CHAPTER 1
Equations, Inequalities, and Mathematical Modeling

Section 1.1  Graphs of Equations

- You should be able to use the point-plotting method of graphing.
- You should be able to find x- and y-intercepts.
  (a) To find the x-intercepts, let y = 0 and solve for x.
  (b) To find the y-intercepts, let x = 0 and solve for y.
- You should be able to test for symmetry.
  (a) To test for x-axis symmetry, replace y with −y.
  (b) To test for y-axis symmetry, replace x with −x.
  (c) To test for origin symmetry, replace x with −x and y with −y.
- You should know the standard equation of a circle with center \((h, k)\) and radius \(r\):
  \[(x - h)^2 + (y - k)^2 = r^2\]

**Vocabulary Check**

1. solution or solution point  2. graph  3. intercepts
4. y-axis  5. circle; \((h, k)\); \(r\)  6. point-plotting

1. \(y = \sqrt{x + 4}\)
   (a) \((0, 2)\): \(2 \neq \sqrt{0 + 4}\)
   \[2 = 2\]
   Yes, the point is on the graph.
   (b) \((5, 3)\): \(3 \neq \sqrt{5 + 4}\)
   \[3 = \sqrt{9}\]
   Yes, the point is on the graph.

3. \(y = 4 - |x - 2|\)
   (a) \((1, 5)\): \(5 \neq 4 - |1 - 2|\)
   \[5 \neq 4 - 1\]
   No, the point is not on the graph.
   (b) \((6, 0)\): \(0 \neq 4 - |6 - 2|\)
   \[0 = 4 - 4\]
   Yes, the point is on the graph.

5. \(y = -2x + 5\)

<table>
<thead>
<tr>
<th>(x)</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>(\frac{5}{2})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y)</td>
<td>7</td>
<td>5</td>
<td>3</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>((x, y))</td>
<td>((-1, 7))</td>
<td>((0, 5))</td>
<td>((1, 3))</td>
<td>((2, 1))</td>
<td>((\frac{5}{2}, 0))</td>
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</table>
7. \( y = x^2 - 3x \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>4</td>
<td>0</td>
<td>-2</td>
<td>-2</td>
<td>0</td>
</tr>
<tr>
<td>( (x, y) )</td>
<td>(-1, 4)</td>
<td>(0, 0)</td>
<td>(1, -2)</td>
<td>(2, -2)</td>
<td>(3, 0)</td>
</tr>
</tbody>
</table>

9. \( y = 16 - 4x^2 \)

- Intercepts:
  - \( x \)-intercepts: \( 0 = 16 - 4x^2 \)
    - \( 4x^2 = 16 \)
    - \( x^2 = 4 \)
    - \( x = \pm 2 \)
    - \( (-2, 0), (2, 0) \)
  - \( y \)-intercept: \( y = 16 - 4(0)^2 = 16 \)
    - \( (0, 0) \)

11. \( y = 2x^3 - 4x^2 \)

- Intercepts:
  - \( 0 = 2x^3 - 4x^2 \)
    - \( 0 = 2x^2(x - 2) \)
    - \( x = 0 \) or \( x = 2 \)
    - \( (0, 0), (2, 0) \)
  - \( y \)-intercept: \( y = 2(0)^3 - 4(0)^2 \)
    - \( y = 0 \)
    - \( (0, 0) \)

13. \( y \)-axis symmetry

![y-axis symmetry](image)

15. Origin symmetry

![Origin symmetry](image)

17. \( x^2 - y = 0 \)

- \( -(x)^2 - y = 0 \Rightarrow x^2 - y = 0 \Rightarrow y \)-axis symmetry
- \( x^2 - (-y) = 0 \Rightarrow x^2 + y = 0 \Rightarrow \) No \( x \)-axis symmetry
- \( -(x)^2 - (-y) = 0 \Rightarrow x^2 + y = 0 \Rightarrow \) No origin symmetry

19. \( y = x^3 \)

- \( y = (-x)^3 \Rightarrow y = -x^3 \Rightarrow \) No \( y \)-axis symmetry
- \( -y = x^3 \Rightarrow y = -x^3 \Rightarrow \) No \( x \)-axis symmetry
- \( -y = (-x)^3 \Rightarrow -y = -x^3 \Rightarrow y = x^3 \Rightarrow \) Origin symmetry

21. \( y = \frac{x}{x^2 + 1} \)

- \( y = \frac{-x}{(-x)^2 + 1} \Rightarrow y = \frac{-x}{x^2 + 1} \Rightarrow \) No \( y \)-axis symmetry
- \( -y = \frac{x}{x^2 + 1} \Rightarrow y = \frac{-x}{x^2 + 1} \Rightarrow \) No \( x \)-axis symmetry
- \( -y = \frac{-x}{(-x)^2 + 1} \Rightarrow -y = \frac{-x}{x^2 + 1} \Rightarrow y = \frac{x}{x^2 + 1} \Rightarrow \) Origin symmetry

23. \( xy^2 + 10 = 0 \)

- \( -(x)(-y)^2 + 10 = 0 \Rightarrow -xy^2 + 10 = 0 \Rightarrow \) No \( y \)-axis symmetry
- \( x(-y)^2 + 10 = 0 \Rightarrow xy^2 + 10 = 0 \Rightarrow \) \( x \)-axis symmetry
- \( -(x)(-y)^2 + 10 = 0 \Rightarrow -xy^2 + 10 = 0 \Rightarrow \) No origin symmetry
25. \( y = -3x + 1 \)
   - \( x \)-intercept: \( \left( \frac{1}{3}, 0 \right) \)
   - \( y \)-intercept: \( (0, 1) \)
   - No axis or origin symmetry

27. \( y = x^2 - 2x \)
   - Intercepts: \( (0, 0), (2, 0) \)
   - No axis or origin symmetry

29. \( y = x^3 + 3 \)
   - Intercepts: \( (0, 3), (\sqrt[3]{3}, 0) \)
   - No axis or origin symmetry

31. \( y = \sqrt{x - 3} \)
   - Domain: \([3, \infty)\)
   - Intercept: \( (3, 0) \)
   - No axis or origin symmetry

33. \( y = |x - 6| \)
   - Intercepts: \( (0, 6), (6, 0) \)
   - No axis or origin symmetry

35. \( x = y^2 - 1 \)
   - Intercepts: \( (0, -1), (0, 1), (-1, 0) \)
   - \( x \)-axis symmetry

37. \( y = 3 - \frac{1}{2}x \)
   - Intercepts: \( (6, 0), (0, 3) \)

39. \( y = x^2 - 4x + 3 \)

41. \( y = \frac{2x}{x - 1} \)

43. \( y = \sqrt{x} \)

Intercepts: \( (3, 0), (1, 0), (0, 3) \)

Intercept: \( (0, 0) \)

Intercept: \( (0, 0) \)
45. \( y = x \sqrt{x + 6} \)

Intercepts: (0, 0), (−6, 0)

47. \( y = |x + 3| \)

Intercepts: (−3, 0), (0, 3)

49. Center: (0, 0); radius: 4
   Standard form:
   \((x - 0)^2 + (y - 0)^2 = 4^2\)
   \(x^2 + y^2 = 16\)

51. Center: (2, −1); radius: 4
   Standard form:
   \((x - 2)^2 + (y + 1)^2 = 4^2\)
   \(x^2 + (y + 1)^2 = 16\)

53. Center: (−1, 2); solution point: (0, 0)
   \((x - (-1))^2 + (y - 2)^2 = r^2\)
   \((0 + 1)^2 + (0 - 2)^2 = r^2 \implies 5 = r^2\)
   Standard form: \((x + 1)^2 + (y - 2)^2 = 5\)

55. Endpoints of a diameter: (0, 0), (6, 8)
   Center: \(\left(\frac{0 + 6}{2}, \frac{0 + 8}{2}\right) = (3, 4)\)
   \((x - 3)^2 + (y - 4)^2 = r^2\)
   \((0 - 3)^2 + (0 - 4)^2 = r^2 \implies 25 = r^2\)
   Standard form: \((x - 3)^2 + (y - 4)^2 = 25\)

57. \(x^2 + y^2 = 25\)
   Center: (0, 0), radius: 5

59. \((x - 1)^2 + (y + 3)^2 = 9\)
   Center: (1, −3), radius: 3

61. \(\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{5}{4}\)
   Center: \(\left(\frac{1}{2}, \frac{1}{2}\right)\), radius: \(\frac{\sqrt{5}}{2}\)

63. \(y = 225,000 - 20,000t, 0 \leq t \leq 8\)

65. (a) \[
\begin{array}{c}
\text{Y}
\end{array}
\]

(c) \[
\begin{array}{c}
\text{Y}
\end{array}
\]

(b) \(2x + 2y = \frac{1040}{3}\)
   \(2y = \frac{1040}{3} - 2x\)
   \(y = \frac{520}{3} - x\)
   \(A = xy = x\left(\frac{520}{3} - x\right)\)

(d) When \(x = y = 86\frac{2}{3}\) yards, the area is a maximum of 7511\(\frac{1}{2}\) square yards.

(e) A regulation NFL playing field is 120 yards long and 53\(\frac{1}{3}\) yards wide. The actual area is 6400 square yards.
67. \( y = -0.0025t^2 + 0.574t + 44.25 \), \( 20 \leq t \leq 100 \)

(c) For the year 1948, let \( t = 48 \): \( y \approx 66.0 \) years.

(d) For the year 2005, let \( t = 105 \): \( y \approx 77.0 \) years.

For the year 2010, let \( t = 110 \): \( y \approx 77.1 \) years.

(e) No. The graph reaches a maximum of \( y = 77.2 \) years when \( t = 114.8 \), or during the year 2014. After this time, the model has life expectancy decreasing, which is not realistic.

69. False. A graph is symmetric with respect to the \( x \)-axis if, whenever \((x, y)\) is on the graph, \((x, -y)\) is also on the graph.

71. The viewing window is incorrect. Change the viewing window. Examples will vary. For example, \( y = x^2 + 20 \) will not appear in the standard window setting.

73. \( 9x^3 + 4x^3 - 7 \)

Terms: \( 9x^3, 4x^3, -7 \)

75. \( \sqrt[3]{18x} - \sqrt[3]{x} = 3\sqrt[3]{x} - \sqrt[3]{x} = 2\sqrt[3]{x} \)

Section 1.2  Linear Equations in One Variable

- You should know how to solve linear equations.
  \[ ax + b = 0, \ a \neq 0 \]
- An identity is an equation whose solution consists of every real number in its domain.
- To solve an equation you can:
  (a) Add or subtract the same quantity from both sides.
  (b) Multiply or divide both sides by the same nonzero quantity.
  (c) Remove all symbols of grouping and all fractions.
  (d) Combine like terms.
  (e) Interchange the two sides.
- Check the answer!
- A “solution” that does not satisfy the original equation is called an extraneous solution.
- Be able to find intercepts algebraically.

Vocabulary Check

1. equation  
2. solve  
3. identities; conditional  
4. \( ax + b = 0 \)  
5. extraneous
1. \(5x - 3 = 3x + 5\)
   (a) \(5(0) - 3 \neq 3(0) + 5\)
   \(-3 \neq 5\)
   \(x = 0\) is not a solution.
   (c) \(5(4) - 3 \neq 3(4) + 5\)
   \(17 = 19\)
   \(x = 4\) is a solution.
   (b) \(5(-5) - 3 \neq 3(-5) + 5\)
   \(-23 \neq -10\)
   \(x = -5\) is not a solution.
   (d) \(5(10) - 3 \neq 3(10) + 5\)
   \(47 \neq 35\)
   \(x = 10\) is not a solution.

3. \(3x^2 + 2x - 5 = 2x^2 - 2\)
   (a) \(3(-3)^2 + 2(-3) - 5 \neq 2(-3)^2 - 2\)
   \(27 - 6 - 5 \neq 18 - 2\)
   \(16 = 16\)
   \(x = -3\) is a solution.
   (c) \(3(4)^2 + 2(4) - 5 \neq 2(4)^2 - 2\)
   \(48 + 8 - 5 \neq 32 - 2\)
   \(51 \neq 30\)
   \(x = 4\) is not a solution.
   (b) \(3(1)^2 + 2(1) - 5 \neq 2(1)^2 - 2\)
   \(3 + 2 - 5 \neq 2 - 2\)
   \(0 = 0\)
   \(x = 1\) is a solution.
   (d) \(3(-5)^2 + 2(-5) - 5 \neq 2(-5)^2 - 2\)
   \(75 - 10 - 5 \neq 50 - 2\)
   \(60 \neq 48\)
   \(x = -5\) is not a solution.

5. \(\frac{5}{2x} - \frac{4}{x} = 3\)
   (a) \(\frac{5}{2(-1/2)} - \frac{4}{(-1/2)} \neq 3\)
   \(-5 + 8 \neq 3\)
   \(3 = 3\)
   \(x = -\frac{1}{2}\) is a solution.
   (c) \(\frac{5}{2(0)} - \frac{4}{0}\) is undefined.
   \(x = 0\) is not a solution.
   (b) \(\frac{5}{2(4)} - \frac{4}{4} \neq 3\)
   \(\frac{5}{8} - 1 \neq 3\)
   \(-\frac{3}{8} \neq 3\)
   \(x = 4\) is not a solution.
   (d) \(\frac{5}{2(1/4)} - \frac{4}{1/4} \neq 3\)
   \(10 - 16 \neq 3\)
   \(-6 \neq 3\)
   \(x = \frac{1}{2}\) is not a solution.

7. \(\sqrt{3x - 2} = 4\)
   (a) \(\sqrt{3(3)} - 2 \neq 4\)
   \(\sqrt{7} \neq 4\)
   \(x = 3\) is not a solution.
   (c) \(\sqrt{3(9)} - 2 \neq 4\)
   \(\sqrt{27} \neq 4\)
   \(x = 9\) is not a solution.
   (b) \(\sqrt{3(2)} - 2 \neq 4\)
   \(\sqrt{4} \neq 4\)
   \(x = 2\) is not a solution.
   (d) \(\sqrt{3(-6)} - 2 \neq 4\)
   \(\sqrt{-20} \neq 4\)
   \(x = -6\) is not a solution.
9. $6x^2 - 11x - 35 = 0$
   (a) $6\left(-\frac{5}{3}\right)^2 - 11\left(-\frac{5}{3}\right) - 35 \neq 0$
   \[ \frac{50}{3} + \frac{55}{3} - \frac{105}{3} \neq 0 \]
   $x = \frac{-5}{3}$ is a solution.

   (b) $6\left(-\frac{2}{3}\right)^2 - 11\left(-\frac{2}{3}\right) - 35 \neq 0$
   \[ \frac{24}{9} + \frac{114}{9} - \frac{1218}{9} \neq 0 \]
   $x = -\frac{2}{3}$ is not a solution.

(c) $6\left(\frac{2}{3}\right)^2 - 11\left(\frac{2}{3}\right) - 35 \neq 0$
   \[ \frac{144}{9} - \frac{77}{9} - \frac{20}{3} \neq 0 \]
   $0 = 0$

   (d) $6\left(\frac{3}{2}\right)^2 - 11\left(\frac{3}{2}\right) - 35 \neq 0$
   \[ \frac{54}{9} - \frac{55}{9} - \frac{105}{9} \neq 0 \]
   $x = \frac{5}{3}$ is not a solution.

11. $2(x - 1) = 2x - 2$ is an identity by the Distributive Property. It is true for all real values of $x$.

13. $-6(x - 3) + 5 = -2x + 10$ is conditional. There are real values of $x$ for which the equation is not true.

15. $4(x + 1) - 2x = 4x + 4 - 2x = 2x + 4 = 2(x + 2)$
   This is an identity by simplification. It is true for all real values of $x$.

19. $3 + \frac{1}{x + 1} = \frac{4x}{x + 1}$ is conditional. There are real values of $x$ for which the equation is not true.

21. $4x + 32 = 83$
   Original equation
   $4x + 32 = 83 - 32$ Subtract 32 from both sides.
   $4x = 51$ Simplify.
   $\frac{4x}{4} = \frac{51}{4}$ Divide both sides by 4.
   $x = \frac{51}{4}$ Simplify.

23. $x + 11 = 15$

   $x + 11 - 11 = 15 - 11$

   $x = 4$

25. $7 - 2x = 25$

   $7 - 7 - 2x = 25 - 7$

   $-2x = 18$

   $\frac{-2x}{-2} = \frac{18}{-2}$

   $x = -9$

27. $8x - 5 = 3x + 20$

   $8x - 3x - 5 = 3x - 3x + 20$

   $5x - 5 = 20$

   $5x - 5 + 5 = 20 + 5$

   $5x = 25$

   $\frac{5x}{5} = \frac{25}{5}$

   $x = 5$

29. $2(x + 5) - 7 = 3(x - 2)$

   $2x + 10 - 7 = 3x - 6$

   $2x + 3 = 3x - 6$

   $2x - 3x + 3 = 3x - 3x - 6$

   $-x + 3 = -6 - 3$

   $-x + 3 - 3 = -6 - 3$

   $-x = -9$

   $x = 9$

31. $x - 3(2x + 3) = 8 - 5x$

   $x - 6x - 9 = 8 - 5x$

   $-5x - 9 = 8 - 5x$

   $-5x + 5x - 9 = 8 - 5x + 5x$

   $-9 \neq 8$

   No solution

33. $\frac{5x}{4} + \frac{1}{2} = x - \frac{1}{2}$

   $4\left(\frac{5x}{4} + \frac{1}{2}\right) = 4\left(x - \frac{1}{2}\right)$

   $4\left(\frac{5x}{4}\right) + 4\left(\frac{1}{2}\right) = 4x - 4\left(\frac{1}{2}\right)$

   $5x + 2 = 4x - 2$

   $x = -4$
35. \( \frac{3}{4}(z + 5) - \frac{1}{4}(z + 24) = 0 \)
\[ 4 \left( \frac{3}{4}(z + 5) - \frac{1}{4}(z + 24) \right) = 4(0) \]
\[ 4(\frac{3}{4})z + 5 - 4(\frac{1}{4})z - 4(24) = 4(0) \]
\[ 6(z + 5) - (z + 24) = 0 \]
\[ 6z + 30 - z - 24 = 0 \]
\[ 5z = -6 \]
\[ z = -\frac{6}{5} \]

39. \( 3(x - 1) = 4 \quad \) or \( 3(x - 1) = 4 \)
\[ x - 1 = \frac{4}{3} \quad \quad 3x - 3 = 4 \]
\[ x = \frac{4}{3} + 1 \quad \quad 3x = 7 \]
\[ x = \frac{7}{3} \quad \quad x = \frac{7}{3} \]

The second way is easier since you are not working with fractions until the end of the solution.

43. \( x + 8 = 2(x - 2) - x \)
\[ x + 8 = 2x - 4 - x \]
\[ x + 8 = x - 4 \]
\[ 8 = -4 \]

Contradiction; no solution

47. \( \frac{5x - 4}{5x + 4} = \frac{2}{3} \)
\[ 3(5x - 4) = 2(5x + 4) \]
\[ 15x - 12 = 10x + 8 \]
\[ 5x = 20 \]
\[ x = 4 \]

51. \( 3 = 2 + \frac{2}{z + 2} \)
\[ 3(z + 2) = \left( 2 + \frac{2}{z + 2} \right) (z + 2) \]
\[ 3z + 6 = 2z + 4 + 2 \]
\[ z = 0 \]

49. \( 10 - \frac{13}{x} = 4 + \frac{5}{x} \)
\[ \frac{10x - 13}{x} = \frac{4x + 5}{x} \]
\[ 10x - 13 = 4x + 5 \]
\[ 6x = 18 \]
\[ x = 3 \]
55. \[ \frac{2}{(x-4)(x-2)} = \frac{1}{x-4} + \frac{2}{x-2} \]
Multiply both sides by \((x-4)(x-2)\).

\[
\begin{align*}
2 &= 1(x-2) + 2(x-4) \\
2 &= x-2 + 2x - 8 \\
2 &= 3x - 10 \\
12 &= 3x \\
4 &= x
\end{align*}
\]

A check reveals that \(x = 4\) is an extraneous solution—it makes the denominator zero. There is no real solution.

57. \[ \frac{1}{x-3} + \frac{1}{x+3} = \frac{10}{x^2 - 9} \]
Multiply both sides by \((x+3)(x-3)\).

\[
\begin{align*}
1(x+3) + 1(x-3) &= 10 \\
2x &= 10 \\
x &= 5
\end{align*}
\]

59. \[ \frac{3}{x^2 - 3x} + \frac{4}{x} = \frac{1}{x-3} \]
Multiply both sides by \(x(x-3)\).

\[
\begin{align*}
3 + 4(x-3) &= x \\
3 + 4x - 12 &= x \\
3x &= 9 \\
x &= 3
\end{align*}
\]

A check reveals that \(x = 3\) is an extraneous solution since it makes the denominator zero, so there is no solution.

61. \( (x + 2)^2 + 5 = (x + 3)^2 \)
\[
\begin{align*}
x^2 + 4x + 4 + 5 &= x^2 + 6x + 9 \\
4x + 9 &= 6x + 9 \\
-2x &= 0 \\
x &= 0
\end{align*}
\]

63. \( (x + 2)^2 - x^2 = 4(x + 1) \)
\[
\begin{align*}
x^2 + 4x + 4 - x^2 &= 4x + 4 \\
4 &= 4
\end{align*}
\]
The equation is an identity; every real number is a solution.

65. \( y = 2(x - 1) - 4 \)
\[
\begin{align*}
0 &= 2(x - 1) - 4 \\
0 &= 2x - 2 - 4 \\
0 &= 2x - 6 \\
6 &= 2x \\
3 &= x \\
x &= 3
\end{align*}
\]
The \(x\)-intercept is at 3. The solution to \(0 = 2(x - 1) - 4\) and the \(x\)-intercept of \(y = 2(x - 1) - 4\) are the same. They are both \(x = 3\). The \(x\)-intercept is \((3, 0)\).

67. \( y = 20 - (3x - 10) \)
\[
\begin{align*}
0 &= 20 - (3x - 10) \\
0 &= 20 - 3x + 10 \\
0 &= 30 - 3x \\
3x &= 30 \\
x &= 10
\end{align*}
\]
The \(x\)-intercept is at 10. The solution to \(0 = 20 - (3x - 10)\) and the \(x\)-intercept of \(y = 20 - (3x - 10)\) are the same. They are both \(x = 10\). The \(x\)-intercept is \((10, 0)\).
69. $y = -38 + 5(9 - x) \quad 0 = -38 + 5(9 - x)$  
\[0 = -38 + 45 - 5x\]  
\[0 = 7 - 5x\]  
\[5x = 7\]  
\[x = \frac{7}{5}\]

The $x$-intercept is at $\frac{7}{5}$. The solution to $0 = -38 + 5(9 - x)$ and the $x$-intercept of $y = -38 + 5(9 - x)$ are the same. They are both $x = \frac{7}{5}$. The $x$-intercept is \(\left(\frac{7}{5}, 0\right)\).

