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You should know the following important facts about lines.

- The graph of \( y = mx + b \) is a straight line. It is called a linear equation in two variables.
  (a) The slope (steepness) is \( m \).
  (b) The \( y \)-intercept is \((0, b)\).

- The slope of the line through \((x_1, y_1)\) and \((x_2, y_2)\) is
  \[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{change in } y}{\text{change in } x} = \frac{\text{rise}}{\text{run}}.
  \]

- (a) If \( m > 0 \), the line rises from left to right.
  (b) If \( m = 0 \), the line is horizontal.
  (c) If \( m < 0 \), the line falls from left to right.
  (d) If \( m \) is undefined, the line is vertical.

- Equations of Lines
  (a) Slope-Intercept Form: \( y = mx + b \)
  (b) Point-Slope Form: \( y - y_1 = m(x - x_1) \)
  (c) Two-Point Form: \( y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) \)
  (d) General Form: \( Ax + By + C = 0 \)
  (e) Vertical Line: \( x = a \)
  (f) Horizontal Line: \( y = b \)

- Given two distinct nonvertical lines
  \( L_1: y = m_1x + b_1 \) and \( L_2: y = m_2x + b_2 \)
  (a) \( L_1 \) is parallel to \( L_2 \) if and only if \( m_1 = m_2 \) and \( b_1 \neq b_2 \).
  (b) \( L_1 \) is perpendicular to \( L_2 \) if and only if \( m_1 = -1/m_2 \).
1. (a) \( m = \frac{3}{4} \). Since the slope is positive, the line rises. Matches \( L_2 \).
   (b) \( m \) is undefined. The line is vertical. Matches \( L_y \).
   (c) \( m = -2 \). The line falls. Matches \( L_1 \).

3. \( y = mx + b \)

5. Two points on the line: \((0, 0)\) and \((4, 6)\)
   \[ \text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{6}{4} = \frac{3}{2} \]

7. Two points on the line: \((0, 8)\) and \((2, 0)\)
   \[ \text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{-8}{2} = -4 \]

9. \( y = 5x + 3 \)
   \[ \text{Slope:} \; m = 5 \]
   \[ \text{y-intercept:} \; (0, 3) \]

11. \( y = -\frac{1}{2}x + 4 \)
    \[ \text{Slope:} \; m = -\frac{1}{2} \]
    \[ \text{y-intercept:} \; (0, 4) \]

13. \( 5x - 2 = 0 \)
    \[ x = \frac{2}{5}, \text{vertical line} \]
    \[ \text{Slope: undefined} \]
    \[ \text{No y-intercept} \]

15. \( 7x + 6y = 30 \)
    \[ y = -\frac{7}{6}x + 5 \]
    \[ \text{Slope:} \; m = -\frac{7}{6} \]
    \[ \text{y-intercept:} \; (0, 5) \]

17. \( y - 3 = 0 \)
    \[ y = 3, \text{horizontal line} \]
    \[ \text{Slope:} \; m = 0 \]
    \[ \text{y-intercept:} \; (0, 3) \]

19. \( x + 5 = 0 \)
    \[ x = -5 \]
    \[ \text{Slope: undefined (vertical line)} \]
    \[ \text{No y-intercept} \]

21. \( m = \frac{6 - (-2)}{1 - (-3)} = \frac{8}{4} = 2 \)

23. \( m = \frac{4 - (-1)}{-6 - (-6)} = \frac{5}{0} \)
    \[ m \text{ is undefined.} \]

25. \( m = \frac{1 - (-\frac{4}{7})}{-\frac{2}{7} - (\frac{11}{2})} = \frac{1}{7} \)
27. \[ m = \frac{1.6 - 3.1}{-5.2 - 4.8} = \frac{-1.5}{-10} = 0.15 \]

Point: (5, -6), Slope: \( m = 1 \)

Since \( m = 1 \), \( y \) increases by 1 for every one unit increase in \( x \). Three additional points are (6, -5), (-7, -4), and (8, -3).

29. Point: (2, 1), Slope: \( m = 0 \)

Since \( m = 0 \), \( y \) does not change. Three points are (0, 1), (3, 1), and (-1, 1).

31. Point: (5, -6), Slope: \( m = 1 \)

Since \( m = 1 \), \( y \) increases by 1 for every one unit increase in \( x \). Three additional points are (6, -5), (7, -4), and (8, -3).

33. Point: (-8, 1), Slope is undefined.

Since \( m \) is undefined, \( x \) does not change. Three points are (-8, 0), (-8, 2), and (-8, 3).

35. Point: (-5, 4), Slope: \( m = 2 \)

Since \( m = 2 = \frac{2}{1} \), \( y \) increases by 2 for every one unit increase in \( x \). Three additional points are (-4, 6), (-3, 8), and (-2, 10).

37. Point: (7, -2), Slope: \( m = \frac{1}{2} \)

Since \( m = \frac{1}{2} \), \( y \) increases by 1 unit for every two unit increase in \( x \). Three additional points are (9, -1), (11, 0), and (13, 1).

39. Point (0, -2); \( m = 3 \)

\[ y + 2 = 3(x - 0) \]
\[ y = 3x - 2 \]

41. Point (-3, 6); \( m = -2 \)

\[ y - 6 = -2(x + 3) \]
\[ y = -2x \]

43. Point (4, 0); \( m = -\frac{1}{3} \)

\[ y - 0 = -\frac{1}{3}(x - 4) \]
\[ y = -\frac{1}{3}x + \frac{4}{3} \]

45. Point (6, -1); \( m \) is undefined.

The line is vertical.

\[ x = 6 \]

47. Point (4, \( \frac{5}{2} \)); \( m = 0 \)

The line is horizontal.

\[ y = \frac{5}{2} \]

49. Point (-5.1, 1.8); \( m = 5 \)

\[ y - 1.8 = 5(x - (-5.1)) \]
\[ y = 5x + 27.3 \]
51. (5, -1) and (-5, 5)
\[ y + 1 = \frac{5 + 1}{-5 - 5}(x - 5) \]
\[ y = -\frac{3}{5}(x - 5) - 1 \]
\[ y = -\frac{3}{5}x + 2 \]

55. \( \left(\frac{2}{5}, \frac{3}{2}\right) \) and \( \left(\frac{1}{2}, \frac{5}{4}\right) \)
\[ y - \frac{1}{2} = \frac{\frac{3}{2} - \frac{1}{2}}{\frac{5}{4} - \frac{1}{2}}(x - 2) \]
\[ y = -\frac{1}{2}(x - 2) + \frac{1}{2} \]
\[ y = -\frac{1}{2}x + \frac{3}{2} \]

59. (1, 0.6) and (-2, -0.6)
\[ y - 0.6 = \frac{-0.6 - 0.6}{-2 - 1}(x - 1) \]
\[ y = 0.4(x - 1) + 0.6 \]
\[ y = 0.4x + 0.2 \]

63. \( \left(\frac{7}{3}, -8\right) \) and \( \left(\frac{7}{3}, 1\right) \)
\[ m = \frac{1 - (-8)}{\frac{7}{3} - \frac{7}{3}} = \frac{9}{0} \] and is undefined.
\[ x = \frac{7}{3} \]
The line is vertical.

53. (-8, 1) and (-8, 7)
Since both points have \( x = -8 \), the slope is undefined, and the line is vertical.
\[ x = -8 \]

57. \( \left(-\frac{1}{10}, -\frac{3}{5}\right) \) and \( \left(\frac{9}{10}, \frac{9}{5}\right) \)
\[ y - \left(-\frac{3}{5}\right) = \frac{-\frac{9}{5} - \left(-\frac{3}{5}\right)}{\frac{9}{10} - \left(-\frac{1}{10}\right)}(x - \left(-\frac{1}{10}\right)) \]
\[ y = -\frac{6}{5}(x + \frac{1}{10}) - \frac{3}{5} \]
\[ y = -\frac{6}{5}x - \frac{18}{25} \]

61. (2, -1) and \( \left(\frac{1}{3}, 1\right) \)
\[ y + 1 = \frac{-1 - (-1)}{\frac{1}{3} - 2}(x - 2) \]
\[ y + 1 = \frac{0}{-\frac{5}{3}} \]
\[ y = -1 \]
The line is horizontal.
65. \( L_1: (0, -1), (5, 9) \)
Slope of \( L_1: m = \frac{9 + 1}{5 - 0} = 2 \)
\( L_2: (0, 3), (4, 1) \)
Slope of \( L_2: m = \frac{1 - 3}{4 - 0} = -\frac{1}{2} \)
\( L_1 \) and \( L_2 \) are perpendicular.

67. \( L_1: (3, 6), (-6, 0) \)
Slope of \( L_1: m = \frac{0 - 6}{-6 - 3} = -\frac{2}{3} \)
\( L_2: (0, -1), \left(\frac{7}{3}, \frac{5}{2}\right) \)
Slope of \( L_2: m = \frac{\frac{7}{3} + 1}{\frac{5}{2} - 0} = \frac{2}{3} \)
\( L_1 \) and \( L_2 \) are parallel.

69. \( 4x - 2y = 3 \)
\( y = 2x - \frac{3}{2} \)
Slope: \( m = 2 \)
(a) \( (2, 1), m = 2 \)
y - 1 = 2(x - 2)
y = 2x - 3
(b) \( (2, 1), m = -\frac{1}{2} \)
y - 1 = -\frac{1}{2}(x - 2)
y = -\frac{1}{2}x + 2

71. \( 3x + 4y = 7 \)
\[ y = -\frac{3}{4}x + \frac{7}{4} \]
Slope: \( m = -\frac{3}{4} \)
(a) \( \left(-\frac{3}{4}, \frac{7}{4}\right), m = -\frac{3}{4} \)
y - \frac{7}{4} = -\frac{3}{4}(x - \left(-\frac{3}{4}\right))
y = -\frac{3}{4}x + \frac{5}{4}
(b) \( \left(-\frac{3}{4}, \frac{7}{4}\right), m = \frac{4}{3} \)
y - \frac{7}{4} = \frac{4}{3}(x - \left(-\frac{3}{4}\right))
y = \frac{4}{3}x + \frac{123}{92}

73. \( y = -3 \)
m = 0
(a) \( (-1, 0) \) and \( m = 0 \)
y = 0
(b) \( (-1, 0) \), \( m \) is undefined.
x = -1

75. \( x = 4 \)
m is undefined.
(a) \( (2, 5) \), \( m \) is undefined. The line is vertical, passing through \( (2, 5) \).
x = 2
(b) \( (2, 5), m = 0 \)
y = 5

77. \( x - y = 4 \)
\[ y = x - 4 \]
Slope: \( m = 1 \)
(a) \( (2.5, 6.8), m = 1 \)
y = 6.8 = 1(x - 2.5)
y = x + 4.3
(b) \( (2.5, 6.8), m = -1 \)
y = 6.8 = (-1)(x - 2.5)
y = -x + 9.3

81. \( \frac{x}{-1/6} + \frac{y}{-2/3} = 1 \)
\[ 6x + \frac{3}{2}y = -1 \]
\[ 12x + 3y + 2 = 0 \]

83. \( \frac{x}{c} + \frac{y}{c} = 1, \ c \neq 0 \)
x + y = c
1 + 2 = c
3 = c
x + y = 3
x + y - 3 = 0

85. \( a) y = 2x \)
(b) \( y = -2x \)
(c) \( y = \frac{1}{2}x \)
(b) and (c) are perpendicular.
87. (a) \( y = -\frac{1}{3}x \)

(b) \( y = -\frac{1}{3}x + 3 \)

(c) \( y = 2x - 4 \)

(a) and (b) are parallel. (c) is perpendicular to (a) and (b).

89. Set the distance between \((4, -1)\) and \((x, y)\) equal to the distance between \((-2, 3)\) and \((x, y)\).

\[
\sqrt{(x - 4)^2 + (y - (-1))^2} = \sqrt{(x - (-2))^2 + (y - 3)^2}
\]

\[
(x - 4)^2 + (y + 1)^2 = (x + 2)^2 + (y - 3)^2
\]

\[
x^2 - 8x + 16 + y^2 + 2y + 1 = x^2 + 4x + 4 + y^2 - 6y + 9
\]

\[
-8x + 2y + 17 = 4x - 6y + 13
\]

\[
0 = 12x - 8y - 4
\]

\[
0 = 4(3x - 2y - 1)
\]

\[
0 = 3x - 2y - 1
\]

This line is the perpendicular bisector of the line segment connecting \((4, -1)\) and \((-2, 3)\).

91. Set the distance between \((3, \frac{3}{2})\) and \((x, y)\) equal to the distance between \((-7, 1)\) and \((x, y)\).

\[
\sqrt{(x - 3)^2 + \left(y - \frac{3}{2}\right)^2} = \sqrt{(x - (-7))^2 + (y - 1)^2}
\]

\[
(x - 3)^2 + \left(y - \frac{3}{2}\right)^2 = (x + 7)^2 + (y - 1)^2
\]

\[
x^2 - 6x + 9 + y^2 - 5y + \frac{25}{4} = x^2 + 14x + 49 + y^2 - 2y + 1
\]

\[
-6x - 5y + \frac{41}{4} = 14x - 2y + 50
\]

\[
-24x - 20y + 61 = 56x - 8y + 200
\]

\[
80x + 12y + 139 = 0
\]

This line is the perpendicular bisector of the line segment connecting \((3, \frac{3}{2})\) and \((-7, 1)\).

93. (a) \( m = 135 \). The sales are increasing 135 units per year.

(b) \( m = 0 \). There is no change in sales during the year.

(c) \( m = -40 \). The sales are decreasing 40 units per year.

95. (a) \((0, 55,722), (2, 61,768)\): \( m = \frac{61,768 - 55,722}{2 - 0} = 3023 \)

(b) \((2, 61,768), (4, 64,993)\): \( m = \frac{64,993 - 61,768}{4 - 2} = 1612.5 \)

(c) \((4, 64,993), (6, 69,277)\): \( m = \frac{69,277 - 64,993}{6 - 4} = 2142 \)

The average salary increased the most from 1990 to 1992 and the least from 1992 to 1994.

(b) \((0, 55,722), (12, 83,944)\): \( m = \frac{83,944 - 55,722}{12 - 0} = 2351.83 \)

(c) The average salary for senior high school principals increased by $2351.83 per year over the 12 years between 1990 and 2002.
97. $y = \frac{6}{100}x$
$y = \frac{6}{100}(200) = 12$ feet

101. Matches graph (b).

The slope is $-20$, which represents the decrease in the amount of the loan each week. The $y$-intercept is $(0, 200)$, which represents the original amount of the loan.

105. $(5, 0.18), (13, 4.04)$: $m = \frac{4.04 - 0.18}{13 - 5} = 0.4825$
$y - 0.18 = 0.4825(t - 5)$
$y = 0.4825t - 2.2325$
For 2008, use $t = 18$: $y(18) \approx 6.45$
For 2010, use $t = 20$: $y(20) \approx 7.42$

109. (a) $(0, 40.571), (4, 41.289)$:
$m = \frac{41.289 - 40.571}{4 - 0} = 179.5$
$y = 179.5t + 40.571$
(b) For 2008, use $t = 8$: $y(8) = 42,007$ students.
For 2010, use $t = 10$: $y(10) = 42,366$ students.
(c) The slope is $m = 179.5$, which represents the increase in the number of students each year.

111. Sale price = List price $- 15\%$ of the list price
$S = L - 0.15L$
$S = 0.85L$

113. (a) $C = 36,500 + 5.25t + 11.50t$
$= 16.75t + 36,500$
(c) $P = R - C$
$= 27t - (16.75t + 36,500)$
$= 10.25t - 36,500$

115. (a) $x = 10 \text{ m}$
(b) $y = 2(15 + 2x) + 2(10 + 2x) = 8x + 50$

117. Using the points $(0, 875)$ and $(5, 0)$, where the first coordinate represents the year $t$ and the second coordinate represents the value $V$, we have
$m = \frac{0 - 875}{5 - 0} = -175$
$V = -175t + 875, 0 \leq t \leq 5$

119. (a) $(0, 10.25), (27, 36,500)$
(b) $R = 27t$
(d) $0 = 10.25t - 36,500$
$36,500 = 10.25t$
$t \approx 3561$ hours

121. (a) $v = \frac{150}{3650}$
(b) Since $m = 8$, each 1-meter increase in $x$ will increase $y$ by 8 meters.
117. \( C = 0.38m + 120 \)

119. (a) and (b)

![Cellular phone subscribers graph]

(c) Answers will vary. Find two points on your line and then find the equation of the line through your points. Sample answer: \( y = 11.72x - 14.08 \)

121. False. The slope with the greatest magnitude corresponds to the steepest line.

123. Using the Distance Formula, we have \( AB = 6 \), \( BC = \sqrt{40} \), and \( AC = 2 \). Since \( 6^2 + 2^2 = (\sqrt{40})^2 \), the triangle is a right triangle.

