrt{80} - 7\sqrt{500}$.

11. Simplify $(8x^4 - 9x^2 + 2x - 1) - (3x^3 + 5x + 4)$.

12. Multiply $(x - 3)(x^2 + x - 7)$.

13. Multiply $[(x - 2) - y]^2$.

14. Factor $16x^4 - 1$.

15. Factor $6x^2 + 5x - 4$.

16. Factor $x^3 - 64$.

17. Combine and simplify $-\dfrac{3}{x} + \dfrac{x}{x^2 + 2}$.

18. Combine and simplify $\dfrac{x - 3}{4x} \div \dfrac{x^2 - 9}{x^2}$.

19. Simplify $\dfrac{1 - \left(\dfrac{1}{x}\right)}{1 - \dfrac{1}{1 - \left(\dfrac{1}{x}\right)}}$.

20. (a) Plot the points $(-3, 7)$ and $(5, -1)$,

(b) find the distance between the points, and

(c) find the midpoint of the line segment joining the points.