71. $y = 12 - 5x$  
$x$-intercept: $0 = 12 - 5x \Rightarrow 5x = 12 \Rightarrow x = \frac{12}{5}$  
y-intercept: $y = 12 - 5(0) \Rightarrow y = 12$

The $x$-intercept is \(\left(\frac{12}{5}, 0\right)\) and the $y$-intercept is $(0, 12)$.

73. $y = -3(2x + 1)$  
x-intercept: $0 = -3(2x + 1) \Rightarrow 0 = 2x + 1 \Rightarrow x = -\frac{1}{2}$  
y-intercept: $y = -3(2(0) + 1) \Rightarrow y = -3$

The $x$-intercept is \(-\frac{1}{2}, 0\) and the $y$-intercept is $(0, -3)$.

75. $2x + 3y = 10$  
x-intercept: $2x + 3(0) = 10 \Rightarrow 2x = 10 \Rightarrow x = 5$

The $x$-intercept is $(5, 0)$ and the $y$-intercept is \(\left(\frac{10}{3}, 0\right)\).

77. $\frac{2x}{5} + 8 - 3y = 0 \Rightarrow 2x + 40 - 15y = 0$  
Multiply both sides by 5.

x-intercept: $2x + 40 - 15(0) = 0 \Rightarrow 2x + 40 = 0 \Rightarrow x = -20$

y-intercept: $2(0) + 40 - 15y = 0 \Rightarrow 40 - 15y = 0 \Rightarrow y = \frac{40}{15} = \frac{8}{3}$

The $x$-intercept is $(-20, 0)$ and the $y$-intercept is $(0, \frac{8}{3})$.

79. $4y - 0.75x + 1.2 = 0$  
x-intercept: $4(0) - 0.75x + 1.2 = 0 \Rightarrow -0.75x + 1.2 = 0 \Rightarrow x = \frac{1.2}{0.75} = 1.6$

y-intercept: $4y - 0.75(0) + 1.2 = 0 \Rightarrow 4y + 1.2 = 0 \Rightarrow y = \frac{-1.2}{4} = -0.3$

The $x$-intercept is $(1.6, 0)$ and the $y$-intercept is $(0, -0.3)$.

81. $4(x + 1) - ax = x + 5$  
$4x + 4 - ax = x + 5$  
$3x - ax = 1$  
x-intercept: $3x - a\left(\frac{1}{3 - a}\right) = 1$  
$x = \frac{1}{3 - a}, a \neq 3$

83. $6x + ax = 2x + 5$  
$4x + ax = 5$  
x-intercept: $x(4 + a) = 5 \Rightarrow x = \frac{5}{4 + a}, a \neq -4$

85. $19x + \frac{1}{2}ax = x + 9$  
$18x + \frac{1}{2}ax = 9$  
Multiply both sides by $2$.

$x(36 + a) = 18$  
$x = \frac{18}{36 + a}, a \neq -36$

87. $-2ax + 6(x + 3) = -4x + 1$  
$-2ax + 6x + 18 = -4x + 1$  
$-2ax + 10x + 18 = 1$

$-2ax + 10 = 17$  
x-intercept: $x(-2a + 10) = -17 \Rightarrow x = \frac{-17}{-2a + 10}, a \neq 5$
89. \[0.275x + 0.725(500 - x) = 300\]
\[0.275x + 362.5 - 0.725x = 300\]
\[-0.45x = -62.5\]
\[x = \frac{62.5}{0.45}\]
\[\approx 138.889\]

91. \[\frac{2}{7.398} - \frac{4.405}{x} = \frac{1}{x}\]
Multiply both sides by 7.398x.
\[2x - (4.405)(7.398) = 7.398\]
\[2x = (4.405)(7.398) + 7.398\]
\[2x = (5.405)(7.398)\]
\[x = \frac{(5.405)(7.398)}{2} \approx 19.993\]

93. \[471 = 2\pi(25) + 2\pi(5h)\]
\[471 = 50\pi + 10\pi h\]
\[471 - 50\pi = 10\pi h\]
\[h = \frac{471 - 50\pi}{10\pi} = \frac{471 - 50(3.14)}{10(3.14)} \approx 10\]
\[h = 10\text{ feet}\]

95. (a) Female: \[y = 0.432x - 10.44\]
For \(y = 16:\]
\[16 = 0.432x - 10.44\]
\[26.44 = 0.432x\]
\[\frac{26.44}{0.432} = x\]
\[x = 61.2\text{ inches}\]

(b) Male: \[y = 0.449x - 12.15\]
For \(y = 19:\]
\[19 = 0.449x - 12.15\]
\[31.15 = 0.449x\]
\[69.4 \approx x\]

Yes, it is likely that both bones came from the same person because the estimated height of a male with a 19-inch thigh bone is 69.4 inches.

(d) \[0.432x - 10.44 = 0.449x - 12.15\]
\[1.71 = 0.017x\]
\[x = \frac{100.59}{1.71} \approx 59\text{ inches}\]

It is unlikely that a female would be over 8 feet tall, so if a femur of this length was found, it most likely belonged to a very tall male.

97. \[y = 1.64t + 36.8, \ -1 \leq t \leq 12\]

(a) The \(y\)-intercept is \((0, 36.8)\).

(b) Let \(t = 0:\]
\[y = 1.64(0) + 36.8 = 36.8\]
\[y\)-intercept: \((0, 36.8)\)

(c) \[65 = 1.64t + 36.8\]
\[28.2 = 1.64t\]
\[t = \frac{28.2}{1.64} \approx 17.2\]

This corresponds with the year 2017. Explanations will vary.
99. \(10,000 = 0.32m + 2500\)

\[7500 = 0.32m\]

\[
\frac{7500}{0.32} = m
\]

\[m = 23,437.5 \text{ miles}\]

101. False. \(x(3 - x) = 10 \implies 3x - x^2 = 10\)

This is a quadratic equation. The equation cannot be written in the form \(ax + b = 0\).

103. Equivalent equations are derived from the substitution principle and simplification techniques. They have the same solution(s).

2x + 3 = 8 and 2x = 5 are equivalent equations.

105. (a) 

<table>
<thead>
<tr>
<th>(x)</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.2x - 5.8</td>
<td>-9</td>
<td>-5.8</td>
<td>-2.6</td>
<td>0.6</td>
<td>3.8</td>
<td>7</td>
</tr>
</tbody>
</table>

(b) Since the sign changes from negative at 1 to positive at 2, the root is somewhere between 1 and 2.

1 < \(x\) < 2

(c) 

<table>
<thead>
<tr>
<th>(x)</th>
<th>1.5</th>
<th>1.6</th>
<th>1.7</th>
<th>1.8</th>
<th>1.9</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.2x - 5.8</td>
<td>-1</td>
<td>-0.68</td>
<td>-0.36</td>
<td>-0.04</td>
<td>0.28</td>
<td>0.6</td>
</tr>
</tbody>
</table>

(d) Since the sign changes from negative at 1.8 to positive at 1.9, the root is somewhere between 1.8 and 1.9.

1.8 < \(x\) < 1.9

To improve accuracy, evaluate the expression at subintervals within this interval and determine where the sign changes.

107. \[
\frac{x^2 + 5x - 36}{2x^2 + 17x - 9} = \frac{(x + 9)(x - 4)}{(2x - 1)(x + 9)} = \frac{x - 4}{2x - 1}, \quad x \neq -9
\]

109. \(y = 3x - 5\)

Intercepts: \((0, -5), \left(\frac{5}{3}, 0\right)\)

111. \(y = -x^2 - 5x = -x(x + 5)\)

Intercepts: \((0, 0), (-5, 0)\)
Section 1.3  Modeling with Linear Equations

■ You should be able to set up mathematical models to solve problems.
■ You should be able to translate key words and phrases.
(a) Equality:
Equals, equal to, is, are, was, will be, represents
(b) Addition:
Sum, plus, greater, increased by, more than, exceeds, total of
(c) Subtraction:
Difference, minus, less than, decreased by, subtracted from, reduced by, the remainder
(d) Multiplication:
Product, multiplied by, twice, times, percent of
(e) Division:
Quotient, divided by, ratio, per
(f) Consecutive:
Next, subsequent

You should know the following formulas:
(a) Perimeter:
1. Square: \( P = 4s \)
2. Rectangle: \( P = 2l + 2w \)
3. Circle: \( C = 2\pi r \)
4. Triangle: \( P = a + b + c \)
(b) Area:
1. Square: \( A = s^2 \)
2. Rectangle: \( A = lw \)
3. Circle: \( A = \pi r^2 \)
4. Triangle: \( A = \frac{1}{2}bh \)
(c) Volume
1. Cube: \( V = s^3 \)
2. Rectangular solid: \( V = lwh \)
3. Cylinder: \( V = \pi r^2h \)
4. Sphere: \( V = \frac{4}{3}\pi r^3 \)
(d) Simple Interest: \( I = Prt \)
(e) Compound Interest: \( A = P\left(1 + \frac{r}{n}\right)^n \)
(f) Distance: \( d = rt \)
(g) Temperature: \( F = \frac{9}{5}C + 32 \)

■ You should be able to solve word problems. Study the examples in the text carefully.

Vocabulary Check

1. mathematical modeling
2. verbal model; algebraic equation
3. \( A = \pi r^2 \)
4. \( P = 2l + 2w \)
5. \( V = s^3 \)
6. \( V = \pi r^2h \)
7. \( A = P\left(1 + \frac{r}{12}\right)^{12t} \)
8. \( I = Prt \)

1. \( x + 4 \)
The sum of a number and 4
A number increased by 4
3. \( \frac{u}{5} \)
The ratio of a number and 5
The quotient of a number and 5
A number divided by 5
5. \( \frac{y - 4}{5} \)
The difference of a number and 4 is divided by 5.
A number decreased by 4 is divided by 5.
7. \(-3(b + 2)\)
   The product of \(-3\) and the sum of a number and 2.
   Negative 3 is multiplied by a number increased by 2.

9. \(12(x - 5)\)
   The difference of a number and 5 is multiplied by 12 times the number. 12 is multiplied by a number and that product is multiplied by the number decreased by 5.

11. **Verbal Model:** (Sum) = (first number) + (second number)
    **Labels:** Sum = S, first number = n, second number = \(n + 1\)
    **Expression:** \(S = n + (n + 1) = 2n + 1\)

13. **Verbal Model:** Product = (first odd integer)(second odd integer)
    **Labels:** Product = P, first odd integer = \(2n - 1\), second odd integer = \(2n - 1 + 2 = 2n + 1\)
    **Expression:** \(P = (2n - 1)(2n + 1) = 4n^2 - 1\)

15. **Verbal Model:** (Distance) = (rate) \(\cdot\) (time)
    **Labels:** Distance = d, rate = 50 mph, time = t
    **Expression:** \(d = 50t\)

17. **Verbal Model:** (Amount of acid) = 20% \(\cdot\) (amount of solution)
    **Labels:** Amount of acid (in gallons) = A, amount of solution (in gallons) = x
    **Expression:** \(A = 0.20x\)

19. **Verbal Model:** Perimeter = 2(width) + 2(length)
    **Labels:** Perimeter = P, width = x, length = 2(width) = 2x
    **Expression:** \(P = 2x + 2(2x) = 6x\)

21. **Verbal Model:** (Total cost) = (unit cost)(number of units) + (fixed cost)
    **Labels:** Total cost = C, fixed cost = $1200, unit cost = $25, number of units = x
    **Expression:** \(C = 25x + 1200\)

23. **Verbal Model:** Thirty percent of the list price \(L\)
    **Expression:** \(0.30L\)

25. **Verbal Model:** percent of 500 that is represented by the number \(N\)
    **Equation:** \(N = p(500)\), \(p\) is in decimal form

27.

Area = Area of top rectangle + Area of bottom rectangle
\[A = 4x + 8x = 12x\]
29. **Verbal Model:** Sum = (first number) + (second number)

*Labels:* Sum = 525, first number = n, second number = n + 1

*Equations:*  
525 = n + (n + 1)  
525 = 2n + 1  
524 = 2n  
\( n = 262 \)

*Answer:* First number = n = 262, second number = n + 1 = 263

31. **Verbal Model:** Difference = (one number) − (another number)

*Labels:* Difference = 148, one number = 5x, another number = x

*Equation:*  
148 = 5x − x  
148 = 4x  
x = 37  
5x = 185

*Answer:* The two numbers are 37 and 185.

33. **Verbal Model:** Product = (smaller number) · (larger number) = (smaller number)² − 5

*Labels:* Smaller number = n, larger number = n + 1

*Equation:*  
n(n + 1) = n² − 5  
n² + n = n² − 5  
\( n = -5 \)

*Answer:* Smaller number = n = −5, larger number = n + 1 = −4

35. \( x = \text{percent} \cdot \text{number} \)

37. \( 432 = \text{percent} \cdot 1600 \)

39. \( 12 = \frac{1}{2} \% \cdot \text{number} \)

41. **Verbal Model:** Loan payments = 58.6% · Annual income

*Labels:* Loan payments = 13,077.75

Annual income = I

*Equation:*  
13,077.75 = 0.586I  
I = 22,316.98

The family’s annual income is $22,316.98.
43. **Verbal Model:** (2002 price for a gallon of unleaded gasoline) = (percentage increase)(1990 price for a gallon of unleaded gasoline) + (1990 price for a gallon of unleaded gasoline)

**Labels:**
- 2002 price for a gallon of unleaded gasoline: $1.36
- 1990 price for a gallon of unleaded gasoline: $1.16
- Percentage increase: \( p \)

**Equation:**
- \( 1.36 = 1.16p + 1.16 \)
- \( 0.20 = 1.16p \)
- \( 0.172 = p \)

**Answer:** Percentage increase \( \approx 17.2\% \)

45. **Verbal Model:** (2002 price for a pound of tomatoes) = (percentage increase)(1990 price for a pound of tomatoes) + (1990 price for a pound of tomatoes)

**Labels:**
- 2002 price for a pound of tomatoes: $1.66
- 1990 price for a pound of tomatoes: $0.86
- Percentage increase: \( p \)

**Equation:**
- \( 1.66 = 0.86p + 0.86 \)
- \( 0.80 = 0.86p \)
- \( 0.930 = p \)

**Answer:** Percentage increase \( \approx 93\% \)

47. **Verbal Model:** (Sale price) = (list price) − (discount)

**Labels:**
- Sale price = $1210.75, list price = \( L \), discount = 0.165\( L \)

**Equation:**
- \( 1210.75 = L - 0.165L \)
- \( 1210.75 = 0.835L \)
- \( 1450 = L \)

**Answer:** The list price of the pool is $1450.

49. (a) 

(b) \( l = 1.5w \)

(c) \( 25 = 5w \)

51. **Verbal Model:** Average = \( \frac{(test\ #1) + (test\ #2) + (test\ #3) + (test\ #4)}{4} \)

**Labels:**
- Average = 90, test #1 = 87, test #2 = 92, test #3 = 84, test #4 = \( x \)

**Equation:**
- \( 90 = \frac{87 + 92 + 84 + x}{4} \)
- \( 360 = 263 + x \)
- \( x = 97 \)

**Answer:** You must score 97 (or better) on test #4 to earn an A for the course.
53. Rate = \frac{\text{distance}}{\text{time}} = \frac{50 \text{ kilometers}}{\frac{1}{2} \text{ hour}} = 100 \text{ kilometers/hour}

Total time = \frac{\text{total distance}}{\text{rate}} = \frac{300 \text{ kilometers}}{100 \text{ kilometers/hour}} = 3 \text{ hours}

55. (time on first part) + (time on second part) = (Total time)

\begin{align*}
    t_1 + t_2 &= T \\
    \frac{d_1}{r_1} + \frac{d_2}{r_2} &= T \\
    \frac{x}{58} + \frac{317 - x}{52} &= 5.75 \\
    52x + 58(317 - x) &= 5.75(58)(52) \\
    52x + 18,386 - 58x &= 17,342 \\
    -6x &= -1044 \\
    x &= 174 \text{ miles} \\
    t_1 &= \frac{174}{58} = 3 \text{ hours} \\
    t_2 &= \frac{317 - 174}{52} = 2.75 \text{ hours}
\end{align*}

The salesman averaged 58 miles per hour for 3 hours and 52 miles per hour for 2 hours and 45 minutes.

57. (a) Time for the first family: \( t_1 = \frac{d}{r_1} = \frac{160}{42} = 3.81 \text{ hours} \)

Time for the other family: \( t_2 = \frac{d}{r_2} = \frac{160}{50} = 3.2 \text{ hours} \)

(b) \( t = \frac{d}{r} = \frac{100}{42 + 50} = \frac{100}{92} \approx 1.08 \text{ hours} \)

(c) \( d = rt = 42 \left( \frac{160}{42} - \frac{160}{50} \right) = 25.6 \text{ miles} \)

59. Verbal Model: time = \frac{\text{distance}}{\text{rate}}

\text{Labels:} \quad \text{Let} \ x = \text{wind speed, then the rate to the city} = 600 + x, \text{ the rate from the city} = 600 - x, \text{ the distance to the city} = 1500 \text{ kilometers, the distance traveled so far in the return trip} = 1500 - 300 = 1200 \text{ kilometers.}

\text{Equation:} \quad \frac{1500}{600 + x} = \frac{1200}{600 - x} \\
1500(600 - x) = 1200(600 + x) \\
900,000 - 1500x = 720,000 + 1200x \\
180,000 = 2700x \\
66\frac{2}{3} = x

Wind speed: 66\frac{2}{3} \text{ kilometers per hour}

61. Verbal Model: time = \frac{\text{distance}}{\text{rate}}

\text{Equation:} \quad t = \frac{3.84 \times 10^8 \text{ meters}}{3.0 \times 10^8 \text{ meters per second}} \approx 1.28 \text{ seconds}

The radio wave travels from Mission Control to the moon in 1.28 seconds.
63. **Verbal Model:** \[
\frac{\text{height of building}}{\text{length of building's shadow}} = \frac{\text{height of stake}}{\text{length of stake's shadow}}
\]

**Label:** Let \( h \) = height of the building in feet.

**Equation:**
- \( \frac{h}{87} = \frac{4}{1/3} \)
- \( \frac{1}{3}h = 348 \)
- \( h = 1044 \) feet

The Chrysler building is 1044 feet tall.

65. **Verbal Model:** \[
\frac{\text{height of silo}}{\text{length of silo’s shadow}} = \frac{\text{height of person}}{\text{height of person’s shadow}}
\]

**Label:** Let \( x \) = length of person’s shadow.

**Equation:**
- \( \frac{50}{32 + x} = \frac{6}{x} \)
- \( 50x = 6(32 + x) \)
- \( 50x = 192 + 6x \)
- \( 44x = 192 \)
- \( x \approx 4.36 \) feet

67. **Verbal Model:** \((\text{Interest in 3\% fund}) + (\text{interest in 41/2\% fund}) = (\text{total interest})\)

**Labels:** Let \( x \) = amount in the 3\% fund. Then \( 25,000 - x \) = amount in the 41/2\% fund.

**Equation:**
- \( 1000 = 0.03x + 0.045(25,000 - x) \)
- \( 1000 = 0.03x + 1125 - 0.045x \)
- \( -125 = -0.015x \)
- \( x = $833.33 \) at 3\%
- \( 25,000 - x = $16,666.67 \) at 41/2\%

69. **Verbal Model:** \((\text{profit on minivans}) + (\text{profit on SUVs}) = (\text{total profit})\)

**Labels:** Let \( x \) = amount invested in minivans. Then, \( 600,000 - x \) = amount invested in SUVs.

**Equation:**
- \( 0.24x + 0.28(600,000 - x) = 0.25(600,000) \)
- \( 0.24x + 168,000 - 0.28x = 150,000 \)
- \( -0.04x = -18,000 \)
- \( x = 450,000 \)

$450,000 is invested in minivans and \( 600,000 - x = $150,000 \) is invested in SUVs.

71. **Verbal Model:** \((\text{Final concentration})(\text{Amount}) = (\text{Solution 1 concentration})(\text{Amount}) + (\text{Solution 2 concentration})(\text{Amount})\)

**Label:** Let \( x \) = amount of 100\% concentrate

**Equation:**
- \( 0.75(55) = 0.40(55 - x) + 1.00x \)
- \( 41.25 = 22 - 0.40x + 1.00x \)
- \( 41.25 = 0.60x + 22 \)
- \( x = 32.1 \) gallons

Approximately 32.1 gallons of the 100\% concentrate will be needed.
73. Verbal Model: (price per pound of peanuts)(number of pounds of peanuts) + (price per pound of walnuts)(number of pounds of walnuts) = (price per pound of nut mixture)(number of pounds of nut mixture)

Labels: Let \( x \) = number of pounds of $2.49 peanuts. Then \( 100 - x \) = number of pounds of $3.89 walnuts.

Equation: Use 50 pounds of peanuts and 50 pounds of walnuts.

\[
2.49x + 3.89(100 - x) = 3.19(100)
\]

\[
2.49x + 389 - 3.89x = 319
\]

\[-1.40x = -70
\]

\[
x = \frac{-70}{-1.40}
\]

\[
x = 50 \text{ pounds of $2.49 peanuts}
\]

\[
100 - x = 50 \text{ pounds of $3.89 walnuts}
\]

Use 50 pounds of peanuts and 50 pounds of walnuts.

75. Verbal Model: Total cost = (Fixed cost) + (Variable cost per unit) \( \cdot \) (Number of units)

Labels: Total cost = $85,000, variable costs = $9.30x, fixed costs = $10,000

Equation: $85,000 = $10,000 + $9.30x

\[
x = \frac{75,000}{9.3} = 8064.52 \text{ units}
\]

At most the company can manufacture 8064 units.