127. The \( V \)-intercept measures the initial cost and the slope measures annual depreciation.

129. \( y = 8 - 3x \) is a linear equation with slope \( m = -3 \) and \( y \)-intercept \( (0, 8) \). Matches graph (d).

131. \( y = \frac{1}{2}x^2 + 2x + 1 \) is a quadratic equation. Its graph is a parabola with vertex \( (-2, -1) \) and \( y \)-intercept \( (0, 1) \). Matches graph (a).

133. \(-7(3 - x) = 14(x - 1)\)
\[-21 + 7x = 14x - 14\]
\[-7x = 7\]
\[x = -1\]

135. \(2x^2 - 21x + 49 = 0\)
\[(2x - 7)(x - 7) = 0\]
\[2x - 7 = 0 \quad \text{or} \quad x - 7 = 0\]
\[x = \frac{7}{2} \quad \text{or} \quad x = 7\]

137. \(\sqrt{x - 9} + 15 = 0\)
\[\sqrt{x - 9} = -15\]
No real solution
The square root of \( x - 9 \) cannot be negative.

Section 2.2 Functions

- Given a set or an equation, you should be able to determine if it represents a function.
- Know that functions can be represented in four ways: verbally, numerically, graphically, and algebraically.
- Given a function, you should be able to do the following.
  - (a) Find the domain and range.
  - (b) Evaluate it at specific values.
- You should be able to use function notation.
### Vocabulary Check

1. **domain; range; function**
2. **verbally; numerically; graphically; algebraically**
3. **independent; dependent**
4. **piecewise-defined**
5. **implied domain**
6. **difference quotient**

---

1. Yes, the relationship is a function. Each domain value is matched with only one range value.

5. Yes, it does represent a function. Each input value is matched with only one output value.

9. (a) Each element of $A$ is matched with exactly one element of $B$, so it does represent a function.
   - (b) The element 1 in $A$ is matched with two elements, $-2$ and 1 of $B$, so it does not represent a function.
   - (c) Each element of $A$ is matched with exactly one element of $B$, so it does represent a function.
   - (d) The element 2 in $A$ is not matched with an element of $B$, so the relation does not represent a function.

13. $x^2 + y^2 = 4 \Rightarrow y = \pm \sqrt{4 - x^2}$
    No, $y$ is not a function of $x$.

17. $2x + 3y = 4 \Rightarrow y = \frac{1}{3}(4 - 2x)$
    Yes, $y$ is a function of $x$.

21. $y = |4 - x|$
    Yes, $y$ is a function of $x$.

25. $f(x) = 2x - 3$
   - (a) $f(1) = 2(1) - 3 = -1$
   - (b) $f(-3) = 2(-3) - 3 = -9$
   - (c) $f(x - 1) = 2(x - 1) - 3 = 2x - 5$

29. $f(y) = 3 - \sqrt{y}$
   - (a) $f(4) = 3 - \sqrt{4} = 1$
   - (b) $f(0.25) = 3 - \sqrt{0.25} = 2.5$
   - (c) $f(4x^2) = 3 - \sqrt{4x^2} = 3 - 2|x|$

31. $q(x) = \frac{1}{x^2 - 9}$
   - (a) $q(0) = \frac{1}{0^2 - 9} = -\frac{1}{9}$
   - (b) $q(3) = \frac{1}{3^2 - 9}$ is undefined.
   - (c) $q(y + 3) = \frac{1}{(y + 3)^2 - 9} = \frac{1}{y^2 + 6y}$
33. \( f(x) = \frac{|x|}{x} \)
   \( \begin{align*}
(a) \quad f(2) &= \frac{|2|}{2} = 1 \\
(b) \quad f(-2) &= \frac{|-2|}{-2} = -1 \\
(c) \quad f(x - 1) &= \frac{|x - 1|}{x - 1} = \begin{cases} -1, & \text{if } x < 1 \\ 1, & \text{if } x > 1 \end{cases}
\end{align*} \)

37. \( f(x) = \begin{cases} 3x - 1, & x < -1 \\ 4, & -1 \leq x \leq 1 \\ x^2, & x > 1 \end{cases} \)
   \( \begin{align*}
(a) \quad f(-2) &= 3(-2) - 1 = -7 \\
(b) \quad f\left(-\frac{1}{2}\right) &= 4 \\
(c) \quad f(3) &= 3^2 = 9
\end{align*} \)

41. \( h(t) = \frac{1}{2}|t + 3| \)
   \( \begin{align*}
h(-5) &= \frac{1}{2}|-5 + 3| = 1 \\
h(-4) &= \frac{1}{2}|-4 + 3| = \frac{1}{2} \\
h(-3) &= \frac{1}{2}|-3 + 3| = 0 \\
h(-2) &= \frac{1}{2}|-2 + 3| = \frac{1}{2} \\
h(-1) &= \frac{1}{2}|-1 + 3| = 1
\end{align*} \)

\[
\begin{array}{c|ccccc}
   t & -5 & -4 & -3 & -2 & -1 \\
   h(t) & 1 & \frac{1}{2} & 0 & \frac{1}{2} & 1
\end{array}
\]

43. \( f(x) = \begin{cases} -\frac{1}{2}x + 4, & x \leq 0 \\ (x - 2)^2, & x > 0 \end{cases} \)
   \( \begin{align*}
f(-2) &= -\frac{1}{2}(-2) + 4 = 5 \\
f(-1) &= -\frac{1}{2}(-1) + 4 = 4\frac{1}{2} = \frac{9}{2} \\
f(0) &= -\frac{1}{2}(0) + 4 = 4 \\
f(1) &= (1 - 2)^2 = 1 \\
f(2) &= (2 - 2)^2 = 0
\end{align*} \)

45. \( 15 - 3x = 0 \)
   \( 3x = 15 \)
   \( x = 5 \)

47. \( \frac{3x - 4}{5} = 0 \)
   \( 3x - 4 = 0 \)
   \( x = \frac{4}{3} \)

49. \( x^2 - 9 = 0 \)
   \( x^2 = 9 \)
   \( x = \pm 3 \)

51. \( x^3 - x = 0 \)
   \( x(x^2 - 1) = 0 \)
   \( x(x + 1)(x - 1) = 0 \)
   \( x = 0, x = -1, \text{ or } x = 1 \)

53. \( f(x) = g(x) \)
   \( x^2 + 2x + 1 = 3x + 3 \)
   \( x^2 - x - 2 = 0 \)
   \( (x + 1)(x - 2) = 0 \)
   \( x = -1 \text{ or } x = 2 \)

55. \( f(x) = g(x) \)
   \( \sqrt{3x + 1} = x + 1 \)
   \( \sqrt{3x} = x \)
   \( 3x = x^2 \)
   \( 0 = x^2 - 3x \)
   \( 0 = x(x - 3) \)
   \( x = 0 \text{ or } x = 3 \)
57. \( f(x) = 5x^2 + 2x - 1 \)

Since \( f(x) \) is a polynomial, the domain is all real numbers \( x \).

59. \( h(t) = \frac{4}{t} \)

Domain: All real numbers except \( t = 0 \)

61. \( g(y) = \sqrt{y - 10} \)

Domain: \( y - 10 \geq 0 \)
\( y \geq 10 \)

63. \( f(x) = \sqrt{1 - x^2} \)

Domain: \( 1 - x^2 \geq 0 \)

\(-x^2 \geq -1 \)
\( x^2 \leq 1 \)
\( x^2 - 1 \leq 0 \)

Critical numbers: \( x = \pm 1 \)

Test intervals: \( (-\infty, -1), (-1, 1), (1, \infty) \)

Test: Is \( x^2 - 1 \leq 0 \)?

Domain: \( -1 \leq x \leq 1 \)

67. \( f(x) = \frac{\sqrt{x - 1}}{x - 4} \)

Domain: \( s - 1 \geq 0 \implies s \geq 1 \) and \( s \neq 4 \)

The domain consists of all real numbers \( s \), such that \( s \geq 1 \) and \( s \neq 4 \).

75. By plotting the points, we have a parabola, so

\( g(x) = cx^2 \). Since \((-4,-32)\) is on the graph, we have

\(-32 = c(-4)^2 \implies c = -2 \). Thus, \( g(x) = -2x^2 \).

79. \( f(x) = x^2 - x + 1 \)

\( f(2 + h) = (2 + h)^2 - (2 + h) + 1 \)

\( = 4 + 4h + h^2 - 2 - h + 1 \)

\( = h^2 + 3h + 3 \)

\( f(2) = (2)^2 - 2 + 1 = 3 \)

\( f(2 + h) - f(2) = h^2 + 3h \)

\( \frac{f(2 + h) - f(2)}{h} = \frac{h^2 + 3h}{h} = h + 3, \ h \neq 0 \)

81. \( f(x) = x^3 + 3x \)

\( f(x + h) = (x + h)^3 + 3(x + h) \)

\( = x^3 + 3x^2h + 3xh^2 + h^3 + 3x + 3h \)

\( \frac{f(x + h) - f(x)}{h} = \frac{(x^3 + 3x^2h + 3xh^2 + h^3 + 3x + 3h) - (x^3 + 3x)}{h} \)

\( = \frac{h(3x^2 + 3xh + h^2 + 3)}{h} \)

\( = 3x^2 + 3xh + h^2 + 3, \ h \neq 0 \)
83. \( g(x) = \frac{1}{x^2} \)

\[
g(x) - g(3) = \frac{1}{x^2} - \frac{1}{9} \]

\[
= \frac{9 - x^2}{9x^2(x - 3)}
\]

\[
= \frac{-(x + 3)(x - 3)}{9x^2(x - 3)}
\]

\[
= \frac{-x + 3}{9x^2}, \; x \neq 0, 3
\]

85. \( f(x) = \sqrt{3x} \)

\[
\frac{f(x) - f(5)}{x - 5} = \frac{\sqrt{3x} - 5}{x - 5}
\]

\[
A = \left(\frac{P}{4}\right)^2 = \frac{P^2}{16}
\]

87. \( A = s^2 \) and \( P = 4s \) \( \Rightarrow \) \( \frac{P}{4} = s \)

89. (a) Height, \( x \) | Volume, \( V \)

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>484</td>
</tr>
<tr>
<td>2</td>
<td>800</td>
</tr>
<tr>
<td>3</td>
<td>972</td>
</tr>
<tr>
<td>4</td>
<td>1024</td>
</tr>
<tr>
<td>5</td>
<td>980</td>
</tr>
<tr>
<td>6</td>
<td>864</td>
</tr>
</tbody>
</table>

The volume is maximum when \( x = 4 \) and \( V = 1024 \) cubic centimeters.

91. \( A = \frac{1}{2}bh = \frac{1}{2}xy \)

Since \((0, y), (2, 1),\) and \((x, 0)\) all lie on the same line, the slopes between any pair are equal.

\[
\frac{1 - y}{2 - 0} = \frac{0 - 1}{x - 2}
\]

\[
\frac{1 - y}{2} = \frac{-1}{x - 2}
\]

\[
y = \frac{2}{x - 2} + 1
\]

\[
y = \frac{x}{x - 2}
\]

Therefore,

\[
A = \frac{1}{2}\left(\frac{x}{x - 2}\right) = \frac{x^2}{2(x - 2)}.
\]

The domain of \( A \) includes \( x \)-values such that \( x^2/[2(x - 2)] > 0 \). Using methods of Section 1.8 we find that the domain is \( x > 2 \).

93. \( y = -\frac{1}{10}x^2 + 3x + 6 \)

\[y(30) = -\frac{1}{10}(30)^2 + 3(30) + 6 = 6 \text{ feet}\]

If the child holds a glove at a height of 5 feet, then the ball will be over the child’s head since it will be at a height of 6 feet.
95. \( p(t) = \begin{cases} 0.182t^2 + 0.57t + 27.3, & 0 \leq t \leq 7 \\ 2.50t + 21.3, & 8 \leq t \leq 12 \end{cases} \)

<table>
<thead>
<tr>
<th>Year</th>
<th>Function Value</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990</td>
<td>( p(0) = 27.3 )</td>
<td>$27,300</td>
</tr>
<tr>
<td>1991</td>
<td>( p(1) = 28.052 )</td>
<td>$28,052</td>
</tr>
<tr>
<td>1992</td>
<td>( p(2) = 29.168 )</td>
<td>$29,168</td>
</tr>
<tr>
<td>1993</td>
<td>( p(3) = 30.648 )</td>
<td>$30,648</td>
</tr>
<tr>
<td>1994</td>
<td>( p(4) = 32.492 )</td>
<td>$32,492</td>
</tr>
<tr>
<td>1995</td>
<td>( p(5) = 34.7 )</td>
<td>$34,700</td>
</tr>
<tr>
<td>1996</td>
<td>( p(6) = 37.272 )</td>
<td>$37,272</td>
</tr>
<tr>
<td>1997</td>
<td>( p(7) = 40.208 )</td>
<td>$40,208</td>
</tr>
<tr>
<td>1998</td>
<td>( p(8) = 41.3 )</td>
<td>$41,300</td>
</tr>
<tr>
<td>1999</td>
<td>( p(9) = 43.8 )</td>
<td>$43,800</td>
</tr>
<tr>
<td>2000</td>
<td>( p(10) = 46.3 )</td>
<td>$46,300</td>
</tr>
<tr>
<td>2001</td>
<td>( p(11) = 48.8 )</td>
<td>$48,800</td>
</tr>
<tr>
<td>2002</td>
<td>( p(12) = 51.3 )</td>
<td>$51,300</td>
</tr>
</tbody>
</table>

99. (a) \( R = n(\text{rate}) = n[8.00 - 0.05(n - 80)], \ n \geq 80 \)

\[
R = 12.00n - 0.05n^2 = 12n - \frac{n^2}{20} = \frac{240n - n^2}{20}, \ n \geq 80
\]

(b) 

<table>
<thead>
<tr>
<th>( n )</th>
<th>90</th>
<th>100</th>
<th>110</th>
<th>120</th>
<th>130</th>
<th>140</th>
<th>150</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R(n) )</td>
<td>$675</td>
<td>$700</td>
<td>$715</td>
<td>$720</td>
<td>$715</td>
<td>$700</td>
<td>$675</td>
</tr>
</tbody>
</table>

The revenue is maximum when 120 people take the trip.

101. (a) \( (3000)^2 + h^2 = d^2 \)

\[
h = \sqrt{d^2 - (3000)^2}
\]

Domain: \( d \geq 3000 \) (since both \( d \geq 0 \) and \( d^2 - (3000)^2 \geq 0 \))

103. False. The range is \([-1, \infty)\).

105. The domain is the set of inputs of the function, and the range is the set of outputs.

107. (a) Yes. The amount that you pay in sales tax will increase as the price of the item purchased increases.

(b) No. The length of time that you study the night before an exam does not necessarily determine your score on the exam.
Chapter 2 Functions and Their Graphs

109. \( \frac{t}{3} + \frac{t}{5} = 1 \)
15. \( \frac{t}{3} + \frac{t}{5} = 15(1) \)
5t + 3t = 15
8t = 15
t = \frac{15}{8}

113. \((-2, -5)\) and \((4, -1)\)

\( m = \frac{-1 - (-5)}{4 - (-2)} = \frac{4}{6} = \frac{2}{3} \)

\( y = \frac{2}{3}x + \frac{4}{3} \)

3y + 15 = 2x + 4

2x - 3y - 11 = 0

Section 2.3 Analyzing Graphs of Functions

- You should be able to determine the domain and range of a function from its graph.
- You should be able to use the vertical line test for functions.
- You should be able to find the zeros of a function.
- You should be able to determine when a function is constant, increasing, or decreasing.
- You should be able to approximate relative minimums and relative maximums from the graph of a function.
- You should know that (a) odd if \( f(-x) = -f(x) \) and (b) even if \( f(-x) = f(x) \).

Vocabulary Check

1. ordered pairs 2. vertical line test 3. zeros 4. decreasing
5. maximum 6. average rate of change; secant 7. odd 8. even

1. Domain: \((-\infty, -1)\) \cup [1, \infty) 
   Range: \([0, \infty)\)

3. Domain: \([-4, 4]\) 
   Range: \([0, 4]\)

5. (a) \( f(-2) = 0 \) (b) \( f(-1) = -1 \) (c) \( f\left(\frac{1}{2}\right) = 0 \) (d) \( f(1) = -3 \)

7. (a) \( f(-2) = -3 \) (b) \( f(1) = 0 \) (c) \( f(0) = 1 \) (d) \( f(2) = -3 \)

9. \( y = \frac{1}{2}x^2 \)

11. \( x - y^2 = 1 \) \( \Rightarrow \) \( y = \pm \sqrt{x - 1} \)

\( y \) is not a function of \( x \). Some vertical lines cross the graph twice.
13. \( x^2 = 2xy - 1 \)
   A vertical line intersects the graph just once, so \( y \) is a function of \( x \).