77. \( A = \frac{1}{2}bh \)

\[
2A = bh
\]

\[
\frac{2A}{b} = h
\]

79. \( S = C + RC \)

\[
S = C(1 + R)
\]

\[
\frac{S}{1 + R} = C
\]

81. \( V = \frac{4}{3}\pi a^2 b \)

\[
\frac{3V}{\pi a^2} = b
\]

\[
\frac{3V}{4\pi a^2} = b
\]

83. \( h = v_0t + \frac{1}{2}at^2 \)

85. \( C = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} \)

\[
\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}
\]

\[
\frac{1}{C} - \frac{1}{C_2} = \frac{1}{C_1}
\]

\[
\frac{C_2 - C}{CC_2} = \frac{1}{C_1}
\]

\[
\frac{CC_2}{C_2 - C} = C_1
\]

87. \( L = a + (n - 1)d \)

\[
L - a + d = nd
\]

\[
\frac{L - a + d}{d} = n
\]

89. \( W_1x = W_2(L - x) \)

\[
50x = 75(10 - x)
\]

\[
50x = 750 - 75x
\]

\[
125x = 750
\]

\[
x = 6 \text{ feet from 50-pound child.}
\]
You should be able to solve a quadratic equation by factoring, if possible. You should be able to solve a quadratic equation of the form $ax^2 + bx + c = 0$, $a \neq 0$, by extracting square roots. You should be able to solve a quadratic equation by completing the square. You should know and be able to use the Quadratic Formula: For $ax^2 + bx + c = 0$, $a \neq 0$, 

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$ 

You should be able to determine the types of solutions of a quadratic equation by checking the discriminant $b^2 - 4ac$. 

(a) If $b^2 - 4ac > 0$, there are two distinct real solutions. The graph has two $x$-intercepts. 
(b) If $b^2 - 4ac = 0$, there is one repeated real solution. The graph has one $x$-intercept. 
(c) If $b^2 - 4ac < 0$, there is no real solution. The graph has no $x$-intercepts. You should be able to use your calculator to solve quadratic equations. You should be able to solve applications involving quadratic equations. Study the examples in the text carefully.
### Vocabulary Check

1. quadratic equation
2. factoring, extracting square roots, completing the square, and the Quadratic Formula
3. discriminant
4. position equation; \(-16t^2 + v_0t + s_0\); velocity of the object; initial height of the object
5. Pythagorean Theorem

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
</table>
| 1. | \[2x^2 = 3 - 8x\]  
General form: \[2x^2 + 8x - 3 = 0\] | 3. | \[(x - 3)^2 = 3\]  
\[x^2 - 6x + 9 = 3\]  
General form: \[x^2 - 6x + 6 = 0\] | 5. | \[\frac{1}{2}(3x^2 - 10) = 18x\]  
\[3x^2 - 10 = 90x\]  
General form: \[3x^2 - 90x - 10 = 0\] |
| 7. | \[6x^2 + 3x = 0\]  
\[3(x^2 + 1) = 0\]  
\[3x = 0\] or \[2x + 1 = 0\]  
\[x = 0\] or \[x = -\frac{1}{2}\] | 9. | \[x^2 - 2x - 8 = 0\]  
\[(x - 4)(x + 2) = 0\]  
\[x - 4 = 0\] or \[x + 2 = 0\]  
\[x = 4\] or \[x = -2\] | 11. | \[x^2 + 10x + 25 = 0\]  
\[(x + 5)^2 = 0\]  
\[x + 5 = 0\]  
\[x = -5\] |
| 13. | \[3 + 5x - 2x^2 = 0\]  
\[(3 - x)(1 + 2x) = 0\]  
\[3 - x = 0\] or \[1 + 2x = 0\]  
\[x = 3\] or \[x = -\frac{1}{2}\] | 15. | \[x^2 + 4x = 12\]  
\[x^2 + 4x - 12 = 0\]  
\[(x + 6)(x - 2) = 0\]  
\[x + 6 = 0\] or \[x - 2 = 0\]  
\[x = -6\] or \[x = 2\] | 17. | \[\frac{3}{4}x^2 + 8x + 20 = 0\]  
\[4\left(\frac{3}{4}x^2 + 8x + 20\right) = 4(0)\]  
\[3x^2 + 32x + 80 = 0\]  
\[(3x + 20)(x + 4) = 0\]  
\[3x + 20 = 0\] or \[x + 4 = 0\]  
\[x = -\frac{20}{3}\] or \[x = -4\] |
| 19. | \[x^2 + 2ax + a^2 = 0\]  
\[(x + a)^2 = 0\]  
\[x + a = 0\]  
\[x = -a\] | 21. | \[x^2 = 49\]  
\[x = \pm 7\] | 23. | \[x^2 = 11\]  
\[x = \pm \sqrt{11}\] |
| 25. | \[3x^2 = 81\]  
\[x^2 = 27\]  
\[x = \pm 3\sqrt{3}\] | 27. | \[(x - 12)^2 = 16\]  
\[x - 12 = \pm 4\]  
\[x = 12 \pm 4\]  
\[x = 16\] or \[x = 8\] | 29. | \[(x + 2)^2 = 14\]  
\[x + 2 = \pm \sqrt{14}\]  
\[x = -2 \pm \sqrt{14}\] |
| 31. | \[(2x - 1)^2 = 18\]  
\[2x - 1 = \pm \sqrt{18}\]  
\[2x = 1 \pm 3\sqrt{2}\]  
\[x = \frac{1 \pm 3\sqrt{2}}{2}\] | 33. | \[(x - 7)^2 = (x + 3)^2\]  
\[x - 7 = \pm (x + 3)\]  
\[x - 7 = x + 3\] or \[x - 7 = -x - 3\]  
\[-7 \neq 3\] or \[2x = 4\]  
\[x = 2\]  
The only solution to the equation is \(x = 2\). |
35. \( x^2 + 4x - 32 = 0 \)
\[ x^2 + 4x = 32 \]
\[ x^2 + 4 + 2^2 = 32 + 2^2 \]
\[ (x + 2)^2 = 36 \]
\[ x + 2 = \pm 6 \]
\[ x = -2 \pm 6 \]
\[ x = 4 \text{ or } x = -8 \]

37. \( x^2 + 12x + 25 = 0 \)
\[ x^2 + 12x = -25 \]
\[ x^2 + 12x + 6^2 = -25 + 6^2 \]
\[ (x + 6)^2 = 11 \]
\[ x + 6 = \pm \sqrt{11} \]
\[ x = -6 \pm \sqrt{11} \]

39. \( 9x^2 - 18x = -3 \)
\[ x^2 - 2x = -\frac{1}{3} \]
\[ x^2 - 2x + 1^2 = -\frac{1}{3} + 1^2 \]
\[ (x - 1)^2 = \frac{2}{3} \]
\[ x - 1 = \pm \sqrt{\frac{2}{3}} \]
\[ x = 1 \pm \sqrt{\frac{2}{3}} \]
\[ x = 1 \pm \frac{\sqrt{6}}{3} \]

41. \( 8 + 4x - x^2 = 0 \)
\[ -x^2 + 4x + 8 = 0 \]
\[ x^2 - 4x = -8 \]
\[ x^2 - 4x + 2^2 = -8 + 2^2 \]
\[ (x - 2)^2 = 12 \]
\[ x - 2 = \pm \sqrt{12} \]
\[ x = 2 \pm 2\sqrt{3} \]

43. \( 2x^2 + 5x - 8 = 0 \)
\[ 2x^2 + \frac{5}{2}x = 4 \]
\[ x^2 + \frac{5}{4}x = 2 \]
\[ x^2 + \frac{5}{4}x + \left(\frac{5}{4}\right)^2 = 2 + \left(\frac{5}{4}\right)^2 \]
\[ \left(x + \frac{5}{4}\right)^2 = \frac{89}{16} \]
\[ x + \frac{5}{4} = \pm \sqrt{\frac{89}{4}} \]
\[ x = -\frac{5}{4} \pm \frac{\sqrt{89}}{4} \]

45. \( \frac{1}{x^2 + 2x + 5} = \frac{1}{x^2 + 2x + 1^2 - 1^2 + 5} \)
\[ = \frac{1}{(x + 1)^2 + 4} \]

49. \( \frac{1}{\sqrt{6x - x^3}} = \frac{1}{\sqrt{-1(x^2 - 6x + 3^3 - 3^3)}} \)
\[ = \frac{1}{\sqrt{-1[(x - 3)^2 - 9]}} \]
\[ = \frac{1}{\sqrt{-(x - 3)^2 + 9}} = \frac{1}{\sqrt{9 - (x - 3)^2}} \]

47. \( \frac{4}{x^2 + 4x - 3} = \frac{4}{x^2 + 4x + 4 - 4 - 3} \)
\[ = \frac{4}{(x + 2)^2 - 7} \]

51. (a) \( y = (x + 3)^2 - 4 \)

(b) The x-intercepts are \((-1, 0)\) and \((-5, 0)\).

(c) \(0 = (x + 3)^2 - 4\)
\[4 = (x + 3)^2\]
\[\pm \sqrt{4} = x + 3\]
\[-3 \pm 2 = x\]
\[x = -1 \text{ or } x = -5\]

(d) The x-intercepts of the graph are solutions to the equation \(0 = (x + 3)^2 - 4.\)
53. (a) \( y = 1 - (x - 2)^2 \)

(b) The \( x \)-intercepts are (1, 0) and (3, 0).

55. (a) \( y = -4x^2 + 4x + 3 \)

(b) The \( x \)-intercepts are \(( -\frac{1}{2}, 0)\) and \((\frac{3}{2}, 0)\).

(d) The \( x \)-intercepts of the graph are solutions to the equation \( 0 = -4x^2 + 4x + 3 \).

57. (a) \( y = x^2 + 3x - 4 \)

(b) The \( x \)-intercepts are (−4, 0) and (1, 0).

59. \( 2x^2 - 5x + 5 = 0 \)
\[ b^2 - 4ac = (-5)^2 - 4(2)(5) = -15 < 0 \]
No real solution

63. \( \frac{1}{4}x^2 - 5x + 25 = 0 \)
\[ b^2 - 4ac = (-5)^2 - 4\left(\frac{1}{4}\right)(25) = -\frac{25}{4} < 0 \]
No real solution

67. \( 2x^2 + x - 1 = 0 \)
\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{1^2 - 4(2)(-1)}}{2(2)} = \frac{-1 \pm 3}{4} = \frac{1}{2} - 1 \]

69. \( 16x^2 + 8x - 3 = 0 \)
\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-8 \pm \sqrt{8^2 - 4(16)(-3)}}{2(16)} = \frac{-8 \pm 16}{32} = \frac{1}{4} \pm \frac{3}{4} \]

71. \( 2 + 2x - x^2 = 0 \)
\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{2^2 - 4(-1)(2)}}{2(-1)} = \frac{-2 \pm 2\sqrt{3}}{-2} = 1 \pm \sqrt{3} \]
73. \( x^2 + 14x + 44 = 0 \)
\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
\[ = \frac{-14 \pm \sqrt{14^2 - 4(144)}}{2(1)} \]
\[ = -7 \pm \sqrt{5} \]

75. \( x^2 + 8x - 4 = 0 \)
\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
\[ = -4 \pm \sqrt{4^2 - 4(-4)} \]
\[ = -4 \pm \sqrt{25} \]
\[ = -4 \pm 5 \]
\[ x = -9 \pm 5 \]

77. \( 12x - 9x^2 = -3 \)
\[ 9x^2 - 12x + 3 = 0 \]
\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
\[ = \frac{-12 \pm \sqrt{12^2 - 4(-9)(3)}}{2(-9)} \]
\[ = \frac{-12 \pm 6\sqrt{3}}{3} \]
\[ = \frac{2}{3} \pm \frac{\sqrt{3}}{3} \]

79. \( 9x^2 + 24x + 16 = 0 \)
\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
\[ = \frac{-24 \pm \sqrt{24^2 - 4(9)(16)}}{2(9)} \]
\[ = \frac{-24 \pm 0}{18} \]
\[ = -4 \]

85. \( 8t = 5 + 2r^2 \)
\[ -2r^2 + 8t - 5 = 0 \]
\[ r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
\[ = \frac{-8 \pm \sqrt{6^2 - 4(-2)(-5)}}{2(-2)} \]
\[ = \frac{-8 \pm 2\sqrt{6}}{4} \]
\[ = \frac{-2 \pm \sqrt{6}}{2} \]

87. \( (y - 5)^2 = 2y \)
\[ y^2 - 12y + 25 = 0 \]
\[ y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
\[ = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(1)(25)}}{2(1)} \]
\[ = \frac{12 \pm 2\sqrt{11}}{2} \]
\[ = 6 \pm \sqrt{11} \]

91. \( 5.1x^2 - 1.7x - 3.2 = 0 \)
\[ x = \frac{1.7 \pm \sqrt{(-1.7)^2 - 4(5.1)(-3.2)}}{2(5.1)} \]
\[ x \approx 0.976, -0.643 \]

93. \(-0.067x^2 - 0.852x + 1.277 = 0 \)
\[ x = \frac{-(-0.852) \pm \sqrt{(-0.852)^2 - 4(-0.067)(1.277)}}{2(-0.067)} \]
\[ x = -14.071, 1.355 \]

95. \( 422x^2 - 506x - 347 = 0 \)
\[ x = \frac{506 \pm \sqrt{(-506)^2 - 4(422)(-347)}}{2(422)} \]
\[ x \approx 1.687, -0.488 \]

97. \( 12.67x^2 + 31.55x + 8.09 = 0 \)
\[ x = \frac{-31.55 \pm \sqrt{(31.55)^2 - 4(12.67)(8.09)}}{2(12.67)} \]
\[ x = -2.200, -0.290 \]
99. \(x^2 - 2x - 1 = 0\) Complete the square.
\[
\begin{align*}
x^2 - 2x &= 1 \\
x^2 - 2x + 1^2 &= 1 + 1^2 \\
(x - 1)^2 &= 2 \\
x - 1 &= \pm \sqrt{2} \\
x &= 1 \pm \sqrt{2}
\end{align*}
\]

103. \(x^2 - x - \frac{11}{4} = 0\) Complete the square.
\[
\begin{align*}
x^2 - x - \frac{11}{4} &= 0 \\
x^2 - x + \left(\frac{1}{2}\right)^2 &= \frac{1}{4} + \left(\frac{3}{2}\right)^2 \\
\left(x - \frac{1}{2}\right)^2 &= \frac{12}{4} \\
x - \frac{1}{2} &= \pm \sqrt{\frac{12}{4}} \\
x &= \frac{1}{2} \pm \sqrt{3}
\end{align*}
\]

107. \(3x + 4 = 2x^2 - 7\) Quadratic Formula
\[
0 = 2x^2 - 3x - 11 \\
x = \frac{-(3) \pm \sqrt{(-3)^2 - 4(2)(-11)}}{2(2)} \\
= \frac{3 \pm \sqrt{97}}{4} \\
= \frac{3 \pm \sqrt{97}}{4}
\]

111. \(S = x^2 + 4sh\)
\[
84 = x^2 + 4s(2) \\
0 = x^2 + 8s - 84 \\
0 = (x + 14)(x - 6) \\
x = -14 \text{ or } x = 6
\]
Since \(x\) must be positive, we have \(x = 6\) inches. The dimensions of the box are \(6\) inches \(\times\) \(6\) inches \(\times\) \(2\) inches.

113. \((200 - 2x)(100 - 2x) = \frac{1}{2}(100)(200)\)
\[
\begin{align*}
20,000 - 600x + 4x^2 &= 10,000 \\
4x^2 - 600x + 10,000 &= 0 \\
4(x^2 - 150x + 2500) &= 0
\end{align*}
\]
Thus, \(a = 1, b = -150,\) and \(c = 2500.\)
113. —CONTINUED—

\[
x = \frac{150 \pm \sqrt{(-150)^2 - 4(1)(2500)}}{2(1)} \approx \frac{150 \pm 111.8034}{2}
\]

\[
x = \frac{150 + 111.8034}{2} \approx 130.902 \text{ feet}
\]

\[
x = \frac{150 - 111.8034}{2} \approx 19.098 \text{ feet}
\]

The person must go around the lot \(19.098 \text{ feet} \div 24 \text{ inches} = \frac{19.098 \text{ feet}}{2 \text{ feet}} = 9.5 \text{ times.}

115. \( s = -16t^2 + 32,000 \)

(a) \(-16t^2 + 32,000 = 0\)

\[
t^2 = 2000
\]

\[
t = \sqrt{2000} 
\]

\[
= 20.5 \text{ seconds}
\]

\[
= 44.72 \text{ seconds}
\]

(b) **Model:** \((\text{Rate}) \cdot (\text{time}) = (\text{distance})\)

**Labels:**
- Rate = 500 miles per hour
- Time = \(\frac{2\sqrt{3}}{3600} \approx 0.0124 \text{ hour}\)
- Distance = \(d\)

**Equation:** \(d = 500(0.0124) = 6.2 \text{ miles}\)

The bomb will travel approximately 6.2 miles horizontally.

117. \( s = -16t^2 + v_0t + s_0 \)

(a) \( v_0 = 100 \text{ mph} = \frac{100(5280)}{3600} = 146.7 \text{ ft/sec} \)

\[
s_0 = 6\frac{1}{4} \text{ feet}
\]

\[
s = -16t^2 + 146.7t + 6\frac{1}{4}
\]

(b) When \( t = 3: \) \( s(3) = 302.25 \text{ feet} \)

When \( t = 4: \) \( s(4) = 336.92 \text{ feet} \)

When \( t = 5: \) \( s(5) = 339.58 \text{ feet} \)

During the interval \( 3 \leq t \leq 5, \) the baseball’s speed decreased due to gravity.

(c) The ball hits the ground when \( s = 0. \)

\[-16t^2 + 146.7t + 6\frac{1}{4} = 0\]

By the Quadratic Formula, \( t \approx -0.042 \) or \( t \approx 9.209. \)

Assuming that the ball is not caught and drops to the ground, it will be in the air for approximately 9.209 seconds.

119. \( P = -0.0081t^2 + 0.417t + 1.99, \; 7 \leq t \leq 13 \)

(a)

<table>
<thead>
<tr>
<th>( t )</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P )</td>
<td>$4.51</td>
<td>$4.81</td>
<td>$5.09</td>
<td>$5.35</td>
<td>$5.60</td>
<td>$5.83</td>
<td>$6.04</td>
</tr>
</tbody>
</table>

The average admission price reached or surpassed $5.00 in 1999.

(b) 

\[-0.0081t^2 + 0.417t + 1.99 = 5.00\]

\[-0.0081t^2 + 0.417t - 3.01 = 0\]

By the Quadratic Formula, \( t \approx 8.68 \) or 42.80. Since \( 7 \leq t \leq 13, \) we choose \( t \approx 8.68 = 9 \)
 which corresponds to 1999.

(c) For 2008, let \( t = 18: \) \( P(18) = $6.87. \) Answers will vary.
121. $x^2 + x^2 = 5^2$

$2x^2 = 25$

$x^2 = \frac{25}{2}$

$x = \sqrt{\frac{25}{2}}$

$= \frac{5}{\sqrt{2}}$

$= \frac{5 \sqrt{2}}{2} \approx 3.54$ centimeters

Each leg in the right triangle is approximately 3.54 centimeters.

123. $d_N = (3 \text{ hours})(r + 50 \text{ mph})$

$d_E = (3 \text{ hours})(r \text{ mph})$

$d_N^2 + d_E^2 = 2440^2$

$9(r + 50)^2 + 9r^2 = 2440^2$

$18r^2 + 900r - 5,931,100 = 0$

$r = \frac{-900 \pm \sqrt{900^2 - 4(18)(-5,931,100)}}{2(18)} = \frac{-900 \pm 60\sqrt{118,847}}{36}$

Using the positive value for $r$, we have one plane moving northbound at $r + 50 = 600$ miles per hour and one plane moving eastbound at $r = 550$ miles per hour.

125. $x(20 - 0.0002x) = 500,000$

$0 = 0.0002x^2 - 20x + 500,000$

$0 = x^2 - 100,000x + 2,500,000,000$

$0 = (x - 50,000)^2$

$x = 50,000$ units

127. $0.125x^2 + 20x + 500 = 14,000$

$0.125x^2 + 20x - 13,500 = 0$

$x = \frac{-20 \pm \sqrt{20^2 - 4(0.125)(-13,500)}}{2(0.125)}$

Using the positive value for $x$, we have

$x = \frac{-20 + \sqrt{7,150}}{0.25} \approx 258$ units.

129. $800 + 0.04x + 0.002x^2 = 1680$

$0.002x^2 + 0.04x - 880 = 0$

$x = \frac{-0.04 \pm \sqrt{(0.04)^2 - 4(0.002)(-880)}}{2(0.002)}$

$= \frac{-0.04 \pm \sqrt{7.0416}}{0.004}$

Choosing the positive value for $x$, we have

$x = \frac{-0.04 + \sqrt{7.0416}}{0.004} \approx 653$ units.
131. \( M = 1.835r^2 + 3.58r + 333.0 \), \( 5 \leq r \leq 13 \)

\[
\begin{array}{|c|c|c|c|c|c|c|c|c|c|}
\hline
r & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 \\
\hline
M \text{ (in billions)} & $396.78 & $420.54 & $447.98 & $479.08 & $513.86 & $552.30 & $594.42 & $640.20 & $689.66 \\
\hline
\end{array}
\]

The total money in circulation reached or surpassed $600 billion in 2001.

(b) \( 1.835r^2 + 3.58r + 333.0 = 600 \)

\[ 1.835r^2 + 3.58r - 267.0 = 0 \]

By the Quadratic Formula, \( t \approx 11.1 \) or \( -13.1 \). Since \( 5 \leq r \leq 13 \), we choose \( t \approx 11.1 \) which corresponds to 2001.

(c) For 2008, let \( t = 18 \): \( M(18) = $991.98 \) billion

Answers will vary.

133. \( \left(\text{Distance from Oklahoma City to New Orleans}\right)^2 = \left(\text{Distance from Oklahoma City to Austin}\right)^2 + \left(\text{Distance from New Orleans to Austin}\right)^2 \)

Distance from Oklahoma City to New Orleans = 560

Distance from Oklahoma City to Austin = \( x \)

Distance from New Orleans to Austin = 1348 - 560 - \( x \) = 788 - \( x \)

\[
560^2 = x^2 + (788 - x)^2
\]

\[
313,600 = x^2 + 620,944 - 1576x + x^2
\]

\[
0 = 2x^2 - 1576x + 307,344
\]

\[
x = \frac{1576 \pm \sqrt{1576^2 - 4(2)(307,344)}}{2(2)} \approx 354.5 \text{ or } 433.5
\]

The other two distances are 354.5 miles and 433.5 miles.

135. False

\[ b^2 - 4ac = (-1)^2 - 4(-3)(-10) < 0, \]

so, the quadratic equation has no real solutions.

139. \( 3(x + 4)^2 + (x + 4) - 2 = 0 \)

(a) Let \( u = x + 4 \)

\[
3u^2 + u - 2 = 0
\]

\[
(3u - 2)(u + 1) = 0
\]

\[
3u - 2 = 0 \quad \text{or} \quad u + 1 = 0
\]

\[
u = \frac{2}{3} \quad \text{or} \quad u = -1
\]

\[
x + 4 = \frac{2}{3} \quad \text{or} \quad x + 4 = -1
\]

\[
x = - \frac{10}{3} \quad \text{or} \quad x = -5
\]

141. -3 and 6

One possible equation is:

\[ (x - (-3))(x - 6) = 0 \]

\[ (x + 3)(x - 6) = 0 \]

\[ x^2 - 3x - 18 = 0 \]

Any non-zero multiple of this equation would also have these solutions.

137. The student should have subtracted 15\( x \) from both sides so that the equation is equal to zero. By factoring out an \( x \), there are two solutions, \( x = 0 \) and \( x = 6 \).

143. 8 and 14

One possible equation is:

\[ (x - 8)(x - 14) = 0 \]

\[ x^2 - 22x + 112 = 0 \]

Any non-zero multiple of this equation would also have these solutions.
145. \( 1 + \sqrt{2} \) and \( 1 - \sqrt{2} \)

One possible equation is:
\[
\begin{align*}
[x - (1 + \sqrt{2})][x - (1 - \sqrt{2})] &= 0 \\
[(x - 1) - \sqrt{2}][(x - 1) + \sqrt{2}] &= 0 \\
(x - 1)^2 - (\sqrt{2})^2 &= 0 \\
x^2 - 2x + 1 - 2 &= 0 \\
x^2 - 2x - 1 &= 0
\end{align*}
\]

Any non-zero multiple of this equation would also have these solutions.

149. \( 7x^4 + (-7x^4) = 0 \) by the Additive Inverse Property.

153. \( (x + 4)(x^2 - x + 2) = x(x^2 - x + 2) + 4(x^2 - x + 2) \)
\[
= x^3 - x^2 + 2x + 4x^2 - 4x + 8 \\
= x^3 + 3x^2 - 2x + 8
\]

157. \( x^3 + 5x^2 - 2x - 10 = x^2(x + 5) - 2(x + 5) \)
\[
= (x + 5)(x^2 - 2)
\]

147. \( (10x)y = 10(xy) \) by the Associative Property of Multiplication.

151. \( (x + 3)(x - 6) = x^2 - 6x + 3x - 18 \)
\[
= x^2 - 3x - 18
\]

155. \( x^3 - 27x^2 = x^2(x^3 - 27) \)
\[
= x^2(x^3 - 3^3) \\
= x^2(x - 3)(x^2 + 3x + 9)
\]

159. Answers will vary.

### Section 1.5 Complex Numbers

- **Standard form:** \( a + bi \).
  - If \( b = 0 \), then \( a + bi \) is a real number.
  - If \( a = 0 \) and \( b \neq 0 \), then \( a + bi \) is a pure imaginary number.
- **Equality of Complex Numbers:** \( a + bi = c + di \) if and only if \( a = c \) and \( b = d \)
- **Operations on complex numbers**
  - (a) Addition: \( (a + bi) + (c + di) = (a + c) + (b + d)i \)
  - (b) Subtraction: \( (a + bi) - (c + di) = (a - c) + (b - d)i \)
  - (c) Multiplication: \( (a + bi)(c + di) = (ac - bd) + (ad + bc)i \)
  - (d) Division: \( \frac{a + bi}{c + di} = \frac{a + bi}{c + di} \cdot \frac{c - di}{c - di} = \frac{ac + bd}{c^2 + d^2} + \frac{bc - ad}{c^2 + d^2}i \)
- **The complex conjugate of** \( a + bi \) **is** \( a - bi \):
  \( (a + bi)(a - bi) = a^2 + b^2 \)
- **The additive inverse of** \( a + bi \) **is** \( -a - bi \).
- **\( \sqrt{-a} = \sqrt{a}i \) for** \( a > 0 \).
## Vocabulary Check

1. (a) iii  
   (b) i  
   (c) ii  

3. principal square

2. \(\sqrt{-1}; -1\)

4. complex conjugates

---

1. \(a + bi = -10 + 6i\)
   \(a = -10\)
   \(b = 6\)

2. \((a - 1) + (b + 3)i = 5 + 8i\)
   \(a - 1 = 5 \implies a = 6\)
   \(b + 3 = 8 \implies b = 5\)

3. \(4 + \sqrt{-9} = 4 + 3i\)

4. \(2 - \sqrt{-27} = 2 - \sqrt{27}i\)
   \(= 2 - 3\sqrt{3}i\)

5. \(\sqrt{-75} = 5\sqrt{3}i\)

6. \((5 + i) + (6 - 2i) = 11 - i\)

7. \(-6i + i^2 = -6i - 1\)
   \(= -1 - 6i\)

8. \((8 - i) - (4 - i) = 8 - i - 4 + i\)
   \(= 4\)

9. \((-2 + \sqrt{-8}) + (5 - \sqrt{-50}) = -2 + 2\sqrt{2}i + 5 - 5\sqrt{2}i\)
   \(= 3 - 3\sqrt{2}i\)

10. \((1 + i)(3 - 2i) = 3 - 2i + 3i - 2i^2\)
    \(= 3 + i + 2 = 5 + i\)

11. \((\sqrt{14} + \sqrt{10}i)(\sqrt{14} - \sqrt{10}i) = 14 - 10i^2\)
    \(= 14 + 10 = 24\)

12. \((4 + 5i)^2 = 16 + 40i + 25i^2\)
    \(= 16 + 40i - 25\)
    \(= -9 + 40i\)

13. \((2 + 3i)^2 + (2 - 3i)^2 = 4 + 12i + 9i^2 + 4 - 12i + 9i^2\)
    \(= 4 + 12i - 9 + 4 - 12i - 9\)
    \(= -10\)

14. \(\left(-1 - \sqrt{5}i\right)\left(-1 + \sqrt{5}i\right) = \left(-1\right)^2 - \left(\sqrt{5}i\right)^2\)
    \(= 1 + 5 = 6\)

15. \(\frac{5}{i} = -i\)

16. \(\frac{5}{i} \cdot -i = -5i\)

17. \(\frac{5}{i} = -5i\)

18. The complex conjugate of \(-1 - \sqrt{5}i\) is \(-1 + \sqrt{5}i\).

19. The complex conjugate of \(2 + \sqrt{5}i\) is \(-2 - \sqrt{5}i\).

20. The complex conjugate of \(\sqrt{-20} = 2\sqrt{5}i\) is \(-2\sqrt{5}i\).

---

43. The complex conjugate of \(\sqrt{8}\) is \(-\sqrt{8}\).

\(\sqrt{8}\)(\(-\sqrt{8}\)) = 8
47. \[ \frac{2}{4 - 5i} = \frac{2}{4 - 5i} \cdot \frac{4 + 5i}{4 + 5i} = \frac{2(4 + 5i)}{16 + 25} = \frac{8 + 10i}{41} = \frac{8}{41} + \frac{10}{41}i \]

51. \[ \frac{6 - 5i}{i} = \frac{6 - 5i}{i} \cdot \frac{-i}{-i} = \frac{-6i + 5i^2}{1} = -5 - 6i \]

55. \[ \frac{2}{1 + i} - \frac{3}{1 - i} = \frac{2(1 - i) - 3(1 + i)}{(1 + i)(1 - i)} = \frac{2 - 2i - 3 - 3i}{1 + 1} = \frac{-1 - 5i}{2} = \frac{-1}{2} - \frac{5}{2}i \]

59. \[ \sqrt{-6} \cdot \sqrt{-3} = (\sqrt{6})(\sqrt{3}) = \sqrt{18}i^2 = (2\sqrt{3})(-1) = -2\sqrt{3} \]

61. \[ (\sqrt{-10})^2 = (\sqrt{10}i)^2 = 10i^2 = -10 \]

63. \[ (3 + \sqrt{-5})(7 - \sqrt{-10}) = (3 + \sqrt{5}i)(7 - \sqrt{10}i) = 21 - 3\sqrt{10}i + 7\sqrt{5}i - \sqrt{50}i^2 = (21 + 7\sqrt{5}) + (7\sqrt{5} - 3\sqrt{10})i \]

65. \[ x^2 - 2x + 2 = 0; \ a = 1, \ b = -2, \ c = 2 \]
\[ x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)} = \frac{2 \pm \sqrt{-4}}{2} = \frac{2 \pm 2i}{2} = 1 \pm i \]

67. \[ 4x^2 + 16x + 17 = 0; \ a = 4, \ b = 16, \ c = 17 \]
\[ x = \frac{-16 \pm \sqrt{(16)^2 - 4(4)(17)}}{2(4)} = \frac{-16 \pm \sqrt{-16}}{8} = \frac{-16 \pm 4i}{8} = -2 \pm \frac{1}{2}i \]
69. \(4x^2 + 16x + 15 = 0; \ a = 4, \ b = 16, \ c = 15\)
\[x = \frac{-16 \pm \sqrt{(16)^2 - 4(4)(15)}}{2(4)}\]
\[= \frac{-16 \pm \sqrt{16}}{8} = \frac{-16 \pm 4}{8}\]
\[x = \frac{-12}{8} = -\frac{3}{2} \text{ or } x = \frac{-20}{8} = -\frac{5}{2}\]

73. \(1.4x^2 - 2x - 10 = 0\)
Multiply both sides by 5.
\[7x^2 - 10x - 50 = 0\]
\[x = \frac{10 \pm \sqrt{100 - 4(7)(-50)}}{2(7)}\]
\[= \frac{10 \pm \sqrt{1500}}{14} = \frac{10 \pm 10\sqrt{15}}{14}\]
\[= \frac{5 \pm 5\sqrt{15}}{7} = \frac{5}{7} \pm \frac{5\sqrt{15}}{7}\]

77. \(-5i^3 = -5i^2i\)
\[= -5(-1)(-1)i\]
\[= -5i\]

79. \((\sqrt{-75})^3 = (5\sqrt{3})^3\)
\[= 125(3\sqrt{3})(-1)i\]
\[= -375\sqrt{3}i\]

81. \(\frac{1}{i^3} = \frac{-1}{-i} = \frac{i}{-i^2} = \frac{i}{1} = i\)

83. (a) \(z_1 = 9 + 16i, z_2 = 20 - 10i\)

(b) \(\frac{1}{z} = \frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{9 + 16i} + \frac{1}{20 - 10i} = \frac{20 - 10i + 9 + 16i}{(9 + 16i)(20 - 10i)} = \frac{29 + 6i}{340 + 230i}\)
\[z = \left(\frac{340 + 230i}{29 + 6i}\right)\left(\frac{29 - 6i}{29 - 6i}\right) = \frac{11,240 + 4630i}{877} = \frac{11,240}{877} + \frac{4630}{877}i\]

85. (a) \(2^4 = 16\)

(b) \((-2)^4 = 16\)

(c) \((2i)^4 = 2^4i^4 = 16(1) = 16\)

(d) \((-2i)^4 = (-2)^4i^4 = 16(1) = 16\)

87. False, if \(b = 0\) then \(a + bi = a - bi = a\).
That is, if the complex number is real, the number equals its conjugate.

89. False
\[i^{14} + i^{50} + i^{109} + i^{61} = (i^2)^7(i^2) - (i^2)^{13}(i^2) - (i^2)^{27}(i) + (i^2)^{15}(i)\]
\[= (1)^7 + (1)^{13}(-1) - (1)^{15}(-1) - (1)^{27}(i) + (1)^{15}(i)\]
\[= 1 + (-1) + 1 - i + i = 1\]
91. \((a_1 + b_1i)(a_2 + b_2i) = a_1a_2 + a_1b_2i + a_2b_1i + b_1b_2i^2\)
   \[= (a_1a_2 - b_1b_2) + (a_1b_2 + a_2b_1)i\]

The complex conjugate of this product is \((a_1a_2 - b_1b_2) - (a_1b_2 + a_2b_1)i\).

The product of the complex conjugates is:
\[(a_1 - b_1i)(a_2 - b_2i) = a_1a_2 - a_1b_2i - a_2b_1i + b_1b_2i^2\]
\[= (a_1a_2 - b_1b_2) - (a_1b_2 + a_2b_1)i\]

Thus, the complex conjugate of the product of two complex numbers is the product of their complex conjugates.

93. \((4 + 3x) + (8 - 6x - x^2) = -x^2 - 3x + 12\)

95. \((3x - \frac{1}{2})(x + 4) = 3x^2 + 12x - \frac{1}{2}x - 2\)
   \[= 3x^2 + \frac{21}{2}x - 2\]

97. \(-x - 12 = 19\)
   \[-x = 31\]
   \[x = -31\]

99. \(4(5x - 6) - 3(6x + 1) = 0\)
   \[20x - 24 - 18x - 3 = 0\]
   \[2x - 27 = 0\]
   \[2x = 27\]
   \[x = \frac{27}{2}\]

101. \(V = \frac{4}{3}\pi a^2 b\)
   \[3V = 4\pi a^2 b\]
   \[\frac{3V}{4\pi b} = a^2\]
   \[\sqrt{\frac{3V}{4\pi b}} = a\]
   \[a = \frac{1}{2} \sqrt{\frac{3V}{\pi b}} = \frac{\sqrt{3V\pi b}}{2\pi b}\]

103. Let \(x = \#\) liters withdrawn and replaced.
   \[0.50(5 - x) + 1.00x = 0.60(5)\]
   \[2.50 - 0.50x + 1.00x = 3.00\]
   \[0.50x = 0.50\]
   \[x = 1\ \text{liter}\]

Section 1.6 Other Types of Equations

- You should be able to solve certain types of nonlinear or nonquadratic equations by rewriting them in a form in which you can factor, extract square roots, complete the square, or use the Quadratic Formula.
- For equations involving radicals or rational exponents, raise both sides to the same power.
- For equations that are of the quadratic type, \(ax^2 + bx + c = 0, \ a \neq 0\), use either factoring, the Quadratic Formula, or completing the square.
- For equations with fractions, multiply both sides by the least common denominator to clear the fractions.
- For equations involving absolute value, remember that the expression inside the absolute value can be positive or negative.
- Always check for extraneous solutions.

Vocabulary Check
1. polynomial
2. extraneous
3. quadratic type
1. \(4x^4 - 18x^2 = 0\)
\[2x^2(2x^2 - 9) = 0\]
\[2x^2 = 0 \implies x = 0\]
\[2x^2 - 9 = 0 \implies x = \pm \frac{3\sqrt{2}}{2}\]

5. \(x^3 + 216 = 0\)
\[x^3 + 6^3 = 0\]
\[(x + 6)(x^2 - 6x + 36) = 0\]
\[x + 6 = 0 \implies x = -6\]
\[x^2 - 6x + 36 = 0 \implies x = 3 \pm 3\sqrt{3}i\]

7. \(5x^3 + 30x^2 + 45x = 0\)
\[5x(x^2 + 6x + 9) = 0\]
\[5x(x + 3)^2 = 0\]
\[5x = 0 \implies x = 0\]
\[x + 3 = 0 \implies x = -3\]

9. \(x^3 - 3x^2 - x + 3 = 0\)
\[x^3(x - 3) - (x - 3) = 0\]
\[(x - 3)(x^2 - 1) = 0\]
\[(x - 3)(x + 1)(x - 1) = 0\]
\[x - 3 = 0 \implies x = 3\]
\[x + 1 = 0 \implies x = -1\]
\[x - 1 = 0 \implies x = 1\]

11. \(x^4 - x^3 + x - 1 = 0\)
\[x^3(x - 1) + (x - 1) = 0\]
\[(x - 1)(x^3 + 1) = 0\]
\[x - 1)(x + 1)(x^2 - x + 1) = 0\]
\[x - 1 = 0 \implies x = 1\]
\[x + 1 = 0 \implies x = -1\]
\[x^2 - x + 1 = 0 \implies x = \frac{1}{2} \pm \frac{\sqrt{3}}{2}i\]

13. \(x^4 - 4x^2 + 3 = 0\)
\[(x^2 - 3)(x^2 - 1) = 0\]
\[(x + \sqrt{3})(x - \sqrt{3})(x + 1)(x - 1) = 0\]
\[x + \sqrt{3} = 0 \implies x = -\sqrt{3}\]
\[x - \sqrt{3} = 0 \implies x = \sqrt{3}\]
\[x + 1 = 0 \implies x = -1\]
\[x - 1 = 0 \implies x = 1\]

15. \(4x^4 - 65x^2 + 16 = 0\)
\[(4x^2 - 1)(x^2 - 16) = 0\]
\[(2x + 1)(2x - 1)(x + 4)(x - 4) = 0\]
\[2x + 1 = 0 \implies x = -\frac{1}{2}\]
\[2x - 1 = 0 \implies x = \frac{1}{2}\]
\[x + 4 = 0 \implies x = -4\]
\[x - 4 = 0 \implies x = 4\]

3. \(x^4 - 81 = 0\)
\[(x^2 + 9)(x - 3)(x - 3) = 0\]
\[x^2 + 9 = 0 \implies x = \pm 3i\]
\[x + 3 = 0 \implies x = -3\]
\[x - 3 = 0 \implies x = 3\]
17. \[ x^6 + 7x^5 - 8 = 0 \] 
\[(x^3 + 8)(x^3 - 1) = 0 \]
\[(x + 2)(x^2 - 2x + 4)(x - 1)(x^2 + x + 1) = 0 \]
\[ x + 2 = 0 \implies x = -2 \]
\[ x^2 - 2x + 4 = 0 \implies x = 1 \pm \sqrt{3}i \]
\[ x - 1 = 0 \implies x = 1 \]
\[ x^2 + x + 1 = 0 \implies x = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i \]

19. \[ \frac{1}{3x^2} + \frac{8}{x} + 15 = 0 \]
\[ 1 + 8x + 15x^2 = 0 \]
\[(1 + 3x)(1 + 5x) = 0 \]
\[ 1 + 3x = 0 \implies x = -\frac{1}{3} \]
\[ 1 + 5x = 0 \implies x = -\frac{1}{5} \]

23. \[ 3x^{1/3} + 2x^{2/3} = 5 \]
\[ 2x^{2/3} + 3x^{1/3} - 5 = 0 \]
\[ 2(x^{1/3})^2 + 3x^{1/3} - 5 = 0 \]
\[ (2x^{1/3} + 5)(x^{1/3} - 1) = 0 \]
\[ 2x^{1/3} + 5 = 0 \implies x^{1/3} = -\frac{5}{2} \implies x = \left(-\frac{5}{2}\right)^3 = -\frac{125}{8} \]
\[ x^{1/3} - 1 = 0 \implies x^{1/3} = 1 \implies x = (1)^3 = 1 \]

25. \( y = x^3 - 2x^2 - 3x \)
(a) 
\[ \text{Graph} \]
(b) \( x \)-intercepts: \((-1, 0), (0, 0), (3, 0)\)

(c) \[ 0 = x^3 - 2x^2 - 3x \]
\[ 0 = x(x + 1)(x - 3) \]
\[ x = 0 \]
\[ x + 1 = 0 \implies x = -1 \]
\[ x - 3 = 0 \implies x = 3 \]

(d) The \( x \)-intercepts of the graph are the same as the solutions to the equation.

27. \( y = x^4 - 10x^2 + 9 \)
(a) 
\[ \text{Graph} \]
(b) \( x \)-intercepts: \((\pm 1, 0), (\pm 3, 0)\)

(c) \[ 0 = x^4 - 10x^2 + 9 \]
\[ 0 = (x^2 - 1)(x^2 - 9) \]
\[ 0 = (x + 1)(x - 1)(x + 3)(x - 3) \]
\[ x + 1 = 0 \implies x = -1 \]
\[ x - 1 = 0 \implies x = 1 \]
\[ x + 3 = 0 \implies x = -3 \]
\[ x - 3 = 0 \implies x = 3 \]

(d) The \( x \)-intercepts of the graph are the same as the solutions to the equation.
29. \(\sqrt{2x} - 10 = 0\) 
\[\sqrt{2x} = 10\]  
\[2x = 100\]  
\[x = 50\]

31. \(\sqrt{x} - 10 - 4 = 0\) 
\[\sqrt{x} - 10 = 4\]  
\[x - 10 = 16\]  
\[x = 26\]

33. \(\sqrt[3]{2x + 5} + 3 = 0\) 
\[\sqrt[3]{2x + 5} = -3\]  
\[2x + 5 = -27\]  
\[x = -16\]

35. \(-\sqrt{26 - 11x} + 4 = x\) 
\[4 - x = \sqrt{26 - 11x}\]  
\[16 - 8x + x^2 = 26 - 11x\]  
\[x^2 + 3x - 10 = 0\]  
\[(x + 5)(x - 2) = 0\]  
\[x + 5 = 0 \implies x = -5\]  
\[x - 2 = 0 \implies x = 2\]

37. \(\sqrt{x + 1} = \sqrt{3x + 1}\) 
\[x + 1 = 3x + 1\]  
\[-2x = 0\]  
\[x = 0\]

39. \(\sqrt{x} - \sqrt{x - 5} = 1\) 
\[\sqrt{x} = 1 + \sqrt{x - 5}\]  
\[(\sqrt{x})^2 = (1 + \sqrt{x - 5})^2\]  
\[x = 1 + 2\sqrt{x - 5} + x - 5\]  
\[4 = 2\sqrt{x - 5}\]  
\[2 = \sqrt{x - 5}\]  
\[4 = x - 5\]  
\[9 = x\]

41. \(\sqrt{x + 5} + \sqrt{x - 5} = 10\) 
\[\sqrt{x + 5} = 10 - \sqrt{x - 5}\]  
\[(\sqrt{x + 5})^2 = (10 - \sqrt{x - 5})^2\]  
\[x + 5 = 100 - 20\sqrt{x - 5} + x - 5\]  
\[-90 = -20\sqrt{x - 5}\]  
\[9 = 2\sqrt{x - 5}\]  
\[81 = 4(x - 5)\]  
\[81 = 4x - 20\]  
\[101 = 4x\]  
\[\frac{101}{4} = x\]

43. \(\sqrt{x + 2} - \sqrt{2x - 3} = -1\) 
\[\sqrt{x + 2} = \sqrt{2x - 3} - 1\]  
\[(\sqrt{x + 2})^2 = (\sqrt{2x - 3} - 1)^2\]  
\[x + 2 = 2x - 3 - 2\sqrt{2x - 3} + 1\]  
\[x + 2 = 2x - 2 - 2\sqrt{2x - 3}\]  
\[-x + 4 = -2\sqrt{2x - 3}\]  
\[x - 4 = 2\sqrt{2x - 3}\]  
\[(x - 4)^2 = (2\sqrt{2x - 3})^2\]  
\[x^2 - 8x + 16 = 4(2x - 3)\]  
\[x^2 - 8x + 16 = 8x - 12\]  
\[x^2 - 16x + 28 = 0\]  
\[(x - 2)(x - 14) = 0\]  
\[x - 2 = 0 \implies x = 2, \text{ extraneous}\]  
\[x - 14 = 0 \implies x = 14\]

45. \((x - 5)^{3/2} = 8\) 
\[(x - 5)^3 = 8^2\]  
\[x - 5 = \sqrt[3]{64}\]  
\[x = 5 + 4 = 9\]
47. \((x + 3)^{2/3} = 8\)
\((x + 3)^2 = 8^3\)
\(x + 3 = \pm \sqrt[3]{8^3}\)
\(x + 3 = \pm \sqrt[3]{512}\)
\(x = -3 \pm 16\sqrt{2}\)

51. \(3x(x - 1)^{1/2} + 2(x - 1)^{1/2} = 0\)
\((x - 1)^{1/2}[3x + 2(x - 1)] = 0\)
\((x - 1)^{1/2}(5x - 2) = 0\)
\((x - 1)^{1/2} = 0 \Rightarrow x - 1 = 0 \Rightarrow x = 1\)
\(5x - 2 = 0 \Rightarrow x = \frac{2}{5}, \text{extraneous}\)

53. \(y = \sqrt{11x - 30} - x\)
(a)
(b) \(x\)-intercepts: \((5, 0), (6, 0)\)
(c) \(0 = \sqrt{11x - 30} - x\)
\(x = \sqrt{11x - 30}\)
\(x^2 = 11x - 30\)
\(x^2 - 11x + 30 = 0\)
\((x - 5)(x - 6) = 0\)
\(x - 5 = 0 \Rightarrow x = 5\)
\(x - 6 = 0 \Rightarrow x = 6\)
(d) The \(x\)-intercepts of the graph are the same as the solutions to the equation.