15. \( 2x^2 - 7x - 30 = 0 \)
   \( (2x + 5)(x - 6) = 0 \)
   \( 2x + 5 = 0 \) or \( x - 6 = 0 \)
   \( x = -\frac{5}{2} \) or \( x = 6 \)

17. \( \frac{x}{9x^2 - 4} = 0 \)
   \( x = 0 \)

19. \( \frac{1}{3}x^3 - x = 0 \)
   \( x^3 - 2x = 2(0) \)
   \( x(x^2 - 2) = 0 \)
   \( x = 0 \) or \( x^2 - 2 = 0 \)
   \( x^2 = 2 \)
   \( x = \pm \sqrt{2} \)

21. \( 4x^3 - 24x^2 - x + 6 = 0 \)
   \( 4x^2(x - 6) - 1(x - 6) = 0 \)
   \( (x - 6)(4x^2 - 1) = 0 \)
   \( (x - 6)(2x + 1)(2x - 1) = 0 \)
   \( x - 6 = 0, \ 2x + 1 = 0, \ 2x - 1 = 0 \)
   \( x = 6, \ \ x = -\frac{1}{2}, \ \ x = \frac{1}{2} \)

23. \( \sqrt{2x} - 1 = 0 \)
   \( \sqrt{2x} = 1 \)
   \( 2x = 1 \)
   \( x = \frac{1}{2} \)

25. (a) \( 3 + \frac{5}{x} = 0 \)
   \( 3x + 5 = 0 \)
   \( x = -\frac{5}{3} \)

29. (a) \( \frac{3x - 1}{x - 6} = 0 \)
   \( 3x - 1 = 0 \)
   \( x = \frac{1}{3} \)

31. \( f(x) = \frac{1}{3}x \)
   \( f \) is increasing on \((-\infty, \infty)\).

33. \( f(x) = x^3 - 3x^2 + 2 \)
   \( f \) is increasing on \((-\infty, 0)\) and \((2, \infty)\).
   \( f \) is decreasing on \((0, 2)\).

37. \( f(x) = |x + 1| + |x - 1| \)
   \( f \) is increasing on \((1, \infty)\).
   \( f \) is constant on \((-1, 1)\).
   \( f \) is decreasing on \((-\infty, -1)\).

39. \( f(x) = 3 \)
   Constant on \((-\infty, \infty)\)

   \[
   \begin{array}{c|ccccc}
   x & -2 & -1 & 0 & 1 & 2 \\
   f(x) & 3 & 3 & 3 & 3 & 3 \\
   \end{array}
   \]
41. \( g(s) = \frac{s^3}{4} \)
(a) Decreasing on \((-\infty, 0)\); Increasing on \((0, \infty)\)
(b) \[
\begin{array}{c|cccc}
  s & -4 & -2 & 0 & 2 & 4 \\
g(s) & 4 & 1 & 0 & 1 & 4 \\
\end{array}
\]

43. \( f(t) = -t^4 \)
(a) Increasing on \((-\infty, 0)\); Decreasing on \((0, \infty)\)
(b) \[
\begin{array}{c|cccc}
  t & -2 & -1 & 0 & 1 & 2 \\
f(t) & -16 & -1 & 0 & 1 & 16 \\
\end{array}
\]

45. \( f(x) = \sqrt{1 - x} \)
(a) Decreasing on \((-\infty, 1)\)
(b) \[
\begin{array}{c|cccc}
  x & -3 & -2 & -1 & 0 & 1 \\
f(x) & 2 & \sqrt{3} & \sqrt{2} & 1 & 0 \\
\end{array}
\]

47. \( f(x) = x^{3/2} \)
(a) Increasing on \((0, \infty)\)
(b) \[
\begin{array}{c|cccc}
  x & 0 & 1 & 2 & 3 & 4 \\
f(x) & 0 & 1 & 2.8 & 5.2 & 8 \\
\end{array}
\]

49. \( f(x) = (x - 4)(x + 2) \)

51. \( f(x) = -x^2 + 3x - 2 \)

53. \( f(x) = x(x - 2)(x + 3) \)

55. \( f(x) = 4 - x \)

57. \( f(x) = x^2 + x \)

59. \( f(x) = \sqrt{x - 1} \)

Relative minimum: \((1, -9)\)
Relative maximum: \((1.5, 0.25)\)
Relative minimum: \((1.12, -4.06)\)
Relative maximum: \((-1.79, 8.21)\)

\( f(x) \geq 0 \) on \((-\infty, 4] \).
\( f(x) \geq 0 \) on \((-\infty, -1] \) and \([0, \infty) \).
\( f(x) \geq 0 \) on \([1, \infty) \).
61. $f(x) = -(1 + |x|)$
   $f(x)$ is never greater than 0.
   ($f(x) < 0$ for all $x$)

63. $f(x) = -2x + 15$
   \[
   \frac{f(3) - f(0)}{3 - 0} = \frac{9 - 15}{3} = -2
   \]
   The average rate of change from $x_1 = 0$ to $x_2 = 3$ is $-2$.

65. $f(x) = x^2 + 12x - 4$
   \[
   \frac{f(5) - f(1)}{5 - 1} = \frac{81 - 9}{4} = 18
   \]
The average rate of change from $x_1 = 1$ to $x_2 = 5$ is 18.

67. $f(x) = x^3 - 3x^2 - x$
   \[
   \frac{f(3) - f(1)}{3 - 1} = \frac{-3 - (-3)}{2} = 0
   \]
The average rate of change from $x_1 = 1$ to $x_2 = 3$ is 0.

69. $f(x) = -\sqrt{x^2 - 2} + 5$
   \[
   \frac{f(11) - f(3)}{11 - 3} = \frac{2 - 4}{8} = \frac{1}{4}
   \]
The average rate of change from $x_1 = 3$ to $x_2 = 11$ is $-\frac{1}{4}$.

71. $f(x) = x^6 - 2x^2 + 3$
   \[
   f(-x) = (-x)^6 - 2(-x)^2 + 3
   = x^6 - 2x^2 + 3
   = f(x)
   \]
   $f$ is even.

73. $g(x) = x^3 - 5x$
   $g(-x) = (-x)^3 - 5(-x)$
   \[
   = -x^3 + 5x
   = -g(x)
   \]
   $g$ is odd.

75. $f(t) = t^2 + 2t - 3$
   $f(-t) = (-t)^2 + 2(-t) - 3$
   \[
   = t^2 - 2t - 3
   \]
   $\neq f(t), \neq -f(t)$

77. $h = \text{top} - \text{bottom}$
   \[
   = (-x^2 + 4x - 1) - 2
   = -x^2 + 4x - 3
   \]

79. $h = \text{top} - \text{bottom}$
   \[
   = (4x - x^2) - 2x
   = 2x - x^2
   \]

81. $L = \text{right} - \text{left}$
   \[
   = \frac{1}{2}y^2 - 0 = \frac{1}{2}y^2
   \]

83. $L = \text{right} - \text{left}$
   \[
   = 4 - y^2
   \]

85. $L = -0.294x^2 + 97.744x - 664.875$, $20 \leq x \leq 90$

   (a) [Graph]
   \[L = 6000\]

   (b) $L = 2000$ when $x = 29.9645 \approx 30$ watts.

87. (a) For the average salaries of college professors, a scale of $10,000$ would be appropriate.

   (b) For the population of the United States, use a scale of 10,000,000.

   (c) For the percent of the civilian workforce that is unemployed, use a scale of 1%.
89. \( r = 15.639t^3 - 104.75t^2 + 303.5t - 301 \), \( 2 \leq t \leq 7 \)

(a) 

(b) \[ \frac{r(7) - r(2)}{7 - 2} = \frac{2054.927 - 12.112}{5} = 408.563 \]

The average rate of change from 2002 to 2007 is $408.563 billion per year. The estimated revenue is increasing each year at a rapid pace.

91. \( s_0 = 6, v_0 = 64 \)

(a) \( s = -16t^2 + 64t + 6 \)

(b) 

(c) \[ \frac{s(3) - s(0)}{3 - 0} = \frac{54 - 6}{3} = 16 \]

(d) The average rate of change of the height of the object with respect to time over the interval \( t_1 = 0 \) to \( t_2 = 3 \) is 16 feet per second.

(e) \( s(0) = 6, m = 16 \)

Secant line: \( y - 6 = 16(t - 0) \)

\[ y = 16t + 6 \]

(f) 

93. \( v_0 = 120, s_0 = 0 \)

(a) \( s = -16t^2 + 120t \)

(b) 

(c) \[ \frac{s(5) - s(3)}{5 - 3} = \frac{200 - 216}{2} = -8 \]

(d) The average decrease in the height of the object over the interval \( t_1 = 3 \) to \( t_2 = 5 \) is 8 feet per second.

(e) \( s(5) = 200, m = -8 \)

Secant line: \( y - 200 = -8(t - 5) \)

\[ y = -8t + 240 \]

(f) 

95. \( v_0 = 0, s_0 = 120 \)

(a) \( s = -16t^2 + 120 \)

(b) 

(c) \[ \frac{s(2) - s(0)}{2 - 0} = \frac{56 - 120}{2} = -32 \]

(d) On the interval \( t_1 = 0 \) to \( t_2 = 2 \), the height of the object is decreasing at a rate of 32 feet per second.

(e) \( s(0) = 120, m = -32 \)

Secant line: \( y - 120 = -32(t - 0) \)

\[ y = -32t + 120 \]

(f) 

97. False. The function \( f(x) = \sqrt{x^2 + 1} \) has a domain of all real numbers.
99. (a) Even. The graph is a reflection in the x-axis.
(b) Even. The graph is a reflection in the y-axis.
(c) Even. The graph is a vertical translation of $f$.
(d) Neither. The graph is a horizontal translation of $f$.

101. \((-\frac{3}{2}, 4)\)
(a) If $f$ is even, another point is \((\frac{3}{2}, 4)\).
(b) If $f$ is odd, another point is \((\frac{3}{2}, -4)\).

103. \((4, 9)\)
(a) If $f$ is even, another point is \((-4, 9)\).
(b) If $f$ is odd, another point is \((-4, -9)\).

105. (a) $y = x$
\[\begin{array}{c}
\includegraphics[width=2cm]{graph1.pdf}
\end{array}\]
(b) $y = x^2$
\[\begin{array}{c}
\includegraphics[width=2cm]{graph2.pdf}
\end{array}\]
(c) $y = x^3$
\[\begin{array}{c}
\includegraphics[width=2cm]{graph3.pdf}
\end{array}\]
(d) $y = x^4$
\[\begin{array}{c}
\includegraphics[width=2cm]{graph4.pdf}
\end{array}\]
(e) $y = x^5$
\[\begin{array}{c}
\includegraphics[width=2cm]{graph5.pdf}
\end{array}\]
(f) $y = x^6$
\[\begin{array}{c}
\includegraphics[width=2cm]{graph6.pdf}
\end{array}\]

All the graphs pass through the origin. The graphs of the odd powers of $x$ are symmetric with respect to the origin and the graphs of the even powers are symmetric with respect to the y-axis. As the powers increase, the graphs become flatter in the interval $-1 < x < 1$.

107. $x^2 - 10x = 0$
\[x(x - 10) = 0\]
\[x = 0 \text{ or } x = 10\]

109. $x^3 - x = 0$
\[x(x^2 - 1) = 0\]
\[x = 0 \text{ or } x^2 - 1 = 0\]
\[x^2 = 1\]
\[x = \pm 1\]

111. $f(x) = 5x - 8$
(a) $f(9) = 5(9) - 8 = 37$
(b) $f(-4) = 5(-4) - 8 = -28$
(c) $f(x - 7) = 5(x - 7) - 8 = 5x - 35 - 8 = 5x - 43$

113. $f(x) = \sqrt{x - 12} - 9$
(a) $f(12) = \sqrt{12 - 12} - 9 = 0 - 9 = -9$
(b) $f(40) = \sqrt{40 - 12} - 9 = \sqrt{28} - 9 = 2\sqrt{7} - 9$
(c) $f(-\sqrt{36})$ does not exist. The given value is not in the domain of the function.

115. 
\[f(x) = x^2 - 2x + 9\]
\[f(3 + h) = (3 + h)^2 - 2(3 + h) + 9\]
\[= 9 + 6h + h^2 - 6 - 2h + 9\]
\[= h^2 + 4h + 12\]
\[f(3) = 3^2 - 2(3) + 9 = 12\]
\[\frac{f(3 + h) - f(3)}{h} = \frac{(h^2 + 4h + 12) - 12}{h} = \frac{h^2 + 4h}{h} = \frac{h(h + 4)}{h} = h + 4, \; h \neq 0\]
Section 2.4  A Library of Parent Functions

You should be able to identify and graph the following types of functions:
(a) Linear functions like $f(x) = ax + b$
(b) Squaring functions like $f(x) = x^2$
(c) Cubic functions like $f(x) = x^3$
(d) Square root functions like $f(x) = \sqrt{x}$
(e) Reciprocal functions like $f(x) = \frac{1}{x}$
(f) Constant functions like $f(x) = c$
(g) Absolute value functions like $f(x) = |x|$  
(h) Step and piecewise-defined functions like $f(x) = \lfloor x \rfloor$

You should be able to determine the following about these common functions:
(a) Domain and range
(b) $x$-intercept(s) and $y$-intercept
(c) Symmetries
(d) Where it is increasing, decreasing, or constant
(e) If it is odd, even or neither
(f) Relative maximums and relative minimums

Vocabulary Check

1. $f(x) = \lfloor x \rfloor$  
   (g) greatest integer function

2. $f(x) = x$  
   (i) identity function

3. $f(x) = \frac{1}{x}$  
   (h) reciprocal function

4. $f(x) = x^2$  
   (a) squaring function

5. $f(x) = \sqrt{x}$  
   (b) square root function

6. $f(x) = c$  
   (e) constant function

7. $f(x) = |x|$  
   (f) absolute value function

8. $f(x) = x^3$  
   (c) cubic function

9. $f(x) = ax + b$  
   (d) linear function

1. (a) $f(1) = 4$, $f(0) = 6$
   - (1, 4) and (0, 6)
   - $m = \frac{6 - 4}{0 - 1} = 2$
   - $y - 6 = 2(x - 0)$
   - $y = 2x + 6$
   - $f(x) = -2x + 6$
3. (a) \( f(5) = -4, f(-2) = 17 \)
(5, -4) and (-2, 17)
\[ m = \frac{17 - (-4)}{-2 - 5} = \frac{21}{-7} = -3 \]
\[ y - (-4) = -3(x - 5) \]
y + 4 = -3x + 15
\[ y = -3x + 11 \]
f(x) = -3x + 11

(b) 
\[ \begin{array}{c}
\text{Graph of } y = -3x + 11 \\
\end{array} \]

5. (a) \( f(-5) = -1, f(5) = -1 \)
(-5, -1) and (5, -1)
\[ m = \frac{-1 - (-1)}{5 - (-5)} = \frac{0}{10} = 0 \]
y + 1 = 0
\[ y = -1 \]
f(x) = -1

(b) 
\[ \begin{array}{c}
\text{Graph of } y = -1 \\
\end{array} \]

7. (a) \( f\left(\frac{1}{2}\right) = -6, f(4) = -3 \)
\( \left(\frac{1}{2}, -6\right) \) and (4, -3)
\[ m = \frac{-3 - (-6)}{4 - (1/2)} = \frac{3}{7/2} = \frac{6}{7} \]
y - (-3) = \frac{6}{7}(x - 4)
y + 3 = \frac{6}{7}x - \frac{24}{7}
y = \frac{6}{7}x - \frac{45}{7}
f(x) = \frac{6}{7}x - \frac{45}{7}

9. \( f(x) = -x - \frac{3}{4} \)

11. \( f(x) = -\frac{1}{9}x - \frac{5}{2} \)

13. \( f(x) = x^2 - 2x \)

15. \( h(x) = -x^2 + 4x + 12 \)

17. \( f(x) = x^3 - 1 \)

19. \( f(x) = (x - 1)^3 + 2 \)
21. \( f(x) = 4\sqrt{x} \)

22. \( g(x) = 2 - \sqrt{x + 4} \)

23. \( f(x) = \frac{1}{x} \)

24. \( h(x) = \frac{1}{x + 2} \)

25. \( f(x) = \lfloor x \rfloor \)
   (a) \( f(2.1) = 2 \)
   (b) \( f(2.9) = 2 \)
   (c) \( f(-3.1) = -4 \)
   (d) \( f\left(\frac{7}{2}\right) = 3 \)

26. \( h(x) = \lceil x + 3 \rceil \)
   (a) \( h(-2) = \lfloor 1 \rfloor = 1 \)
   (b) \( h\left(\frac{1}{2}\right) = \lceil 3.5 \rceil = 3 \)
   (c) \( h(4.2) = \lceil 7.2 \rceil = 7 \)
   (d) \( h(-21.6) = \lceil -18.6 \rceil = -19 \)

27. \( g(x) = -\lfloor x \rfloor \)

28. \( g(x) = \lfloor x \rfloor - 2 \)

29. \( g(x) = \lceil x + 1 \rceil \)

30. \( f(x) = \begin{cases} 2x + 3, & x < 0 \\ 3 - x, & x \geq 0 \end{cases} \)

31. \( f(x) = \begin{cases} \sqrt{4 + x}, & x < 0 \\ \sqrt{4 - x}, & x \geq 0 \end{cases} \)

32. \( f(x) = \begin{cases} x^2 + 5, & x \leq 1 \\ -x^2 + 4x + 3, & x > 1 \end{cases} \)
49. \( h(x) = \begin{cases} 
4 - x^2, & x < -2 \\
3 + x, & -2 \leq x < 0 \\
x^2 + 1, & x \geq 0
\end{cases} \)

51. \( s(x) = 2\left(\frac{1}{2}x - \left\lfloor \frac{1}{2}x \right\rfloor\right) \)

53. (a) Common function: \( f(x) = |x| \)
(b) \( g(x) = |x + 2| - 1 \)
(c) Sawtooth pattern

55. (a) Common function: \( f(x) = x^3 \)
(b) \( g(x) = (x - 1)^3 - 2 \)
(c) \( \begin{align*} 
\text{(a)} & \\
& \text{Domain: } (-\infty, \infty) \\
& \text{Range: } [0, 2] \\
\end{align*} \)

57. (a) Common function: \( f(x) = c \)
(b) \( g(x) = 2 \)
(c) \( \begin{align*} 
\text{(a)} & \\
& \text{Domain: } (-\infty, \infty) \\
& \text{Range: } [0, 2] \\
\end{align*} \)

59. (a) Common function: \( f(x) = x \)
(b) \( g(x) = x - 2 \)
(c) \( \begin{align*} 
\text{(a)} & \\
& \text{Domain: } (-\infty, \infty) \\
& \text{Range: } [0, 2] \\
\end{align*} \)

61. \( C = 0.60 - 0.42[1 - t], \ t > 0 \)
(a) \( \begin{align*} 
& \text{Cost (in dollars)} \\
& \text{Time (in minutes)}
\end{align*} \)
(b) \( C(12.5) = 5.64 \)

63. \( C = 10.75 - 3.95[1], \ x > 0 \)
(a) \( \begin{align*} 
& \text{Cost of overnight delivery} \\
& \text{Weight (in pounds)}
\end{align*} \)
(b) \( C(10.33) = 10.75 + 3.95(10) = 50.25 \)

65. \( W(h) = \begin{cases} 
12h, & 0 < h \leq 40 \\
18(h - 40) + 480, & h > 40
\end{cases} \)
(a) \( W(30) = 12(30) = 360 \)
\( W(40) = 12(40) = 480 \)
\( W(45) = 18(5) + 480 = 570 \)
\( W(50) = 18(10) + 480 = 660 \)
(b) \( W(h) = \begin{cases} 
12h, & 0 < h \leq 45 \\
18(h - 45) + 540, & h > 45
\end{cases} \)
67. (a) The domain of \( f(x) = -1.97x + 26.3 \) is \( 6 < x \leq 12 \).

One way to see this is to notice that this is the equation of a line with negative slope, so the function values are decreasing as \( x \) increases, which matches the data for the corresponding part of the table. The domain of \( f(x) = 0.505x^2 - 1.47x + 6.3 \) is then \( 1 \leq x \leq 6 \).

(c) \( f(5) = 0.505(5)^2 - 1.47(5) + 6.3 \)

\[ = 0.505(25) - 7.35 + 6.3 = 11.575 \]

\( f(11) = -1.97(11) + 26.3 = 4.63 \)

These values represent the income in thousands of dollars for the months of May and November, respectively.

(d) The model values are very close to the actual values.

<table>
<thead>
<tr>
<th>Month, ( x )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revenue, ( y )</td>
<td>5.2</td>
<td>5.6</td>
<td>6.6</td>
<td>8.3</td>
<td>11.5</td>
<td>15.8</td>
<td>12.8</td>
<td>10.1</td>
<td>8.6</td>
<td>6.9</td>
<td>4.5</td>
<td>2.7</td>
</tr>
<tr>
<td>Model, ( f(x) )</td>
<td>5.3</td>
<td>5.4</td>
<td>6.4</td>
<td>8.5</td>
<td>11.6</td>
<td>15.7</td>
<td>12.5</td>
<td>10.5</td>
<td>8.6</td>
<td>6.6</td>
<td>4.6</td>
<td>2.7</td>
</tr>
</tbody>
</table>

69. False. A piecewise-defined function is a function that is defined by two or more equations over a specified domain. That domain may or may not include \( x \)- and \( y \)-intercepts.

71. For the line through \((0, 6)\) and \((3, 2)\): \( m = \frac{6 - 2}{0 - 3} = -\frac{4}{3} \)

\[ y - 6 = -\frac{4}{3}(x - 0) \Rightarrow y = -\frac{4}{3}x + 6 \]

For the line through \((3, 2)\) and \((8, 0)\): \( m = \frac{2 - 0}{3 - 8} = \frac{2}{5} \)

\[ y - 0 = \frac{2}{5}(x - 8) \Rightarrow y = \frac{2}{5}x + \frac{16}{5} \]

\( f(x) = \begin{cases} 
-\frac{4}{3}x + 6, & 0 \leq x \leq 3 \\
\frac{2}{5}x + \frac{16}{5}, & 3 < x \leq 8 
\end{cases} \)

Note that the respective domains can also be \( 0 \leq x < 3 \) and \( 3 \leq x \leq 8 \).

73. \( 3x + 4 \leq 12 - 5x \)

\( 8x + 4 \leq 12 \)

\( 8x \leq 8 \)

\( x \leq 1 \)

75. \( L_1: (-2, -2) \) and \((2, 10)\)

\[ m_1 = \frac{10 - (-2)}{2 - (-2)} = \frac{12}{4} = 3 \]

\( L_2: (-1, 3) \) and \((3, 9)\)

\[ m_2 = \frac{9 - 3}{3 - (-1)} = \frac{6}{4} = \frac{3}{2} \]

The lines are neither parallel nor perpendicular.
Section 2.5  Transformations of Functions

You should know the basic types of transformations.

Let \( y = f(x) \) and let \( c \) be a positive real number.

1. \( h(x) = f(x) + c \)  
   Vertical shift \( c \) units upward
2. \( h(x) = f(x) - c \)  
   Vertical shift \( c \) units downward
3. \( h(x) = f(x - c) \)  
   Horizontal shift \( c \) units to the right
4. \( h(x) = f(x + c) \)  
   Horizontal shift \( c \) units to the left
5. \( h(x) = -f(x) \)  
   Reflection in the \( x \)-axis
6. \( h(x) = f(-x) \)  
   Reflection in the \( y \)-axis
7. \( h(x) = cf(x), c > 1 \)  
   Vertical stretch
8. \( h(x) = cf(x), 0 < c < 1 \)  
   Vertical shrink
9. \( h(x) = f(cx), c > 1 \)  
   Horizontal shrink
10. \( h(x) = f(cx), 0 < c < 1 \)  
    Horizontal stretch

Vocabulary Check

1. rigid
2. \( -f(x) \); \( f(-x) \)
3. nonrigid
4. horizontal shrink; horizontal stretch
5. vertical stretch; vertical shrink
6. (a) iv  (b) ii  (c) iii  (d) i

1. (a) \( f(x) = |x| + c \)  
   Vertical shifts
   \( c = -1 : f(x) = |x| - 1 \)  
   1 unit down
   \( c = 1 : f(x) = |x| + 1 \)  
   1 unit up
   \( c = 3 : f(x) = |x| + 3 \)  
   3 units up

   \( f(x) = |x - c| \)  
   Horizontal shifts
   \( c = -1 : f(x) = |x + 1| \)  
   1 unit left
   \( c = 1 : f(x) = |x - 1| \)  
   1 unit right
   \( c = 3 : f(x) = |x - 3| \)  
   3 units right

   \( f(x) = |x + 4| + c \)  
   Horizontal shift four units left and a vertical shift
   \( c = -1 : f(x) = |x + 4| - 1 \)  
   1 unit down
   \( c = 1 : f(x) = |x + 4| + 1 \)  
   1 unit up
   \( c = 3 : f(x) = |x + 4| + 3 \)  
   3 units up
3. (a) \( f(x) = [x] + c \)
   \( c = -2 : f(x) = [x] - 2 \)
   Vertical shifts
   2 units down

   \( c = 0 : f(x) = [x] \)
   Common function

   \( c = 2 : f(x) = [x] + 2 \)
   2 units up

(b) \( f(x) = [x + c] \)
    Horizontal shifts
    2 units right

   \( c = -2 : f(x) = [x - 2] \)
   Common function

   \( c = 0 : f(x) = [x] \)
   2 units left

(c) \( f(x) = [x - 1] + c \)
    Horizontal shift 1 unit right and a vertical shift
    2 units down

   \( c = -2 : f(x) = [x - 1] - 2 \)
   Common function

   \( c = 0 : f(x) = [x - 1] \)
   2 units up

5. (a) \( y = f(x) + 2 \)
   Vertical shift 2 units upward

   \[ (0, 1) \rightarrow (0, 3) \]

   \[ (1, 2) \rightarrow (3, 4) \]

   \[ (3, 3) \rightarrow (5, 5) \]

(b) \( y = f(x - 2) \)
   Horizontal shift 2 units to the right

   \[ (0, 1) \rightarrow (2, 3) \]

   \[ (1, 2) \rightarrow (3, 4) \]

   \[ (3, 3) \rightarrow (5, 5) \]

(c) \( y = 2f(x) \)
   Vertical stretch (each y-value is multiplied by 2)

   \[ (0, 1) \rightarrow (0, 2) \]

   \[ (1, 2) \rightarrow (2, 4) \]

   \[ (3, 3) \rightarrow (6, 6) \]

(d) \( y = -f(x) \)
   Reflection in the x-axis

   \[ (0, 1) \rightarrow (0, -1) \]

   \[ (1, 2) \rightarrow (1, -2) \]

   \[ (3, -1) \rightarrow (3, 1) \]

(e) \( y = f(x + 3) \)
   Horizontal shift 3 units to the left

   \[ (0, 1) \rightarrow (-3, 1) \]

   \[ (1, 2) \rightarrow (-2, 2) \]

   \[ (-3, -1) \rightarrow (0, -1) \]

(f) \( y = f(-x) \)
   Reflection in the y-axis

   \[ (0, 1) \rightarrow (0, -1) \]

   \[ (1, 2) \rightarrow (-1, 2) \]

   \[ (-3, 1) \rightarrow (3, 1) \]
5. —CONTINUED—

(g) \( y = f\left(\frac{1}{2}x\right) \)
Horizontal stretch (each \( x \)-value is multiplied by 2)

7. (a) \( y = f(x) - 1 \)
Vertical shift 1 unit downward

(b) \( y = f(x - 1) \)
Horizontal shift 1 unit to the right

(c) \( y = f(-x) \)
Reflection about the \( y \)-axis

(d) \( y = f(x + 1) \)
Horizontal shift 1 unit to the left

(e) \( y = -f(x - 2) \)
Reflection about the \( x \)-axis and a horizontal shift 2 units to the right

(f) \( y = \frac{1}{2}f(x) \)
Vertical shrink (each \( y \)-value is multiplied by \( \frac{1}{2} \))

(g) \( y = f(2x) \)
Horizontal shrink (each \( x \)-value is multiplied by \( \frac{1}{2} \))
9. (a) Vertical shift 1 unit downward
   \[ f(x) = x^2 - 1 \]
   (b) Reflection about the x-axis, horizontal shift 1 unit to the left, and a vertical shift 1 unit upward
   \[ f(x) = -(x + 1)^2 + 1 \]
   (c) Reflection about the x-axis, horizontal shift 2 units to the right, and a vertical shift 6 units upward
   \[ f(x) = -(x - 2)^2 + 6 \]
   (d) Horizontal shift 5 units to the right and a vertical shift 3 units downward
   \[ f(x) = (x - 5)^2 - 3 \]

13. Common function: \( f(x) = x^3 \)
    Horizontal shift 2 units to the right: \( y = (x - 2)^3 \)

17. Common function: \( f(x) = \sqrt{x} \)
    Reflection in the x-axis and a vertical shift 1 unit upward:
    \( y = -\sqrt{x} + 1 \)

19. \( g(x) = 12 - x^2 \)
   (a) Common function: \( f(x) = x^2 \)
   (b) Reflection in the x-axis and a vertical shift 12 units upward
   (c) \( g(x) = 12 - f(x) \)

21. \( g(x) = x^3 + 7 \)
   (a) Common function: \( f(x) = x^3 \)
   (b) Vertical shift 7 units upward
   (c) \( g(x) = f(x) + 7 \)

23. \( g(x) = \frac{2}{3}x^2 + 4 \)
   (a) Common function: \( f(x) = x^2 \)
   (b) Vertical shrink of two-thirds, and a vertical shift 4 units upward
   (c) \( g(x) = \frac{2}{3}f(x) + 4 \)

25. \( g(x) = 2 - (x + 5)^2 \)
   (a) Common function: \( f(x) = x^2 \)
   (b) Reflection in the x-axis, horizontal shift 5 units to the left, and a vertical shift 2 units upward
   (c) \( g(x) = 2 - f(x + 5) \)
27. \( g(x) = \sqrt{3x} \)
   (a) Common function: \( f(x) = \sqrt{x} \)
   (b) Horizontal shrink by \( \frac{1}{3} \)
   (c) \( g(x) = f(3x) \)

29. \( g(x) = (x - 1)^3 + 2 \)
   (a) Common function: \( f(x) = x^3 \)
   (b) Horizontal shift 1 unit to the right and a vertical shift 2 units upward
   (c) \( g(x) = f(x - 1) + 2 \)

31. \( g(x) = -|x| - 2 \)
   (a) Common function: \( f(x) = |x| \)
   (b) Reflection in the x-axis; vertical shift 2 units downward
   (c) \( g(x) = -f(x) - 2 \)

33. \( g(x) = -|x + 4| + 8 \)
   (a) Common function: \( f(x) = |x| \)
   (b) Reflection in the x-axis, horizontal shift 4 units to the left, and a vertical shift 8 units upward
   (c) \( g(x) = -f(x + 4) + 8 \)

35. \( g(x) = 3 - \lfloor x \rfloor \)
   (a) Common function: \( f(x) = \lfloor x \rfloor \)
   (b) Reflection in the x-axis and a vertical shift 3 units up
   (c) \( g(x) = 3 - f(x) \)

37. \( g(x) = \sqrt{x - 9} \)
   (a) Common function: \( f(x) = \sqrt{x} \)
   (b) Horizontal shift 9 units to the right
   (c) \( g(x) = f(x - 9) \)
39. $g(x) = \sqrt{7} - x - 2$, or $g(x) = \sqrt{(x - 7)} - 2$
   (a) Common function: $f(x) = \sqrt{x}$
   (b) Reflection in the y-axis, horizontal shift 7 units to the right, and a vertical shift 2 units downward
   (c) $g(x) = f(7 - x) - 2$

41. $g(x) = \sqrt{\frac{x}{2}} - 4$
   (a) Common function: $f(x) = \sqrt{x}$
   (b) Horizontal stretch (each x-value is multiplied by 2) and a vertical shift 4 units down
   (c) $g(x) = f\left(\frac{x}{2}\right) - 4$

43. $f(x) = x^2$ moved 2 units to the right and 8 units down.
   $g(x) = (x - 2)^2 - 8$

45. $f(x) = x^3$ moved 13 units to the right.
   $g(x) = (x - 13)^3$

47. $f(x) = |x|$ moved 10 units up and reflected about the x-axis.
   $g(x) = -(|x| + 10) = -|x| - 10$

49. $f(x) = \sqrt{x}$ moved 6 units to the left and reflected in both the x- and y-axes.
   $g(x) = -\sqrt{-x} + 6$