49. \((x^2 - 5)^{1/2} = 27\)
\((x^2 - 5)^1 = 27^2\)
\((x^2 - 5) = \sqrt{27^2}\)
\(x^2 = 5 + 9\)
\(x^2 = 14\)
\(x = \pm \sqrt{14}\)

55. \(y = \sqrt{7x + 36} - \sqrt{5x + 16} - 2\)
(a)
(b) \(x\)-intercepts: \((0, 0), (4, 0)\)
(c) \(0 = \sqrt{7x + 36} - \sqrt{5x + 16} - 2\)
\(-\sqrt{7x + 36} = -\sqrt{5x + 16} - 2\)
\(\sqrt{7x + 36} = 2 + \sqrt{5x + 16}\)
\((\sqrt{7x + 36})^2 = (2 + \sqrt{5x + 16})^2\)
\(7x + 36 = 4 + 4\sqrt{5x + 16} + 5x + 16\)
\(7x + 36 = 5x + 20 + 4\sqrt{5x + 16}\)
\(2x + 16 = 4\sqrt{5x + 16}\)
\(x + 8 = 2\sqrt{5x + 16}\)
\((x + 8)^2 = (2\sqrt{5x + 16})^2\)
\(x^2 + 16x + 64 = 4(5x + 16)\)
\(x^2 + 16x + 64 = 20x + 64\)
\(x^2 - 4x = 0\)
\(x(x - 4) = 0\)
\(x = 0\)
\(x - 4 = 0 \Rightarrow x = 4\)
(d) The \(x\)-intercepts of the graph are the same as the solutions to the equation.
57. 
\[ x = \frac{3}{x} + \frac{1}{2} \]

\[ (2x)(x) = (2x)\left(\frac{3}{x}\right) + (2x)\left(\frac{1}{2}\right) \]

\[ 2x^2 = 6 + x \]

\[ 2x^2 - x - 6 = 0 \]

\[ (2x + 3)(x - 2) = 0 \]

\[ 2x + 3 = 0 \Rightarrow x = \frac{-3}{2} \]

\[ x - 2 = 0 \Rightarrow x = 2 \]

58. 
\[ \frac{1}{x} - \frac{1}{x + 1} = 3 \]

\[ x(x + 1)\frac{1}{x} - x(x + 1)\frac{1}{x + 1} = x(x + 1)(3) \]

\[ x + 1 - x = 3x(x + 1) \]

\[ 0 = 3x^2 + 3x \]

\[ a = 3, \ b = 3, \ c = -1 \]

\[ x = \frac{-3 \pm \sqrt{(3)^2 - 4(3)(-1)}}{2(3)} = \frac{-3 \pm \sqrt{21}}{6} \]

59. 
\[ \frac{x}{x^2 - 4} + \frac{1}{x + 2} = 3 \]

\[ (x + 2)(x - 2)\frac{x}{x^2 - 4} + (x + 2)(x - 2)\frac{1}{x + 2} = 3(x + 2)(x - 2) \]

\[ x + x - 2 = 3x^2 - 12 \]

\[ 3x^2 - 2x - 10 = 0 \]

\[ a = 3, \ b = -2, \ c = -10 \]

\[ x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(-10)}}{2(3)} \]

\[ = \frac{2 \pm \sqrt{124}}{6} = \frac{2 \pm 2\sqrt{31}}{6} = \frac{1 \pm \sqrt{31}}{3} \]

60. 
\[ |2x - 1| = 5 \]

\[ 2x - 1 = 5 \Rightarrow x = 3 \]

\[ -(2x - 1) = 5 \Rightarrow x = -2 \]

61. 
\[ \frac{20 - x}{x} = x \]

\[ 20 - x = x^2 \]

\[ 0 = x^2 + x - 20 \]

\[ 0 = (x + 5)(x - 4) \]

\[ x + 5 = 0 \Rightarrow x = -5 \]

\[ x - 4 = 0 \Rightarrow x = 4 \]

62. 
\[ |x| = x^2 + x - 3 \]

First equation:

\[ x = x^2 + x - 3 \]

\[ x^2 - 3 = 0 \]

\[ x = \pm \sqrt{3} \]

Second equation:

\[ -x = x^2 + x - 3 \]

\[ x^2 + 2x - 3 = 0 \]

\[ (x - 1)(x + 3) = 0 \]

\[ x - 1 = 0 \Rightarrow x = 1 \]

\[ x + 3 = 0 \Rightarrow x = -3 \]

Only \( x = \sqrt{3} \) and \( x = -3 \) are solutions to the original equation. \( x = -\sqrt{3} \) and \( x = 1 \) are extraneous.
69. \( |x + 1| = x^2 - 5 \)

**First equation:**

\[
x + 1 = x^2 - 5 \\
x^2 - x - 6 = 0 \\
(x - 3)(x + 2) = 0
\]

\[
x - 3 = 0 \Rightarrow x = 3 \\
x + 2 = 0 \Rightarrow x = -2
\]

Only \( x = 3 \) and \( x = \frac{-1 - \sqrt{17}}{2} \) are solutions to the original equation. \( x = -2 \) and \( x = \frac{-1 + \sqrt{17}}{2} \) are extraneous.

**Second equation:**

\[
-(x + 1) = x^2 - 5 \\
x - 1 = x^2 - 5 \\
x^2 + x - 4 = 0
\]

\[
x = \frac{-1 + \sqrt{17}}{2}
\]

71. \( y = \frac{1}{x} - \frac{4}{x - 1} - 1 \)

(a) 

(b) \( x \)-intercept: \((-1, 0)\)

(c) 

\[
0 = \frac{1}{x} - \frac{4}{x - 1} - 1
\]

\[
0 = (x - 1) - 4x - x(x - 1)
\]

\[
0 = x - 1 - 4x - x^2 + x
\]

\[
0 = -x^2 - 2x - 1
\]

\[
0 = x^2 + 2x + 1
\]

\[
0 = (x + 1)^2
\]

\[
x + 1 = 0 \Rightarrow x = -1
\]

(d) The \( x \)-intercept of the graph is the same as the solution to the equation.

73. \( y = |x + 1| - 2 \)

(a) 

(b) \( x \)-intercepts: \((1, 0), (-3, 0)\)

(c) 

\[
0 = |x + 1| - 2
\]

\[
x + 1 = 2 \quad \text{OR} \quad -(x + 1) = 2
\]

\[
x = 1 \quad \text{OR} \quad x = 1
\]

\[
-x = 2 \quad \text{OR} \quad x = 2
\]

\[
x = 3 \quad \text{OR} \quad x = 3
\]

(d) The \( x \)-intercepts of the graph are the same as the solutions to the equation.

75. \( 3.2x^4 - 1.5x^2 - 2.1 = 0 \)

\[
x^2 = \frac{1.5 \pm \sqrt{1.5^2 - 4(3.2)(-2.1)}}{2(3.2)}
\]

Using the positive value for \( x^2 \), we have:

\[
x = \pm \sqrt{\frac{1.5 + \sqrt{29.13}}{6.4}} = \pm 1.038.
\]

77. \( 1.8x - 6\sqrt{x} - 5.6 = 0 \) \hspace{1cm} 79. \(-2 \text{ and } 5\)

Given equation \( 1.8(\sqrt{x})^2 - 6\sqrt{x} - 5.6 = 0 \)

Use the Quadratic Formula with \( a = 1.8, b = -6, \text{ and } c = -5.6. \)

\[
\sqrt{x} = \frac{6 \pm \sqrt{36 - 4(1.8)(-5.6)}}{2(1.8)} = \frac{6 \pm 8.7361}{3.6}
\]

Considering only the positive value for \( \sqrt{x} \), we have:

\[
\sqrt{x} \approx 4.0934
\]

\[
x \approx 16.756.
\]

One possible equation is:

\[
(x - 2)(x - 5) = 0
\]

\[
x + 2)(x - 5) = 0
\]

\[
x^2 - 3x - 10 = 0
\]

Any non-zero multiple of this equation would also have these solutions.
81. \( \frac{-7}{3} \) and \( \frac{6}{7} \)

One possible equation is:
\[
x = \frac{-7}{3} \implies 3x = -7 \implies 3x + 7 = \text{a factor.}
\]
\[
x = \frac{6}{7} \implies 7x = 6 \implies 7x - 6 = \text{a factor.}
\]
(3x + 7)(7x - 6) = 0
21x^2 + 31x - 42 = 0

Any non-zero multiple of this equation would also have these solutions.

83. \( \sqrt{3} \), \( -\sqrt{3} \), and 4

One possible equation is:
\[
(x - \sqrt{3})(x + \sqrt{3})(x - 4) = 0
\]
\[
(x^2 - 3)(x - 4) = 0
\]
\[
x^3 - 4x^2 - 3x + 12 = 0
\]
Any non-zero multiple of this equation would also have these solutions.

85. \(-1, 1, i, \text{ and } -i\)

One possible equation is:
\[
(x - (-1))(x - 1)(x - i)(x - (-i)) = 0
\]
\[
(x + 1)(x - 1)(x - i)(x + i) = 0
\]
\[
(x^2 - 1)(x^2 + 1) = 0
\]
\[
x^4 - 1 = 0
\]
Any non-zero multiple of this equation would also have these solutions.

87. Let \( x \) = the number of students in the original group. Then, \( \frac{1700}{x} \) = the original cost per student.

When six more students join the group, the cost per student becomes \( \frac{1700}{x - 7.5} \).

Model: \((\text{Cost per student}) \cdot (\text{Number of students}) = \text{Total cost}\)
\[
\left( \frac{1700}{x} - 7.5 \right)(x + 6) = 1700
\]
\[
(3400 - 15x)(x + 6) = 3400x
\]
Multiply both sides by 2x to clear fractions.
\[
-15x^2 - 90x + 20,400 = 0
\]
\[
x = \frac{90 \pm \sqrt{(-90)^2 - 4(-15)(20,400)}}{2(-15)} = \frac{90 \pm 1110}{-30}
\]
Using the positive value for \( x \) we conclude that the original number was \( x = 34 \) students.

89. Model: \( \text{Time} = \frac{\text{Distance}}{\text{Rate}} \)

Labels: Let \( x = \) average speed of the plane. Then we have a travel time of \( t = \frac{145}{x} \). If the average speed is increased by 40 mph, then
\[
t - \frac{12}{60} = \frac{145}{x + 40}
\]
\[
t = \frac{145}{x + 40} + \frac{1}{5}
\]
Now, we equate these two equations and solve for \( x \).

Equation:
\[
\frac{145}{x} = \frac{145}{x + 40} + \frac{1}{5}
\]
\[
145(5)(x + 40) = 145(5)x + x(x + 40)
\]
\[
725x + 29,000 = 725x + x^2 + 40x
\]
\[
0 = x^2 + 40x - 29,000
\]
Using the positive value for \( x \) found by the Quadratic Formula, we have \( x = 151.5 \) mph and \( x + 40 = 191.5 \) mph. The airspeed required to obtain the decrease in travel time is 191.5 miles per hour.
91. \[ A = P \left( 1 + \frac{r}{n} \right)^n \]

\[
3052.49 = 2500 \left( 1 + \frac{0.04}{12} \right)^{12(5)}
\]

\[
1.220996 = \left( 1 + \frac{r}{12} \right)^{60}
\]

\[
(1.220996)^{1/60} = 1 + \frac{r}{12}
\]

\[
[(1.220996)^{1/60} - 1](12) = r
\]

\[ r \approx 0.04 = 4\%
\]

93. \( M = 463.97 + 111.6\sqrt{t}, 4 \leq t \leq 12 \)

(a) \( 463.97 + 111.6\sqrt{t} = 816 \)

\[ t = \left( \frac{352.03}{111.6} \right)^2 \]

\[ t \approx 9.95 \]

The number of medical doctors reached 816,000 late during the year 1999.

(b) \( 463.97 + 111.6\sqrt{t} = 900 \)

\[ t = \left( \frac{436.03}{111.6} \right)^2 \]

\[ t \approx 15.27 \]

The model predicts the number of medical doctors will reach 900,000 during the year 2005.

The actual number of medical doctors in 2005 is about 800,000.

95. \( T = 75.82 - 2.11x + 43.51\sqrt{x}, 5 \leq x \leq 40 \)

<table>
<thead>
<tr>
<th>Absolute Pressure, ( x )</th>
<th>Temperature ( T )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>162.56</td>
</tr>
<tr>
<td>10</td>
<td>192.31</td>
</tr>
<tr>
<td>15</td>
<td>212.68</td>
</tr>
<tr>
<td>20</td>
<td>228.20</td>
</tr>
<tr>
<td>25</td>
<td>240.62</td>
</tr>
<tr>
<td>30</td>
<td>250.83</td>
</tr>
<tr>
<td>35</td>
<td>259.38</td>
</tr>
<tr>
<td>40</td>
<td>266.60</td>
</tr>
</tbody>
</table>

(b) \( T = 212^\circ \) when \( x = 15 \) pounds per square inch.

97. \( 37.55 = 40 - \sqrt{0.01x + 1} \)

\[ \sqrt{0.01x + 1} = 2.45 \]

\[ 0.01x + 1 = 6.0025 \]

\[ 0.01x = 5.0025 \]

\[ x = 500.25 \]

Rounding \( x \) to the nearest whole unit yields \( x \approx 500 \) units.
99. **Model:** 
\[
\text{(Distance from home to 1st)}^2 + \text{(distance from 1st to 2nd)}^2 = \text{(distance from home to 2nd)}^2
\]

**Labels:** Distance from home to 1st = x, distance from 1st to 2nd = x, distance from home to 2nd = 127.5

**Equation:** 
\[
x^2 + x^2 = (127.5)^2
\]
\[
2x^2 = 16,256.25
\]
\[
x^2 = \frac{16,256.25}{2}
\]
\[
x = \pm\sqrt{8128.125} \approx \pm 90
\]

The distance between bases is approximately 90 feet.

101. 
\[
S = 8\pi\sqrt{64 + h^2}
\]

(a) 
\[
\begin{array}{c|cccccc}
 h & 8 & 9 & 10 & 11 & 12 & 13 \\
 S & 284.3 & 302.6 & 321.9 & 341.8 & 362.5 & 383.6 \\
\end{array}
\]

When \(S = 350\), \(h = 11.4\).

(b) 
\[
\begin{array}{c|cccccc}
 h & 8 & 9 & 10 & 11 & 12 & 13 \\
 S & 284.3 & 302.6 & 321.9 & 341.8 & 362.5 & 383.6 \\
\end{array}
\]

\(S = 350\) when \(h\) is between 11 and 12 inches.

103. **Model:** 
\[
\left(\text{Portion done by first person}\right) + \left(\text{portion done by second person}\right) = \left(\text{work done}\right)
\]

**Labels:** Work done = 1, rate of first person = \(\frac{1}{r}\), time worked by first person = 12,

rate of second person = \(\frac{1}{r + 3}\), time worked by second person = 12

**Equation:** 
\[
\frac{12}{r} + \frac{12}{r + 3} = 1
\]
\[
r(r + 3)\frac{12}{r} + r(r + 3)\frac{12}{r + 3} = r(r + 3)
\]
\[
12r + 36 + 12r = r^2 + 3r
\]
\[
0 = r^2 - 21r - 36
\]
\[
r = \frac{21 \pm \sqrt{(21)^2 - 4(1)(-36)}}{2(1)}
\]
\[
r \approx 23
\]
(Choose the positive value for \(r\).)

It would take approximately 23 hours and 26 hours individually.

105. 
\[
v = \sqrt{\frac{gR}{\mu_s}}
\]
\[
v^2 = \frac{gR}{\mu_s}
\]
\[
v^2\mu_s = gR
\]
\[
\frac{v^2\mu_s}{R} = g
\]
109. The distance between (1, 2) and \((x, -10)\) is 13.
\[
\sqrt{(x - 1)^2 + (-10 - 2)^2} = 13
\]
\[
(x - 1)^2 + (-12)^2 = 13^2
\]
\[
x^2 - 2x + 1 + 144 = 169
\]
\[
x^2 - 2x - 24 = 0
\]
\[
(x + 4)(x - 6) = 0
\]
\[
x + 4 = 0 \implies x = -4
\]
\[
x - 6 = 0 \implies x = 6
\]
Both \((-4, -10)\) and \((6, -10)\) are a distance of 13 from \((1, 2)\).

111. The distance between \((0, 0)\) and \((8, y)\) is 17.
\[
\sqrt{(8 - 0)^2 + (y - 0)^2} = 17
\]
\[
(8)^2 + (y)^2 = 17^2
\]
\[
64 + y^2 = 289
\]
\[
y^2 = 225
\]
\[
y = \pm \sqrt{225} = \pm 15
\]
Both \((8, 15)\) and \((8, -15)\) are a distance of 17 from \((0, 0)\).

113. \(9 + |9 - a| = b\)
\[
|9 - a| = b - 9
\]
\[
9 - a = b - 9 \quad \text{OR} \quad 9 - a = -(b - 9)
\]
\[
-a = b - 18 \quad 9 - a = -b + 9
\]
\[
a = 18 - b \quad -a = -b
\]
\[
a = b
\]
Thus, \(a = 18 - b\) or \(a = b\). From the original equation we know that \(b \geq 9\).
Some possibilities are: \(b = 9, a = 9\)
\(b = 10, a = 8 \text{ or } a = 10\)
\(b = 11, a = 7 \text{ or } a = 11\)
\(b = 12, a = 6 \text{ or } a = 12\)
\(b = 13, a = 5 \text{ or } a = 13\)
\(b = 14, a = 4 \text{ or } a = 14\)

115. \(20 + \sqrt{20 - a} = b\)
\[
\sqrt{20 - a} = b - 20
\]
\[
20 - a = b^2 - 40b + 400
\]
\[
-a = b^2 - 40b + 380
\]
\[
a = -b^2 + 40b - 380
\]
This formula gives the relationship between \(a\) and \(b\). From the original equation we know that \(a \leq 20\) and \(b \geq 20\). Choose a \(b\) value, where \(b \geq 20\) and then solve for \(a\), keeping in mind that \(a \leq 20\).
Some possibilities are: \(b = 20, a = 20\)
\(b = 21, a = 19\)
\(b = 22, a = 16\)
\(b = 23, a = 11\)
\(b = 24, a = 4\)
\(b = 25, a = -5\)

117. \[
\frac{8}{3x} + \frac{3}{2x} = \frac{16}{6x} + \frac{9}{6x} = \frac{25}{6x}
\]

119. \[
\frac{2}{z + 2} - \left(3 - \frac{2}{z}\right) = \frac{2}{z + 2} - 3 + \frac{2}{z}
\]
\[
= \frac{2z - 3(z + 2)}{z(z + 2)} + \frac{2(z + 2)}{z(z + 2)}
\]
\[
= \frac{2z - 3z^2 - 6z + 2z + 4}{z(z + 2)}
\]
\[
= \frac{-3z^2 - 2z + 4}{z(z + 2)}
\]

121. \(x^2 - 22x + 121 = 0\)
\[
(x - 11)^2 = 0
\]
\[
x - 11 = 0
\]
\[
x = 11
\]
## Section 1.7  Linear Inequalities in One Variable

- You should know the properties of inequalities.
  (a) Transitive: $a < b$ and $b < c$ implies $a < c$.
  (b) Addition: $a < b$ and $c < d$ implies $a + c < b + d$.
  (c) Adding or Subtracting a Constant: $a \pm c < b \pm c$ if $a < b$.
  (d) Multiplying or Dividing a Constant: For $a < b$,
    1. If $c > 0$, then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$.
    2. If $c < 0$, then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$.

- You should be able to solve absolute value inequalities.
  (a) $|x| < a$ if and only if $-a < x < a$.
  (b) $|x| > a$ if and only if $x < -a$ or $x > a$.

### Vocabulary Check

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>1.</td>
<td>solution set</td>
<td>2.</td>
</tr>
<tr>
<td>4.</td>
<td>equivalent</td>
<td>5.</td>
</tr>
</tbody>
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<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>1. Interval: $[-1, 5]$</td>
<td>3. Interval: $(11, \infty)$</td>
<td>5. Interval: $(-\infty, -2)$</td>
</tr>
<tr>
<td>(a) Inequality: $-1 \leq x \leq 5$</td>
<td>(a) Inequality: $x &gt; 11$</td>
<td>(a) Inequality: $x &lt; -2$</td>
</tr>
<tr>
<td>(b) The interval is bounded.</td>
<td>(b) The interval is unbounded.</td>
<td>(b) The interval is unbounded.</td>
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<tbody>
<tr>
<td>7. $x &lt; 3$</td>
<td>9. $-3 &lt; x \leq 4$</td>
</tr>
<tr>
<td>Matches (b).</td>
<td>Matches (d).</td>
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<tbody>
<tr>
<td>13. $5x - 12 &gt; 0$</td>
<td>15. $0 &lt; \frac{x - 2}{4} &lt; 2$</td>
<td></td>
</tr>
<tr>
<td>(a) $x = 3$</td>
<td>(a) $x = 4$</td>
<td>(d) $x = \frac{7}{2}$</td>
</tr>
<tr>
<td>$5(3) - 12 &gt; 0$</td>
<td>$\frac{3}{4} - 2 &gt; 0$</td>
<td>$0 \leq \frac{7 - 2}{4} &lt; 2$</td>
</tr>
<tr>
<td>$3 &gt; 0$</td>
<td>$0 \leq 0 - 2 &lt; 2$</td>
<td>$0 \leq \frac{3}{8} &lt; 2$</td>
</tr>
<tr>
<td>Yes, $x = 3$ is a solution.</td>
<td>Yes, $x = 4$ is a solution.</td>
<td>Yes, $x = \frac{7}{2}$ is a solution.</td>
</tr>
</tbody>
</table>

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<thead>
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<tbody>
<tr>
<td>(b) $x = -3$</td>
<td>(b) $x = 10$</td>
<td>(c) $x = 0$</td>
</tr>
<tr>
<td>$5(-3) - 12 &gt; 0$</td>
<td>$\frac{10 - 2}{4} &lt; 2$</td>
<td>$0 \leq \frac{0 - 2}{4} &lt; 2$</td>
</tr>
<tr>
<td>$-27 &gt; 0$</td>
<td>$0 \leq \frac{1}{2} &lt; 2$</td>
<td>$0 \leq \frac{0}{4} &lt; 2$</td>
</tr>
<tr>
<td>No, $x = -3$ is not a solution.</td>
<td>Yes, $x = \frac{5}{2}$ is a solution.</td>
<td>No, $x = \frac{7}{2}$ is not a solution.</td>
</tr>
</tbody>
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<tbody>
<tr>
<td>(c) $x = \frac{5}{2}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$5\left(\frac{5}{2}\right) - 12 &gt; 0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{5}{2} &gt; 0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yes, $x = \frac{5}{2}$ is a solution.</td>
<td></td>
<td></td>
</tr>
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<tbody>
<tr>
<td>(d) $x = \frac{3}{2}$</td>
<td></td>
</tr>
<tr>
<td>$5\left(\frac{3}{2}\right) - 12 &gt; 0$</td>
<td></td>
</tr>
<tr>
<td>$\frac{15}{2} &gt; 0$</td>
<td></td>
</tr>
<tr>
<td>No, $x = \frac{3}{2}$ is not a solution.</td>
<td></td>
</tr>
</tbody>
</table>
17. $|x - 10| \geq 3$
   (a) $x = 13$
   $|13 - 10| \geq 3$
   $3 \geq 3$
   Yes, $x = 13$ is a solution.
   (b) $x = -1$
   $|-1 - 10| \geq 3$
   $11 \geq 3$
   Yes, $x = -1$ is a solution.
   (c) $x = 14$
   $|14 - 10| \geq 3$
   $4 \geq 3$
   Yes, $x = 14$ is a solution.
   (d) $x = 9$
   $|9 - 10| \geq 3$
   $1 \not\geq 3$
   No, $x = 9$ is not a solution.

19. $4x < 12$
   $\frac{1}{2}(4x) < \frac{1}{2}(12)$
   $x < 3$

21. $-2x > -3$
   $-\frac{1}{2}(-2x) < (-\frac{1}{2})(-3)$
   $x < \frac{3}{2}$

23. $x - 5 \geq 7$
   $x \geq 12$

25. $2x + 7 < 3 + 4x$
   $-2x < -4$
   $x > 2$

27. $2x - 1 \geq 1 - 5x$
   $7x \geq 2$
   $x \geq \frac{2}{7}$

29. $4 - 2x < 3(3 - x)$
   $4 - 2x < 9 - 3x$
   $x < 5$

31. $\frac{5}{3}x - 6 \leq x - 7$
   $-\frac{2}{3}x \leq -1$
   $x \geq 4$

33. $\frac{1}{2}(8x + 1) \geq 3x + \frac{5}{2}$
   $4x + \frac{1}{2} \geq 3x + \frac{5}{2}$
   $x \geq 2$

35. $3.6x + 11 \geq -3.4$
   $3.6x \geq -14.4$
   $x \geq -4$

37. $1 < 2x + 3 < 9$
   $-2 < 2x < 6$
   $-1 < x < 3$

39. $-4 < \frac{2x - 3}{3} < 4$
   $-12 < 2x - 3 < 12$
   $-9 < 2x < 15$
   $-\frac{9}{2} < x < \frac{15}{2}$

41. $\frac{3}{4} > x + 1 > \frac{1}{4}$
   $-\frac{1}{4} > x > -\frac{3}{4}$
   $-\frac{3}{4} < x < -\frac{1}{4}$

43. $3.2 \leq 0.4x - 1 \leq 4.4$
   $4.2 \leq 0.4x \leq 5.4$
   $10.5 \leq x \leq 13.5$

45. $|x| < 6$
   $-6 < x < 6$

47. $\left|\frac{x}{2}\right| > 1$
   $\frac{x}{2} < -1$ or $\frac{x}{2} > 1$
   $x < -2$ or $x > 2$
49. \(|x - 5| < -1\)
   No solution. The absolute value of a number cannot be less than a negative number.

51. \(|x - 20| \leq 6\)
   \(-6 \leq x - 20 \leq 6\)
   \(14 \leq x \leq 26\)

53. \(|3 - 4x| \geq 9\)
   \(3 - 4x \leq -9\) or \(3 - 4x \geq 9\)
   \(-4x \leq -12\) or \(-4x \geq 6\)
   \(x \geq 3\) or \(x \leq -\frac{3}{2}\)

55. \(\frac{|x - 3|}{2} \geq 4\)
   \(\frac{x - 3}{2} \leq -4\) or \(\frac{x - 3}{2} \geq 4\)
   \(x - 3 \leq -8\) or \(x - 3 \geq 8\)
   \(x \leq -5\) or \(x \geq 11\)

57. \(|9 - 2x| - 2 < -1\)
   \(|9 - 2x| < 1\)
   \(-1 < 9 - 2x < 1\)
   \(-10 < -2x < 8\)
   \(5 > x > 4\)
   \(4 < x < 5\)

59. \(2|x + 10| \geq 9\)
   \(|x + 10| \geq \frac{9}{2}\)
   \(x + 10 \leq -\frac{9}{2}\) or \(x + 10 \geq \frac{9}{2}\)
   \(x \leq -\frac{29}{2}\) or \(x \geq -\frac{11}{2}\)

61. \(6x > 12\)
   \(x > 2\)

63. \(5 - 2x \geq 1\)
   \(-2x \geq -4\)
   \(x \leq 2\)

65. \(|x - 8| \leq 14\)
   \(-14 \leq x - 8 \leq 14\)
   \(-6 \leq x \leq 22\)

67. \(2|x + 7| \geq 13\)
   \(|x + 7| \geq \frac{13}{2}\)
   \(x + 7 \leq -\frac{13}{2}\) or \(x + 7 \geq \frac{13}{2}\)
   \(x \leq -\frac{27}{2}\) or \(x \geq -\frac{1}{2}\)

69. \(y = 2x - 3\)
   (a) \(y \geq 1\)
   \(2x - 3 \geq 1\)
   \(2x \geq 4\)
   \(x \geq 2\)

   (b) \(y \leq 0\)
   \(2x - 3 \leq 0\)
   \(2x \leq 3\)
   \(x \leq \frac{3}{2}\)
71. \( y = \frac{-1}{2}x + 2 \)
   (a) \( 0 \leq y \leq 3 \)
   \( 0 \leq -\frac{1}{2}x + 2 \leq 3 \)
   \( -2 \leq -\frac{1}{2}x \leq 1 \)
   \( 4 \geq x \geq -2 \)
   (b) \( y \geq 0 \)
   \( -\frac{1}{2}x + 2 \geq 0 \)
   \( -\frac{1}{2}x \geq -2 \)
   \( x \leq 4 \)

73. \( y = |x - 3| \)
   (a) \( y \leq 2 \)
   \( |x - 3| \leq 2 \)
   \( -2 \leq x - 3 \leq 2 \)
   \( 1 \leq x \leq 5 \)
   (b) \( y \geq 4 \)
   \( |x - 3| \geq 4 \)
   \( x - 3 \leq -4 \) or \( x - 3 \geq 4 \)
   \( x \leq -1 \) or \( x \geq 7 \)

75. \( x - 5 \geq 0 \)
77. \( x + 3 \geq 0 \)
   \( x \geq 5 \)
   \( x \geq -3 \)
   \( [5, \infty) \)
   \( [-3, \infty) \)

79. \( 7 - 2x \geq 0 \)
   \( -2x \geq -7 \)
   \( x \leq \frac{7}{2} \)
   \( x \leq \frac{7}{2} \)

81. \( |x - 10| < 8 \)
   All real numbers within 8 units of 10.

83. The midpoint of the interval \([-3, 3]\) is 0. The interval represents all real numbers \(x\) no more than 3 units from 0.
   \( |x - 0| \leq 3 \)
   \( |x| \leq 3 \)

85. The graph shows all real numbers at least 3 units from 7.
   \( |x - 7| \geq 3 \)

87. All real numbers within 10 units of 12
   \( |x - 12| < 10 \)

89. All real numbers more than 4 units from -3
   \( |x - (-3)| > 4 \)
   \( |x + 3| > 4 \)

91. Let \( x = \) the number of checks written in a month.
Type A account charges: \( 6.00 + 0.25x \)
Type B account charges: \( 4.50 + 0.50x \)
\( 6.00 + 0.25x < 4.50 + 0.50x \)
\( 1.50 < 0.25x \)
\( 6 < x \)
If you write more than six checks a month, then the charges for the type A account are less than the charges for the type B account.

93. \( 1000(1 + r(2)) > 1062.50 \)
   \( 1 + 2r > 1.0625 \)
   \( 2r > 0.0625 \)
   \( r > 0.03125 \)
   \( r > 3.125\% \)

95. \( R > C \)
   \( 115.95x > 95x + 750 \)
   \( 20.95x > 750 \)
   \( x > 35.7995 \)
   \( x \geq 36 \) units
97. Let $x =$ daily sales level (in dozens) of doughnuts.

Revenue: $R = 2.95x$

Cost: $C = 150 + 1.45x$

Profit: $P = R - C$

$= 2.95x - (150 + 1.45x)$

$= 1.50x - 150$

$50 \leq P \leq 200$

$50 \leq 1.50x - 150 \leq 200$

$200 \leq 1.50x \leq 350$

$133\frac{1}{3} \leq x \leq 233\frac{1}{3}$

In whole dozens, $134 \leq x \leq 234$.

99. (a) $y = 0.067x - 5.638$

(b) From the graph we see that $y \geq 3$ when $x \geq 129$.

Algebraically we have:

$3 \leq 0.067x - 5.638$

$8.638 \leq 0.067x$

$x \geq 129$

IQ scores are not a good predictor of GPAs. Other factors include study habits, class attendance, and attitude.

101. $S = 1.05t + 31.0$, $0 \leq t \leq 12$

(a) $32 \leq 1.05t + 31 \leq 42$

$1 \leq 1.05t \leq 11$

$0.95 \leq t \leq 10.48$

Rounding to the nearest year, the average salary was at least $32,000 but not more than $42,000 between 1991 and 2000.

(b) $1.05t + 31 > 48$

$1.05t > 17$

$t > 16$

According to the model, the average salary will exceed $48,000 in 2006.

103. $|x - 10.4| \leq \frac{1}{10}$

$-\frac{1}{10} \leq x - 10.4 \leq \frac{1}{10}$

$-0.0625 \leq s - 10.4 \leq 0.0625$

$10.3375 \leq s \leq 10.4625$

Since $A = s^2$, we have

$(10.3375)^2 \leq area \leq (10.4625)^2$

$106.864 \leq area \leq 109.464$.

105. $|x - 15| \leq \frac{1}{10}$

$-\frac{1}{10} \leq x - 15 \leq \frac{1}{10}$

$14.9 \leq x \leq 15.1$ gallons

$\frac{1}{10}(1.89) = \$0.19$

You might have been undercharged or overcharged by $0.19.

109. $|h - 50| \leq 30$

$-30 \leq h - 50 \leq 30$

$20 \leq h \leq 80$

The minimum relative humidity is 20 and the maximum is 80.

111. False. If $c$ is negative, then $ac \geq bc$.

113. $|x - a| \geq 2$

$x - a \leq -2$

$x \leq a - 2$ or

$x - a \geq 2$

$x \geq a + 2$

Matches (b).
115. \((-4, 2)\) and \((1, 12)\)

\[
d = \sqrt{(1 - (-4))^2 + (12 - 2)^2} = \sqrt{5^2 + 10^2} = \sqrt{125} = 5\sqrt{5}
\]

Midpoint: \(\left(\frac{-4 + 1}{2}, \frac{2 + 12}{2}\right) = \left(-\frac{3}{2}, 7\right)\)

117. \((3, 6)\) and \((-5, -8)\)

\[
d = \sqrt{(-5 - 3)^2 + (-8 - 6)^2} = \sqrt{(-8)^2 + (-14)^2} = \sqrt{260} = 2\sqrt{65}
\]

Midpoint: \(\left(\frac{3 + (-5)}{2}, \frac{6 + (-8)}{2}\right) = (-1, -1)\)

119. 

\(-6(2 - x) - 12 = 36\)

\(-12 + 6x - 12 = 36\)

\(6x - 24 = 36\)

\(6x = 60\)

\(x = 10\)

121. \(14x^2 + 5x - 1 = 0\)

\((7x - 1)(2x + 1) = 0\)

\(7x - 1 = 0\) or \(2x + 1 = 0\)

\(7x = 1\)

\(2x = -1\)

\(x = \frac{1}{7}\)

\(x = -\frac{1}{2}\)

123. \((-3, 10)\)

125. Answers will vary.

Section 1.8 Other Types of Inequalities

- You should be able to solve inequalities.
  
  (a) Find the critical number.
    
    1. Values that make the expression zero
    2. Values that make the expression undefined
  
  (b) Test one value in each test interval on the real number line resulting from the critical numbers.
  
  (c) Determine the solution intervals.

Vocabulary Check

1. critical; test intervals
2. zeroes; undefined values
3. \(P = R - C\)

1. \(x^2 - 3 < 0\)

(a) \(x = 3\)

\((3)^2 - 3 < 0\)

\(6 < 0\)

No, \(x = 3\) is not a solution.

(b) \(x = 0\)

\((0)^2 - 3 < 0\)

\(-3 < 0\)

Yes, \(x = 0\) is a solution.

(c) \(x = \frac{3}{7}\)

\(\left(\frac{3}{7}\right)^2 - 3 < 0\)

\(-\frac{2}{7} < 0\)

Yes, \(x = \frac{3}{7}\) is a solution.

(d) \(x = -5\)

\((-5)^2 - 3 < 0\)

\(22 < 0\)

No, \(x = -5\) is not a solution.
3. \( \frac{x + 2}{x - 4} \geq 3 \)
   
   (a) \( x = 5 \)
   
   \[ \frac{5 + 2}{5 - 4} \geq 3 \]
   
   \[ \frac{7}{1} \geq 3 \]
   
   Yes, \( x = 5 \) is a solution.

   (b) \( x = 4 \)
   
   \[ \frac{4 + 2}{4 - 4} \geq 3 \]
   
   \[ \frac{6}{0} \text{ is undefined.} \]
   
   No, \( x = 4 \) is not a solution.

   (c) \( x = \frac{9}{2} \)
   
   \[ \frac{-9 + 2}{-\frac{9}{2} - 4} \geq 3 \]
   
   \[ \frac{\frac{5}{2}}{\frac{17}{2}} \geq 3 \]
   
   No, \( x = \frac{9}{2} \) is not a solution.

   (d) \( x = \frac{9}{2} \)
   
   \[ \frac{\frac{9}{2} + 2}{\frac{9}{2} - 4} \geq 3 \]
   
   \[ \frac{\frac{17}{2}}{\frac{17}{2}} \geq 3 \]
   
   Yes, \( x = \frac{9}{2} \) is a solution.

5. \( 2x^2 - x - 6 = (2x + 3)(x - 2) \)
   
   \[ 2x + 3 = 0 \Rightarrow x = -\frac{3}{2} \]
   
   \[ x - 2 = 0 \Rightarrow x = 2 \]
   
   Critical numbers: \( x = -\frac{3}{2}, x = 2 \)

7. \( 2 + \frac{3}{x - 5} = \frac{2(x - 5) + 3}{x - 5} \)
   
   \[ x - 7 = 0 \Rightarrow x = 7 \]
   
   \[ x - 5 = 0 \Rightarrow x = 5 \]
   
   Critical numbers: \( x = \frac{7}{2}, x = 5 \)

9. \( x^2 \leq 9 \)
   
   \( x^2 - 9 \leq 0 \)
   
   \( (x + 3)(x - 3) \leq 0 \)
   
   Critical numbers: \( x = \pm 3 \)

   Test intervals: \( (-\infty, -3), (-3, 3), (3, \infty) \)
   
   Test: Is \( (x + 3)(x - 3) \leq 0 \)?

<table>
<thead>
<tr>
<th>Interval</th>
<th>x-Value</th>
<th>Value of ( x^2 - 9 )</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>((-\infty, -3))</td>
<td>(-4)</td>
<td>(16 - 9 = 7)</td>
<td>Positive</td>
</tr>
<tr>
<td>((-3, 3))</td>
<td>(0)</td>
<td>(0 - 9 = -9)</td>
<td>Negative</td>
</tr>
<tr>
<td>((3, \infty))</td>
<td>(4)</td>
<td>(16 - 9 = 7)</td>
<td>Positive</td>
</tr>
</tbody>
</table>

   Solution set: \( [-3, 3] \)

11. \( (x + 2)^2 < 25 \)
    
    \( x^2 + 4x + 4 < 25 \)
    
    \( x^2 + 4x - 21 < 0 \)
    
    \( (x + 7)(x - 3) < 0 \)
    
    Critical numbers: \( x = -7, x = 3 \)
    
    Test intervals: \( (-\infty, -7), (-7, 3), (3, \infty) \)
    
    Test: Is \( (x + 7)(x - 3) < 0 \)?

    | Interval    | x-Value | Value of \( (x + 7)(x - 3) \) | Conclusion |
    |-------------|---------|-------------------------------|------------|
    | \((-\infty, -7)\) | \(-10\) | \((-3)(-13) = 39\)         | Positive   |
    | \((-7, 3)\)        | \(0\)   | \((7)(-3) = -21\)         | Negative   |
    | \((3, \infty)\)     | \(5\)   | \((12)(2) = 24\)         | Positive   |

    Solution set: \( (-7, 3) \)
13. \(x^2 + 4x + 4 \geq 9\)
   \(x^2 + 4x - 5 \geq 0\)
   \((x + 3)(x - 2) < 0\)
   Critical numbers: \(x = -3, x = 2\)
   Test intervals: \((-\infty, -3), (-3, 2), (2, \infty)\)
   Test: Is \((x + 3)(x - 2) < 0?\)

<table>
<thead>
<tr>
<th>Interval</th>
<th>x-Value</th>
<th>Value of ((x + 3)(x - 2))</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>((-\infty, -3))</td>
<td>(x = -4)</td>
<td>((-1)(-5) = 5)</td>
<td>Positive</td>
</tr>
<tr>
<td>((-3, 2))</td>
<td>(x = 0)</td>
<td>((3)(-1) = -3)</td>
<td>Negative</td>
</tr>
<tr>
<td>((2, \infty))</td>
<td>(x = 3)</td>
<td>((6)(1) = 6)</td>
<td>Positive</td>
</tr>
</tbody>
</table>

   Solution set: \((-\infty, -3) \cup (-3, 2)\)

15. \(x^2 + x < 6\)
   \(x^2 + x - 6 < 0\)
   \((x + 3)(x - 2) < 0\)
   Critical numbers: \(x = -3, x = 2\)
   Test intervals: \((-\infty, -3), (-3, 2), (2, \infty)\)
   Test: Is \((x + 3)(x - 2) < 0?\)

<table>
<thead>
<tr>
<th>Interval</th>
<th>x-Value</th>
<th>Value of ((x + 3)(x - 2))</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>((-\infty, -3))</td>
<td>(x = -4)</td>
<td>((-1)(-6) = 6)</td>
<td>Positive</td>
</tr>
<tr>
<td>((-3, 2))</td>
<td>(x = 0)</td>
<td>((3)(-2) = -6)</td>
<td>Negative</td>
</tr>
<tr>
<td>((2, \infty))</td>
<td>(x = 3)</td>
<td>((6)(1) = 6)</td>
<td>Positive</td>
</tr>
</tbody>
</table>

   Solution set: \((-3, 2)\)

17. \(x^2 + 2x - 3 < 0\)
   \((x + 3)(x - 1) < 0\)
   Critical numbers: \(x = -3, x = 1\)
   Test intervals: \((-\infty, -3), (-3, 1), (1, \infty)\)
   Test: Is \((x + 3)(x - 1) < 0?\)

<table>
<thead>
<tr>
<th>Interval</th>
<th>x-Value</th>
<th>Value of ((x + 3)(x - 1))</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>((-\infty, -3))</td>
<td>(x = -4)</td>
<td>((-1)(-5) = 5)</td>
<td>Positive</td>
</tr>
<tr>
<td>((-3, 1))</td>
<td>(x = 0)</td>
<td>((3)(-1) = -3)</td>
<td>Negative</td>
</tr>
<tr>
<td>((1, \infty))</td>
<td>(x = 2)</td>
<td>((5)(1) = 5)</td>
<td>Positive</td>
</tr>
</tbody>
</table>

   Solution set: \((-3, 1)\)

19. \(x^2 + 8x - 5 \geq 0\)
   \(x^2 + 8x - 5 = 0\)
   Complete the square.