51. $f(x) = x^2$
   (a) Reflection in the x-axis and a vertical stretch (each y-value is multiplied by 3)
   $g(x) = -3x^2$
   (b) Vertical shift 3 units upward and a vertical stretch (each y-value is multiplied by 4)
   $g(x) = 4x^2 + 3$

53. $f(x) = |x|
   (a) Reflection in the x-axis and a vertical shrink (each y-value is multiplied by $\frac{1}{2}$)
   $g(x) = -\frac{1}{2}|x|$
   (b) Vertical stretch (each y-value is multiplied by 3) and a vertical shift 3 units downward
   $g(x) = 3|x| - 3$

55. Common function: $f(x) = x^3$
   Vertical stretch (each y-value is multiplied by 2)
   $g(x) = 2x^3$

57. Common function: $f(x) = x^2$
   Reflection in the x-axis; vertical shrink (each y-value is multiplied by $\frac{1}{2}$)
   $g(x) = -\frac{1}{2}x^2$

59. Common function: $f(x) = \sqrt{x}$
   Reflection in the y-axis; vertical shrink (each y-value is multiplied by $\frac{1}{2}$)
   $g(x) = \frac{1}{2}\sqrt{-x}$

61. Common function: $f(x) = x^3$
   Reflection in the x-axis, horizontal shift 2 units to the right and a vertical shift 2 units upward
   $g(x) = -(x - 2)^3 + 2$

63. Common function: $f(x) = \sqrt{x}$
   Reflection in the x-axis and a vertical shift 3 units downward
   $g(x) = -\sqrt{x} - 3$
65. (a) \( g(x) = f(x) + 2 \)  
Vertical shift 2 units upward

(b) \( g(x) = f(x) - 1 \)  
Vertical shift 1 unit downward

(c) \( g(x) = f(-x) \)  
Reflection in the y-axis

(d) \( g(x) = -2f(x) \)  
Reflection in the x-axis and a vertical stretch (each y-value is multiplied by 2)

(e) \( g(x) = f(4x) \)  
Horizontal shrink (each x-value is multiplied by \( \frac{1}{4} \))

(f) \( g(x) = f\left(\frac{1}{2}x\right) \)  
Horizontal stretch (each x-value is multiplied by 2)

67. \( F = f(t) = 20.6 + 0.035t^2 \), \( 0 \leq t \leq 22 \)

(a) A vertical shrink by 0.035 and a vertical shift of 20.6 units upward

(b) \( \frac{f(22) - f(0)}{22 - 0} = \frac{37.54 - 20.6}{22} = 0.77 \)
The average increase in fuel used by trucks was 0.77 billion gallons per year between 1980 and 2002.

(c) \( g(t) = 20.6 + 0.035(t + 10)^2 = f(t + 10) \)
This represents a horizontal shift 10 units to the left.

(d) \( g(20) = 52.1 \) billion gallons
Yes. There are many factors involved here. The number of trucks on the road continues to increase but are more fuel efficient. The availability and the cost of overseas and domestic fuel also plays a role in usage.

69. True, since \( |x| = |-x| \), the graphs of \( f(x) = |x| + 6 \) and \( f(x) = |-x| + 6 \) are identical.

71. (a) The profits were only \( \frac{3}{4} \) as large as expected: \( g(t) = \frac{3}{4} f(t) \)
(b) The profits were $10,000 greater than predicted: \( g(t) = f(t) + 10,000 \)
(c) There was a two-year delay: \( g(t) = f(t - 2) \)
Chapter 2 Functions and Their Graphs

81. Vocabulary Check

1. addition, subtraction, multiplication, division
2. composition
3. \( g(x) \)
4. inner; outer

Section 2.6 Combinations of Functions: Composite Functions

- Given two functions, \( f \) and \( g \), you should be able to form the following functions (if defined):
  1. Sum: \( (f + g)(x) = f(x) + g(x) \)
  2. Difference: \( (f - g)(x) = f(x) - g(x) \)
  3. Product: \( (fg)(x) = f(x)g(x) \)
  4. Quotient: \( (f/g)(x) = f(x)/g(x), g(x) \neq 0 \)
  5. Composition of \( f \) with \( g \): \( (f \circ g)(x) = f(g(x)) \)
  6. Composition of \( g \) with \( f \): \( (g \circ f)(x) = g(f(x)) \)

- Given two functions, \( f(x) = \frac{4}{x} + \frac{4}{1-x} = \frac{4(1-x) + 4x}{x(1-x)} = \frac{4 - 4x + 4x}{x(1-x)} = \frac{4}{x(1-x)} \)

- Given two functions, \( f(x) = x^2 - 6x + 11 \)
  a. \( f(-3) = (-3)^2 - 6(-3) + 11 = 38 \)
  b. \( f(-\frac{1}{2}) = (-\frac{1}{2})^2 - 6(-\frac{1}{2}) + 11 = \frac{1}{4} + 3 + 11 = \frac{57}{4} \)
  c. \( f(x-3) = (x-3)^2 - 6(x-3) + 11 = x^2 - 6x + 9 - 6x + 18 + 11 = x^2 - 12x + 38 \)

- Given two functions, \( f(x) = \frac{2}{11 - x} \)
  Domain: All real numbers except \( x = 11 \)

- Given two functions, \( f(x) = \sqrt{81 - x^2} \)
  
  - Test intervals: \( -\infty, -9 \), \( -9, 9 \), \( 9, \infty \)
  - Critical numbers: \( x = \pm 9 \)
  - Solution: \( [-9, 9] \)

- Given two functions, \( f(x) = x^2 - 9 \)
  - Test: Is \( 81 - x^2 \geq 0 \)?
    - Solution: \( -9 \leq x \leq 9 \)
For Exercises 13–23, \( f(x) = x^2 + 1 \) and \( g(x) = x - 4 \).

13. \((f + g)(2) = f(2) + g(2) = (2^2 + 1) + (2 - 4) = 3 \)

17. \((f - g)(3t) = f(3t) - g(3t) = (3t^2 + 1) - (3t - 4) = 9t^2 - 3t + 5 \)

5. \( f(x) = x + 2, g(x) = x - 2 \)
   (a) \((f + g)(x) = f(x) + g(x) = (x + 2) + (x - 2) = 2x \)
   (b) \((f - g)(x) = f(x) - g(x) = (x + 2) - (x - 2) = 4 \)
   (c) \((fg)(x) = f(x) \cdot g(x) = (x + 2)(x - 2) = x^2 - 4 \)
   (d) \[ \left( \frac{f}{g} \right)(x) = \frac{f(x)}{g(x)} = \frac{x + 2}{x - 2} \]
   Domain: all real numbers except \( x = 2 \)

9. \( f(x) = x^2 + 6, g(x) = \sqrt{1 - x} \)
   (a) \((f + g)(x) = f(x) + g(x) = (x^2 + 6) + \sqrt{1 - x} \)
   (b) \((f - g)(x) = f(x) - g(x) = (x^2 + 6) - \sqrt{1 - x} \)
   (c) \((fg)(x) = f(x) \cdot g(x) = (x^2 + 6)\sqrt{1 - x} \)
   (d) \[ \left( \frac{f}{g} \right)(x) = \frac{f(x)}{g(x)} = \frac{x^2 + 6}{\sqrt{1 - x}} = \frac{(x^2 + 6)\sqrt{1 - x}}{1 - x}. \]
   Domain: \( x < 1 \)

7. \( f(x) = x^2, g(x) = 4x - 5 \)
   (a) \((f + g)(x) = f(x) + g(x) = x^2 + (4x - 5) = x^2 + 4x - 5 \)
   (b) \((f - g)(x) = f(x) - g(x) = x^2 - (4x - 5) = x^2 - 4x + 5 \)
   (c) \((fg)(x) = f(x) \cdot g(x) = x^2(4x - 5) = 4x^3 - 5x^2 \)
   (d) \[ \left( \frac{f}{g} \right)(x) = \frac{f(x)}{g(x)} = \frac{x^2}{4x - 5} \]
   Domain: all real numbers except \( x = \frac{5}{4} \)

11. \( f(x) = \frac{1}{x}, g(x) = \frac{1}{x^2} \)
   (a) \((f + g)(x) = f(x) + g(x) = \frac{1}{x} + \frac{1}{x^2} = \frac{x + 1}{x^2} \)
   (b) \((f - g)(x) = f(x) - g(x) = \frac{1}{x} - \frac{1}{x^2} = \frac{x - 1}{x^2} \)
   (c) \((fg)(x) = f(x) \cdot g(x) = \frac{1}{x} \left( \frac{1}{x^2} \right) = \frac{1}{x^3} \)
   (d) \[ \left( \frac{f}{g} \right)(x) = \frac{f(x)}{g(x)} = \frac{1/x}{1/x^2} = \frac{x^2}{x} = x, \ x \neq 0 \]

15. \((f - g)(0) = f(0) - g(0) = (0^2 + 1) - (0 - 4) = 5 \)

19. \((fg)(6) = f(6)g(6) = (6^2 + 1)(6 - 4) = 74 \)
21. \( \left( \frac{f}{g} \right)(5) = \frac{f(5)}{g(5)} = \frac{5^2 + 1}{5 - 4} = 26 \)

23. \( \left( \frac{f}{g} \right)(-1) - g(3) = \frac{f(-1)}{g(-1)} - g(3) \)
\[ \frac{(-1)^2 + 1}{-1 - 4} - (3 - 4) \]
\[ = \frac{2}{5} + 1 = \frac{3}{5} \]

25. \( f(x) = \frac{1}{2}x, g(x) = x - 1, (f + g)(x) = \frac{1}{2}x - 1 \)

27. \( f(x) = x^2, g(x) = -2x, (f + g)(x) = x^2 - 2x \)

29. \( f(x) = 3x, g(x) = -\frac{x^3}{10}, (f + g)(x) = 3x - \frac{x^3}{10} \)

31. \( f(x) = x^2, g(x) = x - 1 \)
(a) \( (f \cdot g)(x) = f(g(x)) = f(x - 1) = (x - 1)^2 \)
(b) \( (g \cdot f)(x) = g(f(x)) = g(x^2) = x^2 - 1 \)
(c) \( (f \cdot f)(x) = f(f(x)) = f(x^2) = (x^2)^2 = x^4 \)

For 0 \( \leq x \leq 2 \), \( f(x) \) contributes most to the magnitude.

For \( x > 6 \), \( g(x) \) contributes most to the magnitude.

33. \( f(x) = \sqrt{x} - 1, g(x) = x^3 + 1 \)
(a) \( (f \cdot g)(x) = f(g(x)) \)
\[ = f(x^3 + 1) \]
\[ = \sqrt{x^3 + 1} - 1 \]
\[ = \sqrt{x^3} - 1 \]
(b) \( (g \cdot f)(x) = g(f(x)) \)
\[ = g(\sqrt{x} - 1) \]
\[ = (\sqrt{x} - 1)^3 + 1 \]
\[ = (x - 1) + 1 = x \]
(c) \( (f \cdot f)(x) = f(f(x)) \)
\[ = f(\sqrt{x} - 1) \]
\[ = \sqrt{x} - 1 - 1 \]

35. \( f(x) = \sqrt{x + 4} \) Domain: \( x \geq -4 \)
\( g(x) = x^2 \) Domain: all real numbers
(a) \( (f \cdot g)(x) = f(g(x)) = f(x^2) = \sqrt{x^4 + 4} \)
Domain: all real numbers
(b) \( (g \cdot f)(x) = g(f(x)) \)
\[ = g(\sqrt{x + 4}) = (\sqrt{x + 4})^2 = x + 4 \]
Domain: \( x \geq -4 \)

37. \( f(x) = x^2 + 1 \) Domain: all real numbers
\( g(x) = \sqrt{x} \) Domain: \( x \geq 0 \)
(a) \( (f \cdot g)(x) = f(g(x)) = f(\sqrt{x}) = (\sqrt{x})^2 + 1 = x + 1 \)
Domain: \( x \geq 0 \)
(b) \( (g \cdot f)(x) = g(f(x)) = g(x^2 + 1) = \sqrt{x^2 + 1} \)
Domain: all real numbers
39. \( f(x) = |x| \)  
   Domain: all real numbers

   \( g(x) = x + 6 \)  
   Domain: all real numbers

(a) \((f \circ g)(x) = f(g(x)) = f(x + 6) = |x + 6| \)  
   Domain: all real numbers

(b) \((g \circ f)(x) = g(f(x)) = g(|x|) = |x| + 6 \)  
   Domain: all real numbers

43. \((a) (f + g)(3) = f(3) + g(3) = 2 + 1 = 3 \)

(b) \( \left( \frac{f}{g} \right)(2) = \frac{f(2)}{g(2)} = \frac{0}{2} = 0 \)

47. Let \( f(x) = x^2 \) and \( g(x) = 2x + 1 \), then \((f \circ g)(x) = h(x)\).

This is not a unique solution.

51. Let \( f(x) = \frac{1}{x} \) and \( g(x) = x + 2 \), then \((f \circ g)(x) = h(x)\).

This is not a unique solution.

55. \( T(x) = R(x) + B(x) = \frac{3x}{2} + \frac{1}{12}x^2 \)

59. \( A(t) = 3.36t^2 - 59.8t + 735, \ N(t) = 1.95t^2 - 42.2t + 603 \)

(a) \((A + N)(t) = A(t) + N(t) = 5.31t^2 - 102.0t + 1338 \)
   This represents the combined Army and Navy personnel (in thousands) from 1990 to 2002, where \( t = 0 \) corresponds to 1990.
   \((A + N)(4) = 1014.96 \) thousand
   \((A + N)(8) = 861.84 \) thousand
   \((A + N)(12) = 878.64 \) thousand

(b) \((A - N)(t) = A(t) - N(t) = 1.41t^2 - 17.6t + 132 \)
   This represents the number of Army personnel (in thousands) more than the number of Navy personnel from 1990 to 2002, where \( t = 0 \) corresponds to 1990.
   \((A - N)(4) = 84.16 \) thousand
   \((A - N)(8) = 81.44 \) thousand
   \((A - N)(12) = 123.84 \) thousand

41. \( f(x) = \frac{1}{x} \)  
   Domain: all real numbers except \( x = 0 \)

   \( g(x) = x + 3 \)  
   Domain: all real numbers

(a) \((f \circ g)(x) = f(g(x)) = f(x + 3) = \frac{1}{x + 3} \)  
   Domain: all real numbers except \( x = -3 \)

(b) \((g \circ f)(x) = g(f(x)) = g\left(\frac{1}{x}\right) = \frac{1}{x} + 3 \)  
   Domain: all real numbers except \( x = 0 \)

45. \((a) (f \circ g)(2) = f(g(2)) = f(0) = 0 \)

(b) \((g \circ f)(2) = g(f(2)) = g(0) = 4 \)

49. Let \( f(x) = \sqrt[3]{x} \) and \( g(x) = x^2 - 4 \), then \((f \circ g)(x) = h(x)\).

This answer is not unique.

53. Let \( f(x) = \frac{x + 3}{4 + x} \) and \( g(x) = -x^2 \), then \((f \circ g)(x) = h(x)\).

This answer is not unique.

57. \( a \) \( c(t) = \frac{p(t) + h(t) - d(t)}{p(t)} \times 100 \)

(b) \( c(5) \) represents the percent change in the population in the year 2005.
61. \[
\begin{array}{|c|c|c|c|}
\hline
\text{Year} & y_1 & y_2 & y_3 \\
\hline
1995 & 146.2 & 329.1 & 44.8 \\
1996 & 152.0 & 344.1 & 48.1 \\
1997 & 162.2 & 359.9 & 52.1 \\
1998 & 175.2 & 382.0 & 55.6 \\
1999 & 184.4 & 412.1 & 57.8 \\
2000 & 194.7 & 449.0 & 57.4 \\
2001 & 205.5 & 496.1 & 57.8 \\
\hline
\end{array}
\]

(a) \(y_1 = 10.20t + 92.7\)
(b) \(y_2 = 3.357t^2 - 26.46t + 379.5\)
(c) \(y_3 = -0.465t^2 + 9.71t + 7.4\)

63. (a) \(r(t) = \frac{x}{2}\)
(b) \(A(r) = \pi r^2\)
(c) \((A \cdot r) = A(r(x)) = A\left(\frac{x}{2}\right) = \pi \left(\frac{x}{2}\right)^2\)

65. \(A(T(t)) = N(3t + 2)\)
\[
\begin{align*}
&= 10(3t + 2)^2 - 20(3t + 2) + 600 \\
&= 10(9t^2 + 12t + 4) - 60t - 40 + 600 \\
&= 90t^2 + 60t + 600 \\
&= 30(3t^2 + 2t + 20), \quad 0 \leq t \leq 6
\end{align*}
\]

This represents the number of bacteria in the food as a function of time.