   \(x^2 + 8x + 16 = 5 + 16\)
   \((x + 4)^2 = 21\)
   \(x + 4 = \pm \sqrt{21}\)
   \(x = -4 \pm \sqrt{21}\)
   Critical numbers: \(x = -4 \pm \sqrt{21}\)
   Test intervals: \((-\infty, -4 - \sqrt{21}), (-4 - \sqrt{21}, -4 + \sqrt{21}), (-4 + \sqrt{21}, \infty)\)
   Test: Is \(x^2 + 8x - 5 \geq 0?\)

<table>
<thead>
<tr>
<th>Interval</th>
<th>x-Value</th>
<th>Value of (x^2 + 8x - 5)</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>((-\infty, -4 - \sqrt{21}))</td>
<td>(x = -10)</td>
<td>(100 - 80 - 5 = 15)</td>
<td>Positive</td>
</tr>
<tr>
<td>((-4 - \sqrt{21}, -4 + \sqrt{21}))</td>
<td>(x = 0)</td>
<td>(0 + 0 - 5 = -5)</td>
<td>Negative</td>
</tr>
<tr>
<td>((-4 + \sqrt{21}, \infty))</td>
<td>(x = 2)</td>
<td>(4 + 16 - 5 = 15)</td>
<td>Positive</td>
</tr>
</tbody>
</table>

   Solution set: \((-\infty, -4 - \sqrt{21}) \cup (-4 + \sqrt{21}, \infty)\)
21. \[ x^3 - 3x^2 - x + 3 > 0 \]
\[ x^2(x - 3) - 1(x - 3) > 0 \]
\[ (x - 3)(x - 3) > 0 \]
\[ (x + 1)(x - 1)(x - 3) > 0 \]

Critical numbers: \( x = \pm 1, x = 3 \)

Test intervals: \((-\infty, -1), (-1, 1), (1, 3), (3, \infty)\)

Test: Is \((x + 1)(x - 1)(x - 3) > 0?\)

<table>
<thead>
<tr>
<th>Interval</th>
<th>(x)-Value</th>
<th>Value of ((x + 1)(x - 1)(x - 3))</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>((-\infty, -1))</td>
<td>(x = -2)</td>
<td>((-1)(-3)(-5) = -15)</td>
<td>Negative</td>
</tr>
<tr>
<td>((-1, 1))</td>
<td>(x = 0)</td>
<td>((1)(-1)(-3) = 3)</td>
<td>Positive</td>
</tr>
<tr>
<td>((1, 3))</td>
<td>(x = 2)</td>
<td>((3)(1)(-1) = -3)</td>
<td>Negative</td>
</tr>
<tr>
<td>((3, \infty))</td>
<td>(x = 4)</td>
<td>((5)(3)(1) = 15)</td>
<td>Positive</td>
</tr>
</tbody>
</table>

Solution set: \((-1, 1) \cup (3, \infty)\)

23. \[ x^3 - 2x^2 - 9x - 2 \geq -20 \]
\[ x^3 - 2x^2 - 9x + 18 \geq 0 \]
\[ x^2(x - 2) - 9(x - 2) \geq 0 \]
\[ (x - 2)(x^2 - 9) \geq 0 \]
\[ (x - 2)(x + 3)(x - 3) \geq 0 \]

Critical numbers: \( x = 2, x = \pm 3 \)

Test intervals: \((-\infty, -3), (-3, 2), (2, 3), (3, \infty)\)

Test: Is \((x - 2)(x + 3)(x - 3) \geq 0?\)

<table>
<thead>
<tr>
<th>Interval</th>
<th>(x)-Value</th>
<th>Value of ((x - 2)(x + 3)(x - 3))</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>((-\infty, -3))</td>
<td>(x = -4)</td>
<td>((-6)(-1)(-7) = -42)</td>
<td>Negative</td>
</tr>
<tr>
<td>((-3, 2))</td>
<td>(x = 0)</td>
<td>((-2)(3)(-3) = 18)</td>
<td>Positive</td>
</tr>
<tr>
<td>((2, 3))</td>
<td>(x = 2.5)</td>
<td>((0.5)(5.5)(-0.5) = -1.375)</td>
<td>Negative</td>
</tr>
<tr>
<td>((3, \infty))</td>
<td>(x = 4)</td>
<td>((2)(7)(1) = 14)</td>
<td>Positive</td>
</tr>
</tbody>
</table>

Solution set: \([-3, 2] \cup [3, \infty)\)

25. \[ 4x^2 - 4x + 1 \leq 0 \]
\[ (2x - 1)^2 \leq 0 \]

Critical number: \( x = \frac{1}{2} \)

Test intervals: \((-\infty, \frac{1}{2}), (\frac{1}{2}, \infty)\)

Test: Is \((2x - 1)^2 \leq 0?\)

<table>
<thead>
<tr>
<th>Interval</th>
<th>(x)-Value</th>
<th>Value of ((2x - 1)^2)</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>((-\infty, \frac{1}{2}))</td>
<td>(x = 0)</td>
<td>((-1)^2 = 1)</td>
<td>Positive</td>
</tr>
<tr>
<td>((\frac{1}{2}, \infty))</td>
<td>(x = 1)</td>
<td>((1)^2 = 1)</td>
<td>Positive</td>
</tr>
</tbody>
</table>

Solution set: \( x = \frac{1}{2} \)

27. \[ 4x^3 - 6x^2 < 0 \]
\[ 2x^2(2x - 3) < 0 \]

Critical numbers: \( x = 0, x = \frac{3}{2} \)

Test intervals: \((-\infty, 0), (0, \frac{3}{2}), (\frac{3}{2}, \infty)\)

Test: Is \(2x^2(2x - 3) < 0?\)

By testing an \(x\)-value in each test interval in the inequality, we see that the solution set is: \((-\infty, 0) \cup \left(0, \frac{3}{2}\right)\)
29. \( x^3 - 4x \geq 0 \)
\( x(x + 2)(x - 2) \geq 0 \)
Critical numbers: \( x = 0, x = \pm 2 \)
Test intervals: \( (-\infty, -2), (-2, 0), (0, 2), (2, \infty) \)
Test: Is \( x(x + 2)(x - 2) \geq 0 \)?
By testing an \( x \)-value in each test interval in the inequality, we see that the solution set is: \( [-2, 0] \cup [2, \infty) \)

31. \((x - 1)^2(x + 2)^3 \geq 0\)
Critical numbers: \( x = 1, x = -2 \)
Test intervals: \( (-\infty, -2), (-2, 1), (1, \infty) \)
Test: Is \((x - 1)^2(x + 3)^3 \geq 0\)?
By testing an \( x \)-value in each test interval in the inequality, we see that the solution set is: \( [-2, \infty) \)

33. \( y = -x^2 + 2x + 3 \)

(a) \( y \leq 0 \) when \( x \leq -1 \) or \( x \geq 3 \).
(b) \( y \geq 3 \) when \( 0 \leq x \leq 2 \).

35. \( y = \frac{1}{8}x^3 - \frac{1}{2}x \)

(a) \( y \geq 0 \) when \(-2 \leq x \leq 0, 2 \leq x < \infty\).
(b) \( y \leq 6 \) when \( x \leq 4 \).

37. \( \frac{1}{x} - x > 0 \)
\( \frac{1 - x^2}{x} > 0 \)
Critical numbers: \( x = 0, x = \pm 1 \)
Test intervals: \( (-\infty, -1), (-1, 0), (0, 1), (1, \infty) \)
Test: Is \( \frac{1 - x^2}{x} > 0 \)?
By testing an \( x \)-value in each test interval in the inequality, we see that the solution set is: \( (-\infty, -1) \cup (0, 1) \)

39. \( \frac{x + 6}{x + 1} - 2 < 0 \)
\( \frac{x + 6 - 2(x + 1)}{x + 1} < 0 \)
\( \frac{4 - x}{x + 1} < 0 \)
Critical numbers: \( x = -1, x = 4 \)
Test intervals: \( (-\infty, -1), (-1, 4), (4, \infty) \)
Test: Is \( \frac{4 - x}{x + 1} < 0 \)?
By testing an \( x \)-value in each test interval in the inequality, we see that the solution set is: \( (-\infty, -1) \cup (4, \infty) \)

41. \( \frac{3x - 5}{x - 5} > 4 \)
\( \frac{3x - 5}{x - 5} - 4 > 0 \)
\( \frac{3x - 5 - 4(x - 5)}{x - 5} > 0 \)
\( \frac{15 - x}{x - 5} > 0 \)
Critical numbers: \( x = 5, x = 15 \)
Test intervals: \( (-\infty, 5), (5, 15), (15, \infty) \)
Test: Is \( \frac{15 - x}{x - 5} > 0 \)?
By testing an \( x \)-value in each test interval in the inequality, we see that the solution set is: \( (5, 15) \)
43. \[
\frac{4}{x + 5} > \frac{1}{2x + 3}
\]
\[
\frac{4}{x + 5} - \frac{1}{2x + 3} > 0
\]
\[
\frac{4(2x + 3) - (x + 5)}{(x + 5)(2x + 3)} > 0
\]
\[
\frac{7x + 7}{(x + 5)(2x + 3)} > 0
\]

Critical numbers: \( x = -1, x = -5, x = -\frac{3}{2} \)

Test intervals: \((-\infty, -5), \left(-5, -\frac{3}{2}\right), \left(-\frac{3}{2}, -1\right), (-1, \infty)\)

Test: Is \( \frac{7(x + 1)}{(x + 5)(2x + 3)} > 0? \)

By testing an \( x \)-value in each test interval in the inequality, we see that the solution set is: \((-5, -\frac{3}{2}) \cup (-1, \infty)\)

45. \[
\frac{1}{x - 3} \leq \frac{9}{4x + 3}
\]
\[
\frac{1}{x - 3} - \frac{9}{4x + 3} \leq 0
\]
\[
\frac{4x + 3 - 9(x - 3)}{(x - 3)(4x + 3)} \leq 0
\]
\[
\frac{30 - 5x}{(x - 3)(4x + 3)} \leq 0
\]

Critical numbers: \( x = 3, x = -\frac{3}{4}, x = 6 \)

Test intervals: \((-\infty, -\frac{3}{4}), \left(-\frac{3}{4}, -3\right), (3, 6), (6, \infty)\)

Test: Is \( \frac{30 - 5x}{(x - 3)(4x + 3)} \leq 0? \)

By testing an \( x \)-value in each test interval in the inequality, we see that the solution set is: \((-\frac{3}{4}, 3) \cup [6, \infty)\)

47. \[
\frac{x^2 + 2x}{x^2 - 9} \leq 0
\]
\[
\frac{x(x + 2)}{(x + 3)(x - 3)} \leq 0
\]

Critical numbers: \( x = 0, x = -2, x = \pm 3 \)

Test intervals: \((-\infty, -3), (-3, -2), (-2, 0), (0, 3), (3, \infty)\)

Test: Is \( \frac{x(x + 2)}{(x + 3)(x - 3)} \leq 0? \)

By testing an \( x \)-value in each test interval in the inequality, we see that the solution set is: \((-3, -2] \cup [0, 3)\)
49. \[\frac{5}{x-1} - \frac{2x}{x+1} < 1\]
\[\frac{5}{x-1} - \frac{2x}{x+1} - 1 < 0\]
\[\frac{5(x+1) - 2x(x -1) - (x-1)(x + 1)}{(x -1)(x + 1)} < 0\]
\[\frac{5x + 5 - 2x^2 + 2x - x^2 + 1}{(x -1)(x + 1)} < 0\]
\[-3x^2 + 7x + 6 < 0\]
\[-(3x + 2)(x - 3) < 0\]

Critical numbers: \(x = \frac{2}{3}, x = 3, x = \pm 1\)

Test intervals: \((-\infty, -1), \left(-1, \frac{2}{3}\right), \left(\frac{2}{3}, 1\right), (1, 3), (3, \infty)\)

Test: Is \(\frac{-(3x + 2)(x - 3)}{(x -1)(x + 1)} < 0?\)

By testing an \(x\)-value in each test interval in the inequality, we see that the solution set is: \((-\infty, -1) \cup \left(\frac{2}{3}, 1\right) \cup (3, \infty)\)

---

51. \(y = \frac{3x}{x - 2}\)

(a) \(y \leq 0\) when \(0 \leq x < 2\).

(b) \(y \geq 6\) when \(2 < x \leq 4\).

---

52. \(4 - x^2 \geq 0\)

\((2 + x)(2 - x) \geq 0\)

Critical numbers: \(x = \pm 2\)

Test intervals: \((-\infty, -2), (-2, 2), (2, \infty)\)

Test: Is \(4 - x^2 \geq 0?\)

By testing an \(x\)-value in each test interval in the inequality, we see that the domain set is: \([-2, 2]\)

---

53. \(y = \frac{2x^2}{x^2 + 4}\)

(a) \(y \geq 1\) when \(x \leq -2\) or \(x \geq 2\).

This can also be expressed as \(|x| \geq 2\).

(b) \(y \leq 2\) for all real numbers \(x\).

This can also be expressed as \(-\infty < x < \infty\).

---

54. \(x^2 - 7x + 12 \geq 0\)

\((x - 3)(x - 4) \geq 0\)

Critical numbers: \(x = 3, x = 4\)

Test intervals: \((-\infty, 3), (3, 4), (4, \infty)\)

Test: Is \((x - 3)(x - 4) \geq 0?\)

By testing an \(x\)-value in each test interval in the inequality, we see that the domain set is: \((-\infty, 3) \cup [4, \infty)\)
59. \[
\frac{x}{x^2 - 2x - 35} \geq 0
\]
\[
\frac{x}{(x + 5)(x - 7)} \geq 0
\]
Critical numbers: \(x = 0, x = -5, x = 7\)
Test intervals: \((-\infty, -5), (-5, 0), (0, 7), (7, \infty)\)
Test: Is \(\frac{x}{(x + 5)(x - 7)} \geq 0?\)

By testing an x-value in each test interval in the inequality, we see that the solution set is: \((-5, 0] \cup (7, \infty)\)

60. \[
0.4x^2 + 5.26 < 10.2
0.4x^2 - 4.94 < 0
0.4(x^2 - 12.35) < 0
\]
Critical numbers: \(x \approx \pm 3.51\)
Test intervals: \((-\infty, -3.51), (-3.51, 3.51), (3.51, \infty)\)
By testing an x-value in each test interval in the inequality, we see that the solution set is: \((-3.51, 3.51)\)

61. \[
\frac{1}{2.3x - 5.2} > 3.4
\]
\[
\frac{1}{2.3x - 5.2} - 3.4 > 0
\]
\[
1 - 3.4(2.3x - 5.2) > 0
\]
\[
\frac{-7.82x + 18.68}{2.3x - 5.2} > 0
\]
Critical numbers: \(x \approx 2.39, x = 2.26\)
Test intervals: \((-\infty, 2.26), (2.26, 2.39), (2.39, \infty)\)
By testing an x-value in each test interval in the inequality, we see that the solution set is: \((2.26, 2.39)\)

62. \[
s = -16t^2 + v_0t + s_0 = -16t^2 + 160t
\]
(a) \(-16t^2 + 160t = 0\)
\(-16(t - 10) = 0\)
\(t = 0, t = 10\)
It will be back on the ground in 10 seconds.
(b) \(-16t^2 + 160t > 384\)
\(-16t^2 + 160t - 384 > 0\)
\(-16(t^2 - 10t + 24) > 0\)
\(-16(t - 10t + 24 < 0\)
\((t - 4)(t - 6) < 0\)
\(4 < t < 6\) seconds

63. \(-0.5x^2 + 12.5x + 1.6 > 0\)
The zeros are \(x = \frac{-12.5 \pm \sqrt{(12.5)^2 - 4(-0.5)(1.6)}}{2(-0.5)}\).
Critical numbers: \(x \approx -0.13, x \approx 25.13\)
Test intervals: \((-\infty, -0.13), (-0.13, 25.13), (25.13, \infty)\)
By testing an x-value in each test interval in the inequality, we see that the solution set is: \((-0.13, 25.13)\)

64. \[
LW \geq 500
\]
\[
L(50 - L) \geq 500
\]
\(-L^2 + 50L - 500 \geq 0\)
By the Quadratic Formula we have:
Critical numbers: \(L = 25 \pm 5\sqrt{5}\)
Test: Is \(-L^2 + 50L - 500 \geq 0?\)
Solution set: \(25 - 5\sqrt{5} \leq L \leq 25 + 5\sqrt{5}\)
\(13.8\) meters \(\leq L \leq 36.2\) meters
71. \( R = x(75 - 0.0005x) \) and \( C = 30x + 250,000 \)

\[
P = R - C
= (75x - 0.0005x^2) - (30x + 250,000)
= -0.0005x^2 + 45x - 250,000
\]

\[P \geq 750,000\]

\[-0.0005x^2 + 45x - 250,000 \geq 750,000\]

\[-0.0005x^2 + 45x - 1,000,000 \geq 0\]

Critical numbers: \( x = 40,000, x = 50,000 \) (These were obtained by using the Quadratic Formula.)

Test intervals: \((0, 40,000), (40,000, 50,000), (50,000, \infty)\)

By testing \( x \)-values in each test interval in the inequality, we see that the solution set is \([40,000, 50,000]\) or \(40,000 \leq x \leq 50,000\). The price per unit is

\[
p = \frac{R}{x} = 75 - 0.0005x.
\]

For \( x = 40,000, p = 55 \). For \( x = 50,000, p = 50 \). Therefore, for \( 40,000 \leq x \leq 50,000, $50.00 \leq p \leq $55.00 \).

73. \( C = 0.0031t^3 - 0.216t^2 + 5.54t + 19.1, \) \( 0 \leq t \leq 23 \)

(a) \[\text{Graph of } C \text{ vs. } t\]

(b) \begin{tabular}{|c|c|} \hline \( t \) & \( C \) \hline 24 & 70.5 \hline 26 & 71.6 \hline 28 & 72.9 \hline 30 & 74.6 \hline 32 & 76.8 \hline 34 & 79.6 \hline \end{tabular}

\( C \) will be greater than 75% when \( t = 31 \), which corresponds to 2011.

(c) \( C = 75 \) when \( t = 30.41 \).

(d) \begin{tabular}{|c|c|} \hline \( t \) & \( C \) \hline 36 & 83.2 \hline 37 & 85.4 \hline 38 & 87.8 \hline 39 & 90.5 \hline 40 & 93.5 \hline 41 & 96.8 \hline 42 & 100.4 \hline 43 & 104.4 \hline \end{tabular}

\( C \) will be between 85% and 100% when \( t \) is between 37 and 42. These values correspond to the years 2017 to 2022.

(e) \( 85 \leq C \leq 100 \) when \( 36.82 \leq t \leq 41.89 \) or \( 37 \leq t \leq 42 \).

(f) The model is a third-degree polynomial and as \( t \to \infty, C \to \infty \).
75. \[
\frac{1}{R} = \frac{1}{R_1} + \frac{1}{2}
\]
\[
2R_1 = 2R + RR_1
\]
\[
2R_1 = R(2 + R_1)
\]
\[
\frac{2R_1}{2 + R_1} = R
\]
Since \( R \geq 1 \), we have
\[
\frac{2R_1}{2 + R_1} \geq 1
\]
\[
\frac{2R_1}{2 + R_1} - 1 \geq 0
\]
\[
R_1 - 2
\]
Since \( R_1 > 0 \), the only critical number is \( R_1 = 2 \). The inequality is satisfied when \( R_1 \geq 2 \) ohms.

79. \( x^2 + bx + 4 = 0 \)

To have at least one real solution, \( b^2 - 16 \geq 0 \). This occurs when \( b \leq -4 \) or \( b \geq 4 \). This can be written as \((-\infty, -4] \cup [4, \infty)\).

81. \( 3x^2 + bx + 10 = 0 \)

To have at least one real solution, \( b^2 - 4(3)(10) \geq 0 \).
\[
b^2 - 120 \geq 0
\]
\[
(b + \sqrt{120})(b - \sqrt{120}) \geq 0
\]
Critical numbers: \( b = \pm \sqrt{120} = \pm 2\sqrt{30} \)
Test intervals: \((-\infty, -2\sqrt{30}], (-2\sqrt{30}, 2\sqrt{30}), (2\sqrt{30}, \infty)\)
Test: Is \( b^2 - 120 \geq 0 \)?
Solution set: \((-\infty, -2\sqrt{30}] \cup [2\sqrt{30}, \infty)\)

83. (a) If \( a > 0 \) and \( c \leq 0 \), then \( b \) can be any real number. If \( a > 0 \) and \( c > 0 \), then for \( b^2 - 4ac \) to be greater than or equal to zero, \( b \) is restricted to \( b < -2\sqrt{ac} \) or \( b > 2\sqrt{ac} \).

(b) The center of the interval for \( b \) in Exercises 79–82 is 0.

87. \( x^2(x + 3) - 4(x + 3) = (x^2 - 4)(x + 3) \)
\[
= (x + 2)(x - 2)(x + 3)
\]
89. Area = (length)(width)
\[
= (2x + 1)(x)
\]
\[
= 2x^2 + x
\]
Review Exercises for Chapter 1

1. \( y = 3x - 5 \)

<table>
<thead>
<tr>
<th></th>
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<tr>
<td>y</td>
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<td>-5</td>
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\[ y = 3x - 5 \]

3. \( y = x^2 - 3x \)

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<tr>
<td>x</td>
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<tr>
<td>y</td>
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<td>0</td>
<td>4</td>
</tr>
</tbody>
</table>

\[ y = x^2 - 3x \]

5. \( y - 2x - 3 = 0 \)

\( y = 2x + 3 \)

Line with \( x \)-intercept \( \left( -\frac{3}{2}, 0 \right) \)
and \( y \)-intercept \( (0, 3) \)

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<tr>
<td>y</td>
<td>-1</td>
<td>1</td>
<td>3</td>
<td>5</td>
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</table>

7. \( y = \sqrt{5 - x} \)

Domain: \(-\infty, 5\]

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<td></td>
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</tr>
<tr>
<td>y</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
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</table>

9. \( y + 2x^2 = 0 \)

\( y = -2x^2 \) is a parabola.