(b) \(30(3t^2 + 2t + 20) = 1500\)
\[
\begin{align*}
3t^2 + 2t + 20 &= 50 \\
3t^2 + 2t - 30 &= 0
\end{align*}
\]

By the Quadratic Formula, \(t \approx -3.513\) or 2.846.

Choosing the positive value for \(t\), we have \(t \approx 2.846\) hours.

67. (a) \(f(g(x)) = f(0.03x) = 0.03x - 500,000\)
(b) \(g(f(x)) = g(x - 500,000) = 0.03(x - 500,000)\)

\(g(f(x))\) represents your bonus of 3% of an amount over $500,000.

69. False. \((f \cdot g)(x) = 6x + 1\) and \((g \cdot f)(x) = 6x + 6\).

71. Let \(f(x)\) and \(g(x)\) be two odd functions and define \(h(x) = f(x)g(x)\). Then
\[
\begin{align*}
h(-x) &= f(-x)g(-x) \\
&= [-f(x)][-g(x)] \quad \text{since f and g are odd} \\
&= f(x)g(x) \\
&= h(x).
\end{align*}
\]

Thus, \(h(x)\) is even.

Let \(f(x)\) and \(g(x)\) be two even functions and define \(h(x) = f(x)g(x)\). Then
\[
\begin{align*}
h(-x) &= f(-x)g(-x) \\
&= f(x)g(x) \quad \text{since f and g are even} \\
&= h(x).
\end{align*}
\]

Thus, \(h(x)\) is even.
73. \( f(x) = 3x - 4 \)

\[
\frac{f(x + h) - f(x)}{h} = \frac{[3(x + h) - 4] - (3x - 4)}{h} = \frac{3h}{h} = 3, \quad h \neq 0
\]

75. \( f(x) = \frac{4}{x} \)

\[
\frac{f(x + h) - f(x)}{h} = \frac{\frac{4}{x + h} - \frac{4}{x}}{h} = \frac{\frac{4x - 4(x + h)}{x(x + h)}}{h} = \frac{4x - 4x - 4h}{x(x + h)h} \cdot \frac{1}{h} = \frac{-4h}{x(x + h)} \cdot \frac{1}{h} = \frac{-4}{x(x + h)}, \quad h \neq 0
\]

77. Point: \((2, -4)\)

Slope: \(m = 3\)

\[
y - (-4) = 3(x - 2)
\]

\[
y + 4 = 3x - 6
\]

\[
3x - y - 10 = 0
\]

79. Point: \((8, -1)\)

Slope: \(m = -\frac{3}{2}\)

\[
y - (-1) = -\frac{3}{2}(x - 8)
\]

\[
y + 1 = -\frac{3}{2}x + 12
\]

\[
2y + 2 = -3x + 24
\]

\[
3x + 2y - 22 = 0
\]

Section 2.7 Inverse Functions

- Two functions \(f\) and \(g\) are inverses of each other if \(f(g(x)) = x\) for every \(x\) in the domain of \(g\) and \(g(f(x)) = x\) for every \(x\) in the domain of \(f\).

- A function \(f\) has an inverse function if and only if no horizontal line crosses the graph of \(f\) at more than one point.

- The graph of \(f^{-1}\) is a reflection of the graph of \(f\) about the line \(y = x\).

- Be able to find the inverse of a function, if it exists.
  1. Use the Horizontal Line Test to see if \(f^{-1}\) exists.
  2. Replace \(f(x)\) with \(y\).
  3. Interchange \(x\) and \(y\) and solve for \(y\).
  4. Replace \(y\) with \(f^{-1}(x)\).

Vocabulary Check

1. inverse; \(f\)-inverse
2. range; domain
3. \(y = x\)
4. one-to-one
5. Horizontal
1. $f(x) = 6x$
   
   $f^{-1}(x) = \frac{x}{6} = \frac{1}{6}x$
   
   $f(f^{-1}(x)) = f(\frac{x}{6}) = 6(\frac{x}{6}) = x$
   
   $f^{-1}(f(x)) = f^{-1}(6x) = \frac{6x}{6} = x$

5. $f(x) = 3x + 1$
   
   $f^{-1}(x) = \frac{x - 1}{3}$
   
   $f(f^{-1}(x)) = f(\frac{x - 1}{3}) = 3(\frac{x - 1}{3}) + 1 = x$
   
   $f^{-1}(f(x)) = f^{-1}(3x + 1) = \frac{(3x + 1) - 1}{3} = x$

9. The inverse is a line through $(-1, 0)$. Matches graph (c).

13. $f(x) = 2x$, $g(x) = \frac{x}{2}$

   (a) $f(g(x)) = f\left(\frac{x}{2}\right) = 2\left(\frac{x}{2}\right) = x$
   
   $g(f(x)) = g(2x) = \frac{2x}{2} = x$

   (b)

17. $f(x) = \frac{x^3}{8}$, $g(x) = \sqrt[3]{8x}$

   (a) $f(g(x)) = f\left(\sqrt[3]{8x}\right) = \frac{\left(\sqrt[3]{8x}\right)^3}{8} = \frac{8x}{8} = x$
   
   $g(f(x)) = g\left(\frac{x^3}{8}\right) = \sqrt[3]{8\left(\frac{x^3}{8}\right)} = \sqrt[3]{8} = x$

3. $f(x) = x + 9$
   
   $f^{-1}(x) = x - 9$
   
   $f(f^{-1}(x)) = f(x - 9) = (x - 9) + 9 = x$
   
   $f^{-1}(f(x)) = f^{-1}(x + 9) = (x + 9) - 9 = x$

7. $f(x) = \sqrt[3]{x}$

   $f^{-1}(x) = x^3$
   
   $f(f^{-1}(x)) = f(x^3) = \sqrt[3]{x^3} = x$
   
   $f^{-1}(f(x)) = f^{-1}(\sqrt[3]{x}) = (\sqrt[3]{x})^3 = x$

11. The inverse is half a parabola starting at $(1, 0)$. Matches graph (a).

15. $f(x) = 7x + 1$, $g(x) = \frac{x - 1}{7}$

   (a) $f(g(x)) = f\left(\frac{x - 1}{7}\right) = 7\left(\frac{x - 1}{7}\right) + 1 = x$
   
   $g(f(x)) = g(7x + 1) = \frac{(7x + 1) - 1}{7} = x$

   (b)

19. $f(x) = \sqrt{x - 4}$, $g(x) = x^2 + 4$, $x \geq 0$

   (a) $f(g(x)) = f(x^2 + 4)$, $x \geq 0$
   
   $= \sqrt{(x^2 + 4) - 4} = x$
   
   $g(f(x)) = g(\sqrt{x - 4})$
   
   $= (\sqrt{x - 4})^2 + 4 = x$

   (b)
21. \( f(x) = 9 - x^2, x \geq 0; g(x) = \sqrt{9 - x}, x \leq 9 \)
   (a) \( f(g(x)) = f(\sqrt{9 - x}), x \leq 9 \)
   \[ = 9 - (\sqrt{9 - x})^2 = x \]
   \[ g(f(x)) = g(9 - x^2), x \geq 0 \]
   \[ = \sqrt{9 - (9 - x^2)} = x \]

23. \( f(x) = \frac{x - 1}{x + 5}; g(x) = -\frac{5x + 1}{x - 1} \)
   (a) \( f(g(x)) = \left( \frac{5x + 1}{x - 1} \right) \cdot \frac{x - 1}{x - 1} \cdot \frac{x - 1}{x - 1} = \frac{-6x}{-6} = x \)
   \[ g(f(x)) = g\left( \frac{x - 1}{x + 5} \right) \]
   \[ = -\frac{5}{\frac{x - 1}{x + 5} - 1} \cdot \frac{x + 5}{x + 5} = \frac{5(x - 1) + (x + 5)}{(x - 1)(x + 5)} = \frac{6x}{-6} = x \]

25. No, \( \{-2, -1, 1, 0, 2, 1, 2, -2, 3, -6, 4\} \) does not represent a function. -2 and 1 are paired with two different values.

29. Yes, since no horizontal line crosses the graph of \( f \) at more than one point, \( f \) has an inverse.

31. No, since some horizontal lines cross the graph of \( f \) twice, \( f \) does not have an inverse.

33. \( g(x) = \frac{4 - x}{6} \)
   \( g \) passes the horizontal line test, so \( g \) has an inverse.

35. \( h(x) = |x + 4| - |x - 4| \)
   \( h \) does not pass the horizontal line test, so \( h \) does not have an inverse.

37. \( f(x) = -2\sqrt{16 - x^2} \)
   \( f \) does not pass the horizontal line test, so \( f \) does not have an inverse.
39. (a) \( f(x) = 2x - 3 \)
    \[ y = 2x - 3 \]
    \[ x = 2y - 3 \]
    \[ y = \frac{x + 3}{2} \]
    \[ f^{-1}(x) = \frac{x + 3}{2} \]

(c) The graph of \( f^{-1} \) is the reflection of the graph of \( f \) about the line \( y = x \).

(d) The domains and ranges of \( f \) and \( f^{-1} \) are all real numbers.

43. (a) \( f(x) = \sqrt{x} \)
    \[ y = \sqrt{x} \]
    \[ x = \sqrt{y} \]
    \[ y = x^2 \]
    \[ f^{-1}(x) = x^2, \ x \geq 0 \]

(c) The graph of \( f^{-1} \) is the reflection of the graph of \( f \) about the line \( y = x \).

(d) The domains and ranges of \( f \) and \( f^{-1} \) are \([0, \infty)\).

47. (a) \( f(x) = \frac{4}{x} \)
    \[ y = \frac{4}{x} \]
    \[ x = \frac{4}{y} \]
    \[ xy = 4 \]
    \[ y = \frac{4}{x} \]
    \[ f^{-1}(x) = \frac{4}{x} \]

(c) The graph of \( f^{-1} \) is the same as the graph of \( f \).

(d) The domains and ranges of \( f \) and \( f^{-1} \) are all real numbers except for 0.

49. (a) \( f(x) = \frac{1}{x} \)
    \[ y = \frac{1}{x} \]
    \[ x = \frac{1}{y} \]
    \[ xy = 1 \]
    \[ y = \frac{1}{x} \]
    \[ f^{-1}(x) = \frac{2x + 1}{x - 1} \]

(c) The graph of \( f^{-1} \) is the reflection of the graph of \( f \) about the line \( y = x \).

(d) The domain of \( f \) and the range of \( f^{-1} \) is all real numbers except 2. The range of \( f \) and the domain of \( f^{-1} \) is all real numbers except 1.
51. (a) \( f(x) = \sqrt{x^2 - 1} \) 
\[
\begin{align*}
  y &= \sqrt{x^2 - 1} \\
  x &= \sqrt{y^2 - 1} \\
  x^2 &= y^2 - 1 \\
  y &= x^2 + 1 \\
  f^{-1}(x) &= x^2 + 1
\end{align*}
\]
(c) The graph of \( f^{-1} \) is the reflection of the graph of \( f \) about the line \( y = x \).
(d) The domains and ranges of \( f \) and \( f^{-1} \) are all real numbers.

53. (a) \( f(x) = \frac{6x + 4}{4x + 5} \) 
\[
\begin{align*}
  y &= \frac{6x + 4}{4x + 5} \\
  x &= \frac{6y + 4}{4y + 5} \\
  4xy + 5x &= 6y + 4 \\
  4xy - 6y &= -5x + 4 \\
  y(4x - 6) &= -5x + 4 \\
  y &= \frac{-5x + 4}{4x - 6} \\
  f^{-1}(x) &= \frac{-5x + 4}{4x - 6} - \frac{5x - 4}{6 - 4x}
\end{align*}
\]
(c) The graph of \( f^{-1} \) is the graph of \( f \) reflected about the line \( y = x \).
(d) The domain of \( f \) and the range of \( f^{-1} \) is all real numbers except \( -\frac{5}{2} \). The range of \( f \) and the domain of \( f^{-1} \) is all real numbers except \( \frac{5}{2} \).

55. \( f(x) = x^4 \) 
\[
\begin{align*}
  y &= x^4 \\
  x &= y^4 \\
  y &= \pm \sqrt[4]{x}
\end{align*}
\]
This does not represent \( y \) as a function of \( x \). \( f \) does not have an inverse.

57. \( g(x) = \frac{x}{8} \) 
\[
\begin{align*}
  y &= \frac{x}{8} \\
  x &= \frac{y}{8} \\
  y &= 8x
\end{align*}
\]
This is a function of \( x \), so \( g \) has an inverse.
\[
g^{-1}(x) = 8x
\]

59. \( p(x) = -4 \) 
\[
\begin{align*}
  y &= -4
\end{align*}
\]
Since \( y = -4 \) for all \( x \), the graph is a horizontal line and fails the horizontal line test. \( p \) does not have an inverse.

61. \( f(x) = (x + 3)^2, \ x \geq -3 \Rightarrow y \geq 0 \) 
\[
\begin{align*}
  y &= (x + 3)^2, \ x \geq -3, \ y \geq 0 \\
  x &= (y + 3)^2, \ y \geq -3, \ x \geq 0 \\
  \sqrt{x} &= y + 3, \ y \geq -3, \ x \geq 0 \\
  y &= \sqrt{x} - 3, \ x \geq 0, \ y \geq -3
\end{align*}
\]
This is a function of \( x \), so \( f \) has an inverse.
\[
f^{-1}(x) = \sqrt{x} - 3, \ x \geq 0
\]

63. \( f(x) = \begin{cases} 
  x + 3, & x < 0 \\
  6 - x, & x \geq 0 
\end{cases} \)
\[
\begin{align*}
  y &= \sqrt{x + 3} & \text{for } x < 0 \\
  y &= 6 - \sqrt{x} & \text{for } x \geq 0
\end{align*}
\]
The graph fails the horizontal line test, so \( f(x) \) does not have an inverse.
65. \( h(x) = -\frac{4}{x^2} \)

The graph fails the horizontal line test so \( h \) does not have an inverse.

67. \( f(x) = \sqrt{2x + 3} \Rightarrow x \geq -\frac{3}{2}, \ y \geq 0 \)

\[
y = \sqrt{2x + 3}, \ x \geq -\frac{3}{2}, \ y \geq 0 \\
x = \sqrt{2y + 3}, \ y \geq -\frac{3}{2}, \ x \geq 0 \\
x^2 = 2y + 3, \ x \geq 0, \ y \geq -\frac{3}{2} \\
y = \frac{x^2 - 3}{2}, \ x \geq 0, \ y \geq -\frac{3}{2}
\]

This is a function of \( x \), so \( f \) has an inverse.

\[
f^{-1}(x) = \frac{x^2 - 3}{2}, \ x \geq 0
\]

In Exercises 69–73, \( f(x) = \frac{1}{8}x - 3 \), \( f^{-1}(x) = 8(x + 3) \), \( g(x) = x^3 \), \( g^{-1}(x) = \frac{x^3}{2} \).

69. \( (f^{-1} \circ g^{-1})(1) = f^{-1}(g^{-1}(1)) \)

\[
= f^{-1}(\sqrt[3]{1}) \\
= 8(\sqrt[3]{1} + 3) = 32
\]

71. \( (f^{-1} \circ f^{-1})(6) = f^{-1}(f^{-1}(6)) \)

\[
= f^{-1}(8[6 + 3]) = 8[8(6 + 3) + 3] = 600
\]

73. \( (f \circ g)(x) = f(g(x)) = f(x^3) = \frac{1}{8}x^3 - 3 \)

\[
y = \frac{1}{8}x^3 - 3 \\
x = \frac{1}{8}y^3 - 3 \\
x + 3 = \frac{1}{8}y^3 \\
8(x + 3) = y^3 \\
\sqrt[3]{8(x + 3)} = y \\
(f \circ g)^{-1}(x) = 2\sqrt[3]{x + 3}
\]

In Exercises 75 and 77, \( f(x) = x + 4 \), \( f^{-1}(x) = x - 4 \), \( g(x) = 2x - 5 \), \( g^{-1}(x) = \frac{x + 5}{2} \).

75. \( (g^{-1} \circ f^{-1})(x) = g^{-1}(f^{-1}(x)) \)

\[
= g^{-1}(x - 4) \\
= \frac{(x - 4) + 5}{2} \\
= \frac{x + 1}{2}
\]

77. \( (f \circ g)(x) = f(g(x)) \)

\[
= f(2x - 5) \\
= (2x - 5) + 4 \\
= 2x - 1
\]

\[
(f \circ g)^{-1}(x) = \frac{x + 1}{2}
\]

Note: Comparing Exercises 75 and 77, we see that \( (f \circ g)^{-1}(x) = (g^{-1} \circ f^{-1})(x) \).
79. (a) \( f^{-1}(108,209) = 11 \)

(b) \( f^{-1} \) represents the year for a given number of households in the United States.

c) \( y = 1578.68t + 90,183.63 \)

d) \( y = 1578.68t + 90,183.63 \)
\[ t = \frac{y - 90,183.63}{1578.68} \]
\[ f^{-1}(t) = \frac{t - 90,183.63}{1578.68} \]

e) \( f^{-1}(117,022) \approx 17 \)

(f) \( f^{-1}(108,209) \approx 11.418 \)

This is close to the value of 11 in the table.

83. (a) \[ y = 0.03x^2 + 245.50, \ 0 < x < 100 \Rightarrow 245.50 < y < 545.50 \]
\[ x = 0.03y^2 + 245.50 \]
\[ x - 245.50 = 0.03y^2 \]
\[ \frac{x - 245.50}{0.03} = y^2 \]
\[ y = \sqrt{\frac{x - 245.50}{0.03}}, \ 245.50 < x < 545.50 \]
\[ f^{-1}(x) = \sqrt{\frac{x - 245.50}{0.03}} \]

\( x = \) temperature in degrees Fahrenheit

\( y = \) percent load for a diesel engine

85. False. \( f(x) = x^2 \) is even and does not have an inverse.

87. Let \((f \circ g)(x) = y\). Then \( x = (f \circ g)^{-1}(y) \). Also,
\[(f \circ g)(x) = y \implies f(g(x)) = y \]
\[ g(x) = f^{-1}(y) \]
\[ x = g^{-1}(f^{-1}(y)) \]
\[ x = (g^{-1} \circ f^{-1})(y). \]

Since \( f \) and \( g \) are both one-to-one functions,
\[(f \circ g)^{-1} = g^{-1} \circ f^{-1}. \]

88. (a) Yes. Since the values of \( f \) increase each year, no two \( f \)-values are paired with the same \( t \)-value so \( f \) does have an inverse.

(b) \( f^{-1} \) would represent the year that a given number of miles was traveled by motor vehicles.

(c) Since \( f(8) = 2632, f^{-1}(2632) = 8. \)

(d) No. Since the new value is the same as the value given for 2000, \( f \) would not pass the Horizontal Line Test and would not have an inverse.
91. \[ \begin{array}{c|cccc} x & -2 & -1 & 3 & 4 \\ \hline f & 6 & 0 & -2 & -3 \end{array} \]

\[ \begin{array}{c|cccc} x & -3 & -2 & 0 & 6 \\ \hline f^{-1}(x) & 4 & 3 & -1 & -2 \end{array} \]

93. If \( f(x) = k(2 - x - x^2) \) has an inverse and \( f^{-1}(3) = -2 \), then \( f(-2) = 3. \) Thus,
\[ f(-2) = k(2 - (-2) - (-2)^2) = 3 \]
\[ k(2 + 2 + 8) = 3 \]
\[ 12k = 3 \]
\[ k = \frac{1}{4} = \frac{1}{4} \]
So, \( k = \frac{1}{4}. \)

95. \( x^2 = 64 \)
\[ x = \pm \sqrt{64} = \pm 8 \]

97. \( 4x^2 - 12x + 9 = 0 \)
\[ (2x - 3)^2 = 0 \]
\[ 2x - 3 = 0 \]
\[ x = \frac{3}{2} \]

99. \( x^2 - 6x + 4 = 0 \) Complete the square.
\[ x^2 - 6x = -4 \]
\[ x^2 - 6x + 9 = -4 + 9 \]
\[ (x - 3)^2 = 5 \]
\[ x - 3 = \pm \sqrt{5} \]
\[ x = 3 \pm \sqrt{5} \]

101. \( 50 + 5x = 3x^2 \)
\[ 0 = 3x^2 - 5x - 50 \]
\[ 0 = (3x + 10)(x - 5) \]
\[ 3x + 10 = 0 \Rightarrow x = -\frac{10}{3} \]
\[ x - 5 = 0 \Rightarrow x = 5 \]

103. Let \( 2n = \) first positive even integer. Then \( 2n + 2 = \) next positive even integer.
\[ 2n(2n + 2) = 288 \]
\[ 4n^2 + 4n - 288 = 0 \]
\[ 4(n^2 + n - 72) = 0 \]
\[ 4(n + 9)(n - 8) = 0 \]
\[ n + 9 = 0 \Rightarrow n = -9 \] Not a solution since the integers are positive.
\[ n - 8 = 0 \Rightarrow n = 8 \]
So, \( 2n = 16 \) and \( 2n + 2 = 18. \)

Review Exercises for Chapter 2

1. (a) \( m = \frac{3}{2} > 0 \Rightarrow \) The line rises. Matches \( L_2. \)
   (b) \( m = 0 \Rightarrow \) The line is horizontal. Matches \( L_3. \)
   (c) \( m = -3 < 0 \Rightarrow \) The line falls. Matches \( L_1. \)
   (d) \( m = -\frac{1}{2} < 0 \Rightarrow \) The line gradually falls. Matches \( L_4. \)
   (e) \( m = \frac{1}{2} < 0 \Rightarrow \) The line decreases. Matches \( L_4. \)
   (f) \( m = -\frac{1}{2} > 0 \Rightarrow \) The line rises. Matches \( L_2. \)
3. \( y = -2x - 7 \)
   Slope: \( m = -2 = -\frac{2}{1} \)
   \( y \)-intercept: \( (0, -7) \)
5. \(y = 6\)
   Horizontal line, \(m = 0\)
   \(y\)-intercept: \((0, 6)\)

7. \(y = 3x + 13\)
   Slope: \(m = 3 = \frac{3}{1}\)
   \(y\)-intercept: \((0, 13)\)

9. \(y = -\frac{3}{2}x - 1\)
   Slope: \(m = -\frac{3}{2}\)
   \(y\)-intercept: \((0, -1)\)

11. \((-2, 5), (0, t), (1, 1)\) are collinear.

   \[
   \begin{align*}
   \frac{t - 5}{0 - (-2)} &= \frac{1 - 5}{1 - (-2)} \\
   
   \frac{t - 5}{2} &= \frac{-4}{3} \\
   3(t - 5) &= -8 \\
   3t - 15 &= -8 \\
   3t &= 7 \\
   t &= \frac{7}{3}
   \end{align*}
   \]

13. Point: \((2, -1)\)
   Slope: \(m = \frac{1}{4} = \frac{\text{rise}}{\text{run}}\)
   \((2 + 4, -1 + 1) = (6, 0)\)
   \((6 + 4, 0 + 1) = (10, 1)\)
   \((2 - 4, -1 - 1) = (-2, -2)\)

15. \((3, -4), (-7, 1)\)

   \[
   m = \frac{1 - (-4)}{-7 - 3} = \frac{5}{-10} = -\frac{1}{2}
   \]

17. \((-4.5, 6), (2.1, 3)\)

   \[
   m = \frac{3 - 6}{2.1 - (-4.5)} = \frac{-3}{6.6} = \frac{-10}{6} = \frac{-5}{11}
   \]

19. \((0, -5), m = \frac{3}{2}\)

   \[
   \begin{align*}
   y - (-5) &= \frac{3}{2}(x - 0) \\
   y + 5 &= \frac{3}{2}x \\
   y &= \frac{3}{2}x - 5
   \end{align*}
   \]

21. \((10, -3), m = \frac{1}{2}\)

   \[
   \begin{align*}
   y - (-3) &= \frac{1}{2}(x - 10) \\
   y + 3 &= \frac{1}{2}x + 5 \\
   y &= \frac{1}{2}x + 2
   \end{align*}
   \]

23. \((0, 0), (0, 10)\)

   \[
   m = \frac{10 - 0}{0 - 0} = \text{undefined}
   \]

The line is vertical.
\(x = 0\)

25. \((-1, 4), (2, 0)\)

   \[
   m = \frac{0 - 4}{2 - (-1)} = \frac{-4}{3}
   \]

   \[
   \begin{align*}
   y - 4 &= \frac{-4}{3}(x - (-1)) \\
   y - 4 &= \frac{-4}{3}x + \frac{4}{3} \\
   y &= \frac{-4}{3}x + \frac{8}{3}
   \end{align*}
   \]
27. Point: \((3, -2)\)

\[5x - 4y = 8 \implies y = \frac{5}{4}x - 2\] and \(m = \frac{5}{4}\)

(a) Parallel slope: \(m = \frac{5}{4}\)

\[y - (-2) = \frac{5}{4}(x - 3)\]

\[y + 2 = \frac{5}{4}x - \frac{15}{4}\]

\[y = \frac{5}{4}x - \frac{23}{4}\]

(b) Perpendicular slope: \(m = -\frac{4}{5}\)

\[y - (-2) = -\frac{4}{5}(x - 3)\]

\[y + 2 = -\frac{4}{5}x + \frac{12}{5}\]

\[y = -\frac{4}{5}x + \frac{2}{5}\]

29. \((6, 12,500)\) \(m = 850\)

\[y - 12,500 = 850(t - 6)\]

\[y = 850t - 5100\]

\[y = 850t + 7400, \quad 6 \leq t \leq 11\]

31. \((2, 160,000), (3, 185,000)\)

\[m = \frac{185,000 - 160,000}{3 - 2} = 25,000\]

\[S - 160,000 = 25,000(t - 2)\]

\[S = 25,000t + 110,000\]

For the fourth quarter let \(t = 4\). Then we have

\[S = 25,000(4) + 110,000 = 210,000\]

33. \(A = \{10, 20, 30, 40\}\) and \(B = \{0, 2, 4, 6\}\)

(a) 20 is matched with two elements in the range so it is not a function.

(b) Function

(c) Function

(d) 30 is not matched with any element of \(B\) so it is not a function.

35. \(16x - y^4 = 0\)

\[y^4 = 16x\]

\[y = \pm 2\sqrt[4]{x}\]

y is not a function of \(x\). Some \(x\)-values correspond to two \(y\)-values.

39. \(f(x) = x^2 + 1\)

(a) \(f(2) = (2)^2 + 1 = 5\)

(b) \(f(-4) = (-4)^2 + 1 = 17\)

(c) \(f(t^2) = (t^2)^2 + 1 = t^4 + 1\)

(d) \(f(t + 1) = (t + 1)^2 + 1 = t^2 + 2t + 2\)

41. \(h(x) = \begin{cases} 2x + 1, & x \leq -1 \\ x^2 + 2, & x > -1 \end{cases}\)

(a) \(h(-2) = 2(-2) + 1 = -3\)

(b) \(h(-1) = 2(-1) + 1 = -1\)

(c) \(h(0) = 0^2 + 2 = 2\)

(d) \(h(2) = 2^2 + 2 = 6\)
43. \( f(x) = \sqrt{25 - x^2} \)
   Domain: \( 25 - x^2 \geq 0 \)
   \((5 + x)(5 - x) \geq 0\)
   Critical numbers: \( x = \pm 5 \)
   Test intervals: \((-\infty, -5), (-5, 5), (5, \infty)\)
   Test: Is \( 25 - x^2 \geq 0 \)?
   Solution set: Thus, the domain is all real numbers \( x \) such that \(-5 \leq x \leq 5\), or \([-5, 5]\).

45. \( g(s) = \frac{5}{3s - 9} \)
   Domain: All real numbers \( s \) except \( s = 3 \)

47. \( h(x) = \frac{x}{x^2 - x - 6} = \frac{x}{(x + 2)(x - 3)} \)
   Domain: All real numbers \( x \) except \( x = -2, 3 \)

49. \( v(t) = -32t + 48 \)
   (a) \( v(1) = 16 \) feet per second
   (b) \( 0 = -32t + 48 \)
   \( t = \frac{48}{32} = 1.5 \) seconds
   (c) \( v(2) = -16 \) feet per second

51. \( (a) 2x + 2y = 24 \)
   \( y = 12 - x \)
   \( A = xy = x(12 - x) \)
   (b) Since \( x \) and \( y \) cannot be negative, we have \( 0 < x < 12 \). The domain is \( 0 < x < 12 \).

53. \( f(x) = 2x^2 + 3x - 1 \)
   \[ \frac{f(x+h) - f(x)}{h} = \frac{[2(x+h)^2 + 3(x+h) - 1] - (2x^2 + 3x - 1)}{h} \]
   \[ = \frac{2x^2 + 4xh + 2h^2 + 3x + 3h - 1 - 2x^2 - 3x + 1}{h} \]
   \[ = \frac{h(4x + 2h + 3)}{h} \]
   \[ = 4x + 2h + 3, \; h \neq 0 \]

55. \( y = (x - 3)^2 \)
   The graph passes the Vertical Line Test. \( y \) is a function of \( x \).

57. \( x - 4 = y^2 \)
   The graph does not pass the Vertical Line Test. \( y \) is not a function of \( x \).
59. \( f(x) = 3x^2 - 16x + 21 \)
\[
3x^2 - 16x + 21 = 0 \\
(3x - 7)(x - 3) = 0 \\
3x - 7 = 0 \quad \text{or} \quad x - 3 = 0 \\
x = \frac{7}{3} \quad \text{or} \quad x = 3
\]

63. \( f(x) = |x| + |x + 1| \)
- \( f \) is increasing on \((0, \infty)\).
- \( f \) is decreasing on \((-\infty, -1)\).
- \( f \) is constant on \((-1, 0)\).

67. \( f(x) = x^3 - 6x^4 \)
Relative maximum: \((0.125, 0.000488) \approx (0.13, 0.00)\)

71. \( f(x) = 2 - \sqrt{x + 1} \)
\[
\frac{f(7) - f(3)}{7 - 3} = \frac{(2 - \sqrt{8}) - (2 - 2)}{4} = \frac{2 - 2\sqrt{2}}{4} = \frac{1 - \sqrt{2}}{2}
\]
The average rate of change of \( f \) from \( x_1 = 3 \) to \( x_2 = 7 \) is \( \frac{1 - \sqrt{2}}{2} \).

75. \( f(x) = 2x\sqrt{x^2 + 3} \)
\[
f(-x) = 2(-x)\sqrt{(-x)^2 + 3} \\
= -2x\sqrt{x^2 + 3} \\
= -f(x)
\]
f is odd.

61. \( f(x) = \frac{8x + 3}{11 - x} \)
\[
8x + 3 = 0 \\
11 - x = 0 \\
x = \frac{3}{8}
\]

65. \( f(x) = -x^2 + 2x + 1 \)
Relative maximum: \((1, 2)\)

69. \( f(x) = -x^2 + 8x - 4 \)
\[
\frac{f(4) - f(0)}{4 - 0} = \frac{12 - (-4)}{4} = 4
\]
The average rate of change of \( f \) from \( x_1 = 0 \) to \( x_2 = 4 \) is 4.

73. \( f(x) = x^3 + 4x - 7 \)
\[
f(-x) = (-x)^3 + 4(-x) - 7 \\
= -x^3 - 4x - 7 \\
\neq f(x) \\
\neq -f(x)
\]
Neither even nor odd

77. \( f(2) = -6, f(-1) = 3 \)
Points: \((2, -6), (-1, 3)\)
\[
m = \frac{3 - (-6)}{-1 - 2} = \frac{9}{-3} = -3 \\
y - (-6) = -3(x - 2) \\
y + 6 = -3x + 6 \\
y = -3x
\]
79. \( f\left(\frac{-4}{5}\right) = 2, f\left(\frac{11}{5}\right) = 7 \)

Points: \( \left(\frac{-4}{5}, 2\right), \left(\frac{11}{5}, 7\right) \)

\[
m = \frac{7 - 2}{11/5 - (-4/5)} = \frac{5}{3}
\]

\[
y - 2 = \frac{5}{3}\left(x - \left(-\frac{4}{5}\right)\right)
\]

\[
y - 2 = \frac{5}{3}x + \frac{4}{3}
\]

\[
y = \frac{5}{3}x + \frac{10}{3}
\]

81. \( f(x) = 3 - x^2 \)

Intercepts: \( (0, 3), (\pm \sqrt{3}, 0) \)

y-axis symmetry

83. \( f(x) = -\sqrt{x} \)

Domain: \( x \geq 0 \)

Intercepts: \( (0, 0) \)

<table>
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<th>4</th>
<th>9</th>
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<td>-1</td>
<td>-2</td>
<td>-3</td>
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85. \( g(x) = \frac{3}{x} \)

No intercepts

Origin symmetry

<table>
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<tr>
<td>y</td>
<td>-1</td>
<td>-3</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

87. \( f(x) = \lfloor x \rfloor - 2 \)

89. \( f(x) = \begin{cases} 
5x - 3, & x \geq -1 \\ 
-4x + 5, & x < -1 
\end{cases} \)

91. Common function: \( f(x) = x^3 \)

Horizontal shift 4 units to the left and a vertical shift 4 units upward

93. (a) \( f(x) = x^2 \)

(b) \( h(x) = x^2 - 9 \)

Vertical shift 9 units downward

(c)

(d) \( h(x) = f(x) - 9 \)
95. (a) \( f(x) = \sqrt{x} \)
(b) \( h(x) = \sqrt{x - 7} \)
   Horizontal shift 7 units to the right
(c) \( y \)
(d) \( h(x) = f(x - 7) \)

99. (a) \( f(x) = |x| \)
(b) \( h(x) = -|x| + 6 \)
   Reflection in the x-axis and a vertical shift 6 units upward
(c) \( y \)
(d) \( h(x) = -f(x) + 6 \)

103. (a) \( f(x) = [x] \)
(b) \( h(x) = 5[x - 9] \)
   Horizontal shift 9 units to the right and a vertical stretch (each y-value is multiplied by 5)
(c) \( y \)
(d) \( h(x) = 5f(x - 9) \)

107. \( f(x) = x^2 + 3, g(x) = 2x - 1 \)
   (a) \( (f + g)(x) = (x^2 + 3) + (2x - 1) = x^2 + 2x + 2 \)
   (b) \( (f - g)(x) = (x^2 + 3) - (2x - 1) = x^2 - 2x + 4 \)
   (c) \( (fg)(x) = (x^2 + 3)(2x - 1) = 2x^3 - x^2 + 6x - 3 \)
   (d) \( \frac{f}{g}(x) = \frac{x^2 + 3}{2x - 1} \). Domain: \( x \neq \frac{1}{2} \)

97. (a) \( f(x) = x^2 \)
(b) \( h(x) = -(x + 3)^2 + 1 \)
   Reflection in the x-axis, a horizontal shift 3 units to the left, and a vertical shift 1 unit upward
(c) \( y \)
(d) \( h(x) = -f(x + 3) + 1 \)

101. (a) \( f(x) = |x| \)
(b) \( h(x) = -|x - 4| + 6 \)
   Reflection in both the x- and y-axes; horizontal shift of 4 units to the right; vertical shift of 6 units upward
(c) \( y \)
(d) \( h(x) = -f(-(x - 4)) + 6 = -f(-x + 4) + 6 \)

105. (a) \( f(x) = \sqrt{x} \)
(b) \( h(x) = -2\sqrt{x - 4} \)
   Reflection in the x-axis, a vertical stretch (each y-value is multiplied by 2), and a horizontal shift 4 units to the right
(c) \( y \)
(d) \( h(x) = -2f(x - 4) \)
109. \( f(x) = \frac{1}{3}x - 3, \ g(x) = 3x + 1 \)
The domains of \( f(x) \) and \( g(x) \) are all real numbers.
(a) \((f \circ g)(x) = f(g(x))\)
\[= f(3x + 1)\]
\[= \frac{1}{3} (3x + 1) - 3\]
\[= x + \frac{1}{3} - 3\]
\[= x - \frac{8}{3}\]
Domain: all real numbers

(b) \((g \circ f)(x) = g(f(x))\)
\[= g(\frac{1}{3}x - 3)\]
\[= 3(\frac{1}{3}x - 3) + 1\]
\[= x - 9 + 1\]
\[= x - 8\]
Domain: all real numbers

111. \( h(x) = (6x - 5)^3 \)
Answer is not unique.
One possibility: Let \( f(x) = x^3 \) and \( g(x) = 6x - 5 \).
\[f(g(x)) = f(6x - 5) = (6x - 5)^3 = h(x)\]

115. \( f(x) = x - 7 \)
\(f^{-1}(x) = x + 7\)
\[f(f^{-1}(x)) = f(x + 7) = (x + 7) - 7 = x\]
\[f^{-1}(f(x)) = f^{-1}(x - 7) = (x - 7) + 7 = x\]

119. \( f(x) = 4 - \frac{1}{3}x \)
The graph passes the Horizontal Line Test. The function has an inverse.

113. \( v(t) = -31.86t^2 + 233.6t + 2594 \)
\(d(t) = -4.18t^2 + 571.0t - 3706 \)
(a) \((v + d)(t) = v(t) + d(t) = -36.04t^2 + 804.6t - 1112 \)
\((v + d)(t)\) represents the combined factory sales (in millions of dollars) for VCRs and DVD players from 1997 to 2003.
(b) \(v(t)\)
\(d(t)\)
\((v + d)(t)\)

(c) \((v + d)(10) = \$3330\) million

117. The graph passes the Horizontal Line Test.
The function has an inverse.

121. \( h(t) = \frac{2}{t - 3} \)
The graph passes the Horizontal Line Test. The function has an inverse.

123. (a) \( f(x) = \frac{1}{2}x - 3 \)
\[y = \frac{1}{2}x - 3\]
\[x = \frac{1}{2}y - 3\]
\[x + 3 = \frac{1}{2}y\]
\[2(x + 3) = y\]
\[f^{-1}(x) = 2x + 6\]
(c) The graph of \( f^{-1} \) is the reflection of the graph of \( f \) about the line \( y = x \).

(d) The domains and ranges of \( f \) and \( f^{-1} \) are the set of all real numbers.
125. (a) \[ f(x) = \sqrt{x + 1} \]
\[ y = \sqrt{x + 1} \]
\[ x = \sqrt{y + 1} \]
\[ x^2 = y + 1 \]
\[ x^2 - 1 = y \]
\[ f^{-1}(x) = x^2 - 1, \quad x \geq 0 \]

Note: The inverse must have a restricted domain.

(c) The graph of \( f^{-1} \) is the reflection of the graph of \( f \) about the line \( y = x \).

127. \( f(x) = 2(x - 4)^2 \) is increasing on \( [4, \infty) \).

Let \[ f(x) = 2(x - 4)^2, \quad x \geq 4 \text{ and } y \geq 0 \]
\[ y = 2(x - 4)^2 \]
\[ x = 2(y - 4)^2, \quad x \geq 0, \quad y \geq 4 \]
\[ \frac{x}{2} = (y - 4)^2 \]
\[ \sqrt{\frac{x}{2}} = y - 4 \]
\[ \sqrt{\frac{x}{2}} + 4 = y \]
\[ f^{-1}(x) = \sqrt{\frac{x}{2}} + 4, \quad x \geq 0 \]

131. A function from a Set \( A \) to a Set \( B \) is a relation that assigns to each element \( x \) in the Set \( A \) exactly one element \( y \) in the Set \( B \).

133. The basic cubic function, \( y = x^3 \), has the following characteristics. Its domain and range is the set of all real numbers. It is an odd function, thus its graph is symmetric with respect to the origin. It is an increasing function and has an intercept at \((0, 0)\).

The cubic function, \( f(x) = x^3 - 1 \), has the following characteristics. Its domain and range is the set of all real numbers. It is an increasing function with \( x \)-intercept at \((1, 0)\) and \( y \)-intercept at \((0, -1)\). The graph of \( f(x) \) can be obtained by shifting the basic graph of \( y = x^3 \) down one unit.

**Problem Solving for Chapter 2**

1. (a) \( W_1 = 0.07x + 2000 \)
(b) \( W_2 = 0.05x + 2300 \)
(d) If you think you can sell \$20,000 per month, keep your current job with the higher commission rate. For sales over \$15,000 it pays more than the other job.

Point of intersection: \((15,000, 3050)\)

Both jobs pay the same, \$3050, if you sell \$15,000 per month.
3. (a) Let \( f(x) \) and \( g(x) \) be two even functions. Then define \( h(x) = f(x) \pm g(x) \).
\[
\begin{align*}
  h(-x) &= f(-x) \pm g(-x) \\
  &= f(x) \pm g(x) \text{ since } f \text{ and } g \text{ are even} \\
  &= h(x)
\end{align*}
\]
So, \( h(x) \) is also even.

(c) Let \( f(x) \) be odd and \( g(x) \) be even. Then define
\[
\begin{align*}
  h(-x) &= f(-x) \pm g(x) \\
  &= -f(x) \pm g(x) \text{ since } f \text{ is odd and } g \text{ is even} \\
  &\neq h(x) \\
  &\neq -h(x)
\end{align*}
\]
So, \( h(x) \) is neither odd nor even.

(b) Let \( f(x) \) and \( g(x) \) be two odd functions. Then define
\[
\begin{align*}
  h(-x) &= f(-x) \pm g(-x) \\
  &= -f(x) \mp g(x) \text{ since } f \text{ and } g \text{ are odd} \\
  &= -h(x)
\end{align*}
\]
So, \( h(x) \) is also odd. (If \( f(x) \neq g(x) \))

5. \[
\begin{align*}
  f(x) &= a_{2n}x^{2n} + a_{2n-2}x^{2n-2} + \cdots + a_2x^2 + a_0 \\
  f(-x) &= a_{2n}(-x)^{2n} + a_{2n-2}(-x)^{2n-2} + \cdots + a_2(-x)^2 + a_0 \\
  &= a_{2n}x^{2n} + a_{2n-2}x^{2n-2} + \cdots + a_2x^2 + a_0 \\
  &= f(x)
\end{align*}
\]
Therefore, \( f(x) \) is even.

7. (a) April 11: 10 hours
   April 12: 24 hours
   April 13: 24 hours
   April 14: 23.7 hours
   Total: \( 81.7 \) hours

(b) Speed = \( \frac{\text{distance}}{\text{time}} = \frac{2100}{81.7} = \frac{180}{7} = 25 \frac{5}{7} \) mph

(c) \( D = \frac{180}{7} t + 3400 \)

Domain: \( 0 \leq t \leq \frac{1190}{9} \)

Range: \( 0 \leq D \leq 3400 \)

9. (a)–(d) Use \( f(x) = 4x \) and \( g(x) = x + 6 \).
   (a) \( (fg)(x) = f(x + 6) = 4(x + 6) = 4x + 24 \)
   (b) \( (f \circ g)^{-1}(x) = \frac{x - 24}{4} = \frac{1}{4}x - 6 \)
   (c) \( f^{-1}(x) = \frac{1}{4}x \)
   (d) \( g^{-1}(x) = x - 6 \)
   (e) \( f(x) = x^3 + 1 \) and \( g(x) = 2x \)
   \( (f \circ g)(x) = f(2x) = (2x)^3 + 1 = 8x^3 + 1 \)
   \( (f \circ g)^{-1}(x) = \sqrt[3]{\frac{x - 1}{8}} = \frac{1}{2} \sqrt[3]{x - 1} \)
   \( f^{-1}(x) = \sqrt[3]{x - 1} \)
   \( g^{-1}(x) = \frac{1}{2}x \)
   \( (g^{-1} \circ f^{-1})(x) = g^{-1}(\sqrt[3]{x - 1}) = \frac{1}{2} \sqrt[3]{x - 1} \)
   (f) Answers will vary.
   (g) Conjecture: \( (f \circ g)^{-1}(x) = (g^{-1} \circ f^{-1})(x) \)
11. \( H(x) = \begin{cases} 
1, & x \geq 0 \\
0, & x < 0 
\end{cases} \)

![Graph of \( H(x) \)]

(a) \( H(x) - 2 \)

![Graph of \( H(x) - 2 \)]

(b) \( H(x - 2) \)

![Graph of \( H(x - 2) \)]

(c) \( -H(x) \)

![Graph of \( -H(x) \)]

(d) \( H(-x) \)

![Graph of \( H(-x) \)]

(e) \( \frac{1}{2}H(x) \)

![Graph of \( \frac{1}{2}H(x) \)]

(f) \( -H(x - 2) + 2 \)

![Graph of \( -H(x - 2) + 2 \)]

13. \( (f \cdot (g \cdot h))(x) = f((g \cdot h)(x)) \)
\[= f(g(h(x))) \]
\[= (f \cdot g \cdot h)(x) \]
\[((f \cdot g) \cdot h)(x) = (f \cdot g)(h(x)) \]
\[= f(g(h(x))) \]
\[= (f \cdot g \cdot h)(x) \]
15.

<table>
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<th>$x$</th>
<th>$f(x)$</th>
<th>$f^{-1}(x)$</th>
</tr>
</thead>
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<td>$-4$</td>
<td>$-2$</td>
<td></td>
</tr>
<tr>
<td>$-3$</td>
<td>$4$</td>
<td>$1$</td>
</tr>
<tr>
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<td></td>
<td></td>
</tr>
<tr>
<td>$4$</td>
<td></td>
<td>$-3$</td>
</tr>
</tbody>
</table>

(a) $x$ | $f(f^{-1}(x))$
---|-------------------
$-4$ | $f(f^{-1}(-4)) = f(2) = -4$
$-2$ | $f(f^{-1}(-2)) = f(0) = -2$
$0$  | $f(f^{-1}(0)) = f(-1) = 0$
$4$  | $f(f^{-1}(4)) = f(-3) = 4$

(b) $x$ | $(f + f^{-1})(x)$
---|-------------------
$-3$ | $f(-3) + f^{-1}(-3) = 4 + 1 = 5$
$-2$ | $f(-2) + f^{-1}(-2) = 1 + 0 = 1$
$0$  | $f(0) + f^{-1}(0) = -2 + (-1) = -3$
$1$  | $f(1) + f^{-1}(1) = -3 + (-2) = -5$

(c) $x$ | $(f \cdot f^{-1})(x)$
---|-------------------
$-3$ | $f(-3)f^{-1}(-3) = (4)(1) = 4$
$-2$ | $f(-2)f^{-1}(-2) = (1)(0) = 0$
$0$  | $f(0)f^{-1}(0) = (-2)(-1) = 2$
$1$  | $f(1)f^{-1}(1) = (-3)(-2) = 6$

(d) $x$ | $|f^{-1}(x)|$
---|-------------------
$-4$ | $|f^{-1}(-4)| = |2| = 2$
$-3$ | $|f^{-1}(-3)| = |1| = 1$
$0$  | $|f^{-1}(0)| = |-1| = 1$
$4$  | $|f^{-1}(4)| = |-3| = 3$
Practice Test for Chapter 2

1. Find the equation of the line through (2, 4) and (3, −1).

2. Find the equation of the line with slope \( m = \frac{4}{3} \) and \( y \)-intercept \( b = −3 \).

3. Find the equation of the line through (4, 1) perpendicular to the line \( 2x + 3y = 0 \).

4. If it costs a company $32 to produce 5 units of a product and $44 to produce 9 units, how much does it cost to produce 20 units? (Assume that the cost function is linear.)

5. Given \( f(x) = x^2 − 2x + 1 \), find \( f(x − 3) \).

6. Given \( f(x) = 4x − 11 \), find \( \frac{f(x) − f(3)}{x − 3} \).

7. Find the domain and range of \( f(x) = \sqrt{36 − x^2} \).

8. Which equations determine \( y \) as a function of \( x \)?
   (a) \( 6x − 5y + 4 = 0 \)
   (b) \( x^2 + y^2 = 9 \)
   (c) \( y^3 = x^2 + 6 \)

9. Sketch the graph of \( f(x) = x^2 − 5 \).

10. Sketch the graph of \( f(x) = |x + 3| \).

11. Sketch the graph of \( f(x) = \begin{cases} 2x + 1 & \text{if } x \geq 0, \\ x^2 − x & \text{if } x < 0. \end{cases} \)

12. Use the graph of \( f(x) = |x| \) to graph the following.
   (a) \( f(x + 2) \)
   (b) \( −f(x) + 2 \)

13. Given \( f(x) = 3x + 7 \) and \( g(x) = 2x^2 − 5 \), find the following:
   (a) \( (g − f)(x) \)
   (b) \( (fg)(x) \)

14. Given \( f(x) = x^2 − 2x + 16 \) and \( g(x) = 2x + 3 \), find \( f(g(x)) \).

15. Given \( f(x) = x^3 + 7 \), find \( f^{-1}(x) \).
16. Which of the following functions have inverses?
   (a) \( f(x) = |x - 6| \)
   (b) \( f(x) = ax + b, \ a \neq 0 \)
   (c) \( f(x) = x^3 - 19 \)

17. Given \( f(x) = \sqrt{\frac{3 - x}{x}}, \ 0 < x \leq 3 \), find \( f^{-1}(x) \).

Exercises 18–20, true or false?

18. \( y = 3x + 7 \) and \( y = \frac{1}{3}x - 4 \) are perpendicular.

19. \( (f \circ g)^{-1} = g^{-1} \circ f^{-1} \)

20. If a function has an inverse, then it must pass both the Vertical Line Test and the Horizontal Line Test.