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<tr>
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<th>±1</th>
<th>±2</th>
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<tbody>
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<td>x</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y</td>
<td>0</td>
<td>-2</td>
<td>-8</td>
</tr>
</tbody>
</table>

11. \( y = (x - 3)^2 - 4 \)

\( x \)-intercepts: \( 0 = (x - 3)^2 - 4 \) \( \Rightarrow \) \( (x - 3)^2 = 4 \) \( \Rightarrow \) \( x - 3 = \pm 2 \) \( \Rightarrow \) \( x = 3 \pm 2 \) \( \Rightarrow \) \( x = 5 \text{ or } x = 1 \)

\( y \)-intercept: \( y = (0 - 3)^2 - 4 \) \( \Rightarrow \) \( y = 9 - 4 \) \( \Rightarrow \) \( y = 5 \)

The \( x \)-intercepts are \( (1, 0) \) and \( (5, 0) \). The \( y \)-intercept is \( (0, 5) \).

13. \( y = -4x + 1 \)

Intercepts: \( \left( \frac{1}{4}, 0 \right), (0, 1) \)

\( y = -4(-x) + 1 \) \( \Rightarrow \) \( y = 4x + 1 \) \( \Rightarrow \) No \( y \)-axis symmetry

\( -y = -4x + 1 \) \( \Rightarrow \) \( y = 4x - 1 \) \( \Rightarrow \) No \( x \)-axis symmetry

\( -y = -4(-x) + 1 \) \( \Rightarrow \) \( y = -4x - 1 \) \( \Rightarrow \) No origin symmetry
15. $y = 5 - x^2$
   - Intercepts: $(\pm \sqrt{5}, 0), (0, 5)$
   - $y = 5 - (-x)^2 \Rightarrow y = 5 - x^2 \Rightarrow$ y-axis symmetry
   - $y = 5 - x^2 \Rightarrow y = -5 + x^2 \Rightarrow$ No x-axis symmetry
   - $y = 5 - (-x)^2 \Rightarrow y = -5 + x^2 \Rightarrow$ No origin symmetry

17. $y = x^3 + 3$
   - Intercepts: $(-\sqrt[3]{3}, 0), (0, 3)$
   - $y = (-x)^3 + 3 \Rightarrow y = -x^3 + 3 \Rightarrow$ No y-axis symmetry
   - $y = x^3 + 3 \Rightarrow y = -x^3 - 3 \Rightarrow$ No x-axis symmetry
   - $y = (-x)^3 + 3 \Rightarrow y = x^3 - 3 \Rightarrow$ No origin symmetry

19. $y = \sqrt{x + 5}$
   - Domain: $[-5, \infty)$
   - Intercepts: $(-5, 0), (0, \sqrt{5})$
   - $y = \sqrt{-x + 5} \Rightarrow$ No y-axis symmetry
   - $y = \sqrt{x + 5} \Rightarrow y = -\sqrt{x + 5} \Rightarrow$ No x-axis symmetry
   - $y = -\sqrt{-x + 5} \Rightarrow y = -\sqrt{-x + 5} \Rightarrow$ No origin symmetry

21. $x^2 + y^2 = 9$
   - Center: $(0, 0)$
   - Radius: 3

23. $(x + 2)^2 + y^2 = 16$
   - $(x - (-2))^2 + (y - 0)^2 = 4^2$
   - Center: $(-2, 0)$
   - Radius: 4

25. $\left( x - \frac{1}{2} \right)^2 + (y + 1)^2 = 36$
   - $\left( x - \frac{1}{2} \right)^2 + (y - (-1))^2 = 6^2$
   - Center: $(\frac{1}{2}, -1)$
   - Radius: 6

27. Endpoints of a diameter: $(0, 0)$ and $(4, -6)$
   - Center: $(\frac{0 + 4}{2}, \frac{0 + (-6)}{2}) = (2, -3)$
   - Radius: $r = \sqrt{(2 - 0)^2 + (-3 - 0)^2} = \sqrt{4 + 9} = \sqrt{13}$
   - Standard form: $(x - 2)^2 + (y - (-3))^2 = (\sqrt{13})^2$
     - $(x - 2)^2 + (y + 3)^2 = 13$
29. \( F = \frac{5}{2}, 0 \leq x \leq 20 \)

(a) 

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>16</th>
<th>20</th>
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<tbody>
<tr>
<td>( F )</td>
<td>0</td>
<td>5</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>25</td>
</tr>
</tbody>
</table>

(b)  

(c) When \( x = 10 \), \( F = \frac{50}{2} = 25 \) pounds.

31. \( 6 - (x - 2)^2 = 2 + 4x - x^2 \)

\[ 6 - (x^2 - 4x + 4) = 2 + 4x - x^2 \]

\[ 2 + 4x - x^2 = 2 + 4x - x^2 \]

\[ 0 = 0 \quad \text{Identity} \]

All real numbers are solutions.

33. \( -x^3 + x(7 - x) + 3 = x(-x^2 - x) + 7(x + 1) - 4 \)

\[ -x^3 + 7x - x^2 + 3 = -x^3 - x^2 + 7x + 7 - 4 \]

\[ -x^3 - x^2 + 7x + 3 = -x^3 - x^2 + 7x + 3 \]

\[ 0 = 0 \quad \text{Identity} \]

All real numbers are solutions.

35. \( 3x - 2(x + 5) = 10 \)

\[ 3x - 2x - 10 = 10 \]

\[ x = 20 \]

37. \( 4(x + 3) - 3 = 2(4 - 3x) - 4 \)

\[ 4x + 12 - 3 = 8 - 6x - 4 \]

\[ 4x + 9 = -6x + 4 \]

\[ 10x = -5 \]

\[ x = -\frac{1}{2} \]

39. \( \frac{x}{5} - 3 = \frac{2x}{2} + 1 \)

\[ 5\left(\frac{x}{5} - 3\right) = (x + 1)5 \]

\[ x - 15 = 5x + 5 \]

\[ -4x = 20 \]

\[ x = -5 \]

43. \( y = 3x - 1 \)

\( x\)-intercept: \( 0 = 3x - 1 \Rightarrow x = \frac{1}{3} \)

\( y\)-intercept: \( y = 3(0) - 1 \Rightarrow y = -1 \)

The \( x\)-intercept is \( \left(\frac{1}{3}, 0\right) \) and the \( y\)-intercept is \( (0, -1) \).

47. \( y = -\frac{1}{2}x + \frac{2}{3} \)

\( x\)-intercept: \( 0 = -\frac{1}{2}x + \frac{2}{3} \Rightarrow x = 2/3 = \frac{4}{3} \)

\( y\)-intercept: \( y = -\frac{1}{2}(0) + \frac{2}{3} \Rightarrow y = \frac{2}{3} \)

The \( x\)-intercept is \( \left(\frac{4}{3}, 0\right) \) and the \( y\)-intercept is \( \left(0, \frac{2}{3}\right) \).

49. \( 3.8y - 0.5x + 1 = 0 \)

\( x\)-intercept: \( 3.8(0) - 0.5x + 1 = 0 \Rightarrow x = \frac{1}{0.5} = 2 \)

\( y\)-intercept: \( 3.8y - 0.5(0) + 1 = 0 \Rightarrow y = \frac{-1}{3.8} = -\frac{5}{19} \)

The \( x\)-intercept is \( (2, 0) \) and the \( y\)-intercept is \( \left(0, -\frac{5}{19}\right) \).

51. \( 244.92 = 2(3.14)(3)^2 + 2(3.14)(3)h \)

\[ 244.92 = 56.52 + 18.84h \]

\[ 188.40 = 18.84h \]

\[ 10 = h \]

The height is 10 inches.
53. **Verbal Model:** September’s profit + October’s profit = 689,000

**Labels:** Let \( x \) = September’s profit. Then \( x + 0.12x = \) October’s profit.

**Equation:**

\[
x + (x + 0.12x) = 689,000
\]

\[
2.12x = 689,000
\]

\[
x = 325,000
\]

\[
x + 0.12x = 364,000
\]

**Answer:** September profit: $325,000, October profit: $364,000

55. Let \( x \) = height of the streetlight.

By similar triangles we have:

\[
\frac{x}{20} = \frac{6}{5}
\]

\[
5x = 120
\]

\[
x = 24
\]

The streetlight is 24 feet tall.

57. Let \( x \) = the number of original investors.

Each person’s share is \( \frac{90,000}{x} \).

If three more people invest, each person’s share is \( \frac{90,000}{x + 3} \).

Since this is $2500 less than the original cost, we have:

\[
\frac{90,000}{x} - 2500 = \frac{90,000}{x + 3}
\]

\[
90,000(x + 3) - 2500x(x + 3) = 90,000x
\]

\[
90,000x(1 + 3) - 2500x^2 - 7500x = 90,000x
\]

\[
-2500x^2 - 7500x + 270,000 = 0
\]

\[
-2500(x^2 + 3x - 108) = 0
\]

\[
-2500(x + 12)(x - 9) = 0
\]

\[
x = 12, \text{ extraneous or } x = 9
\]

There are currently nine investors.

59. Let \( x \) = the number of liters of pure antifreeze.

30% of \((10 - x)\) + 100% of \(x\) = 50% of 10

\[
0.30(10 - x) + 1.00x = 0.50(10)
\]

\[
3 - 0.30x + 1.00x = 5
\]

\[
0.70x = 2
\]

\[
x = \frac{2}{0.70} = \frac{20}{7} = 2\frac{6}{7} \text{ liters}
\]

61. \( V = \frac{1}{3} \pi r^2 h \)

\[
3V = \pi r^2 h
\]

\[
\frac{3V}{\pi r^2} = h
\]

63.

<table>
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<th></th>
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<th>time</th>
<th>distance</th>
</tr>
</thead>
<tbody>
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<td>1st car</td>
<td>40 mph</td>
<td>( t )</td>
<td>40t</td>
</tr>
<tr>
<td>2nd car</td>
<td>55 mph</td>
<td>( t )</td>
<td>55t</td>
</tr>
</tbody>
</table>

\[
55t - 40t = 10
\]

\[
15t = 10
\]

\[
t = \frac{2}{3} \text{ hour or 40 minutes}
\]

65. \( 15 + x - 2x^2 = 0 \)

\[
0 = 2x^2 - x - 15
\]

\[
0 = (2x + 5)(x - 3)
\]

\[
x + 5 = 0 \implies x = \frac{-5}{2}
\]

\[
x - 3 = 0 \implies x = 3
\]
67. \(6 = 3x^2\)
\[2 = x^2\]
\[\pm \sqrt{2} = x\]

69. \((x + 4)^2 = 18\)
\[x + 4 = \pm \sqrt{18}\]
\[x = -4 \pm 3\sqrt{2}\]

71. \(x^2 - 12x + 30 = 0\)
\[x^2 - 12x = -30\]
\[(x - 6)^2 = 6\]
\[x - 6 = \pm \sqrt{6}\]
\[x = 6 \pm \sqrt{6}\]

73. \(-2x^2 - 5x + 27 = 0\)
\[2x^2 + 5x - 27 = 0\]
\[x = \frac{-5 \pm \sqrt{25 + 4(2)(27)}}{4(2)}\]
\[= \frac{-5 \pm \sqrt{241}}{4}\]

75. \(M = 500x(20 - x)\)
(a) \(500x(20 - x) = 0\) when \(x = 0\) feet and \(x = 20\) feet.
(b) \(b\)

(c) The bending moment is greatest when \(x = 10\) feet.

77. \(6 + \sqrt{-4} = 6 + 2i\)

81. \((7 + 5i) + (-4 + 2i) = (7 - 4) + (5i + 2i) = 3 + 7i\)

85. \((10 - 8i)(2 - 3i) = 20 - 30i - 16i + 24i^2\]
\[= -4 - 46i\]

89. \(\frac{4}{2 - 3i} + \frac{2}{1 + i} = \frac{4}{2 - 3i} \cdot \frac{2 + 3i}{2 + 3i} + \frac{2}{1 + i} \cdot \frac{1 - i}{1 - i}\]
\[= \frac{8 + 12i - 2}{4 + 9} + \frac{2 - 2i}{1 + 1}\]
\[= \frac{8}{13} + \frac{12i}{13} + 1 - i\]
\[= \left(\frac{8}{13} + 1\right) + \left(\frac{12}{13} - 1\right)i\]
\[= \frac{21}{13} - \frac{1}{13}i\]

91. \(3x^2 + 1 = 0\)
\[3x^2 = -1\]
\[x^2 = \frac{-1}{3}\]
\[x = \pm \sqrt{\frac{-1}{3}}\]
\[= \pm \frac{i\sqrt{3}}{3}\]

93. \(x^2 - 2x + 10 = 0\)
\[x^2 - 2x + 1 = -9\]
\[(x - 1)^2 = -9\]
\[x - 1 = \pm \sqrt{-9}\]
\[x = 1 \pm 3i\]

95. \(5x^4 - 12x^3 = 0\)
\[x^3(5x - 12) = 0\]
\[x^3 = 0\] or \(5x - 12 = 0\)
\[x = 0\] or \(x = \frac{12}{5}\)
97. \[ x^4 - 5x^2 + 6 = 0 \]
\[ (x^2 - 2)(x^2 - 3) = 0 \]
\[ x^2 - 2 = 0 \quad \text{or} \quad x^2 - 3 = 0 \]
\[ x^2 = 2 \quad \text{or} \quad x^2 = 3 \]
\[ x = \pm \sqrt{2} \quad \text{or} \quad x = \pm \sqrt{3} \]

99. \[ \sqrt{x + 4} = 3 \]
\[ (\sqrt{x + 4})^2 = (3)^2 \]
\[ x + 4 = 9 \]
\[ x = 5 \]

101. \[ \sqrt{2x + 3} + \sqrt{x - 2} = 2 \]
\[ (\sqrt{2x + 3})^2 = (2 - \sqrt{x - 2})^2 \]
\[ 2x + 3 = 4 - 4\sqrt{x - 2} + x - 2 \]
\[ x + 1 = -4\sqrt{x - 2} \]
\[ (x + 1)^2 = (-4\sqrt{x - 2})^2 \]
\[ x^2 + 2x + 1 = 16(x - 2) \]
\[ x^2 - 14x + 33 = 0 \]
\[ (x - 3)(x - 11) = 0 \]
\[ x = 3, \text{ extraneous or} \quad x = 11, \text{ extraneous} \]

No solution

103. \[ (x - 1)^{2/3} - 25 = 0 \]
\[ (x - 1)^{2/3} = 25 \]
\[ (x - 1)^2 = 25^3 \]
\[ x - 1 = \pm \sqrt[3]{25^3} \]
\[ x = 1 \pm 125 \]
\[ x = 126 \text{ or } x = -124 \]

105. \[ (x + 4)^{1/2} + 5(x + 4)^{1/2} = 0 \]
\[ (x + 4)^{1/2}[1 + 5(x + 4)] = 0 \]
\[ (x + 4)^{1/2}(5x^2 + 20x + 1) = 0 \]
\[ (x + 4)^{1/2} = 0 \quad \text{or} \quad 5x^2 + 20x + 1 = 0 \]
\[ x = -4 \]
\[ x = \frac{-20 \pm \sqrt{400 - 20}}{10} \]
\[ x = \frac{-20 \pm \sqrt{400 - 20}}{10} \]
\[ x = \frac{-20 \pm \sqrt{95}}{10} \]
\[ x = -2 \pm \frac{\sqrt{95}}{5} \]

107. \[ \frac{5}{x} = 1 + \frac{3}{x + 2} \]
\[ 5(x + 2) = (x + 2)(x + 3x) \]
\[ 5x + 10 = x^2 + 2x + 3x \]
\[ 10 = x^2 \]
\[ \pm \sqrt{10} = x \]

109. \[ \frac{3}{x + 2} - \frac{1}{x} = \frac{1}{5x} \]
\[ 3(5x) - 5(x + 2) = x + 2 \]
\[ 15x - 5x - 10 = x + 2 \]
\[ 9x = 12 \]
\[ x = \frac{4}{3} \]

111. \[ |x - 5| = 10 \]
\[ x - 5 = -10 \quad \text{or} \quad x - 5 = 10 \]
\[ x = -5 \quad \text{or} \quad x = 15 \]
113. \[ |x^2 - 3| = 2x \]

\[ x^2 - 3 = 2x \quad \text{or} \quad x^2 - 3 = -2x \]

\[ x^2 - 2x - 3 = 0 \quad x^2 + 2x - 3 = 0 \]

\[ (x - 3)(x + 1) = 0 \quad (x + 3)(x - 1) = 0 \]

\[ x = 3 \quad \text{or} \quad x = -1 \quad x = -3 \quad \text{or} \quad x = 1 \]

The only solutions to the original equation are \( x = 3 \) or \( x = 1 \). (\( x = -3 \) and \( x = -1 \) are extraneous.)

115. \[ 29.95 = 42 - \sqrt{0.001x + 2} \]

\[ -12.05 = -\sqrt{0.001x + 2} \]

\[ 0.001x + 2 = 137. \]

\[ 0.001x = 145.2025 \]

\[ x = 143,202.5 \]

\[ \approx 143,203 \text{ units} \]

117. Interval: \((-7, 2]\]

Inequality: \(-7 < x \leq 2\]

The interval is bounded.

119. Interval: \((-\infty, -10]\]

Inequality: \(x \leq -10\]

The interval is unbounded.

121. \[ 9x - 8 \leq 7x + 16 \]

\[ 2x \leq 24 \]

\[ x \leq 12 \]

\[ (-\infty, 12] \]

123. \[ 4(5 - 2x) \leq \frac{1}{2}(8 - x) \]

\[ 20 - 8x \leq 4 - \frac{1}{2}x \]

\[ -\frac{15}{2}x \leq -16 \]

\[ x \geq \frac{32}{15} \]

\[ \left[ \frac{32}{15}, \infty \right) \]

125. \[ -19 < 3x - 17 \leq 34 \]

\[ -\frac{3}{5} < x \leq 17 \]

\[ (-\frac{3}{5}, 17] \]

127. \[ |x| \leq 4 \]

131. If the side is 19.3 cm, then with the possible error of 0.5 cm we have:

\[ 18.8 \leq \text{side} \leq 19.8 \]

\[ 353.44 \text{ cm}^2 \leq \text{area} \leq 392.04 \text{ cm}^2 \]

133. \[ x^2 - 6x - 27 < 0 \]

\[ (x + 3)(x - 9) < 0 \]

Critical numbers: \( x = -3, x = 9 \)

Test intervals: \((-\infty, -3), (-3, 9), (9, \infty)\)

Test: Is \((x + 3)(x - 9) < 0?\)

By testing an \( x \)-value in each test interval in the inequality, we see that the solution set is: \((-3, 9)\)

135. \[ 6x^2 + 5x < 4 \]

\[ 6x^2 + 5x - 4 < 0 \]

\[ (3x + 4)(2x - 1) < 0 \]

Critical numbers: \( x = -\frac{4}{3}, x = \frac{1}{2} \)

Test intervals: \((-\infty, -\frac{4}{3}), (-\frac{4}{3}, \frac{1}{2}), (\frac{1}{2}, \infty)\)

Test: Is \((3x + 4)(2x - 1) < 0?\)

By testing an \( x \)-value in each test interval in the inequality, we see that the solution set is: \((-\frac{4}{3}, \frac{1}{2})\)

139. \[ \frac{x + 8}{x + 5} - 2 < 0 \]

\[ \frac{x + 8 - 2(x + 5)}{x + 5} < 0 \]

\[ \frac{-x - 2}{x + 5} < 0 \]

Critical numbers: \( x = -2, x = -5 \)

Test intervals: \((-\infty, -5), (-5, -2), (-2, \infty)\)

By testing an \( x \)-value in each test interval in the inequality, we see that the solution set is: \((-\infty, -5) \cup (-2, \infty)\)
141. \[
\frac{2}{x+1} \leq \frac{3}{x-1}
\]
\[
\frac{2(x-1) - 3(x+1)}{(x+1)(x-1)} \leq 0
\]
\[
\frac{2x - 2 - 3x - 3}{(x+1)(x-1)} \leq 0
\]
\[
\frac{-(x+5)}{(x+1)(x-1)} \leq 0
\]
Critical numbers: \(x = -5\), \(x = \pm 1\)
Test intervals: \((-\infty, -5), (-5, -1), (-1, 1), (1, \infty)\)
Test: Is \[-\frac{(x+5)}{(x+1)(x-1)} \leq 0?\]
By testing an \(x\)-value in each test interval in the inequality, we see that the solution set is: \([-5, -1) \cup (1, \infty)\)

145. \(5000(1 + r)^2 > 5500\)
\[(1 + r)^2 > 1.1\]
\[1 + r > 1.0488\]
\[r > 0.488\]
\[r > 4.9\%\]

149. Rational equations, equations involving radicals, and absolute value equations, may have “solutions” that are extraneous. So checking solutions, in the original equations, is crucial to eliminate these extraneous values.

Problem Solving for Chapter 1

1.

3. (a) \(A = \pi ab\)
\[a + b = 20 \implies b = 20 - a, \text{ thus:} \]
\[A = \pi a(20 - a)\]

(b) \[
\begin{array}{cccccc}
  a & 4 & 7 & 10 & 13 & 16 \\
  A & 64\pi & 91\pi & 100\pi & 91\pi & 64\pi \\
\end{array}
\]
3. —CONTINUED—

(c) \[300 = \pi a (20 - a)\]
\[300 = 20 \pi a - \pi a^2\]
\[\pi a^2 - 20 \pi a + 300 = 0\]
\[a = \frac{20 \pi \pm \sqrt{(-20 \pi)^2 - 4 \pi (300)}}{2 \pi}\]
\[= \frac{20 \pi \pm \sqrt{400 \pi^2 - 1200\pi}}{2 \pi}\]
\[= \frac{20 \pi \pm 20 \sqrt{\pi (\pi - 3)}}{2 \pi}\]
\[= 10 \pm \frac{10}{\pi} \sqrt{\pi (\pi - 3)}\]
\[a = 12.123 \quad \text{or} \quad a = 7.877\]

(d) \[A = \pi a (20 - a)\]

(e) The \(a\)-intercepts occur at \(a = 0\) and \(a = 20\). Both yield an area of 0. When \(a = 0, b = 20\) and you have a vertical line of length 40. Likewise when \(a = 20, b = 0\) and you have a horizontal line of length 40. They represent the minimum and maximum values of \(a\).

(f) The maximum value of \(A\) is \(100 \pi \approx 314.159\). This occurs when \(a = b = 10\) and the ellipse is actually a circle.

5. \[h = \left(\sqrt{h_0} - \frac{2 \pi d^2 \sqrt{3}}{lw} t\right)^2\]

\[l = 60^\circ, w = 30^\circ, h_0 = 25^\circ, d = 2^\circ\]

\[h = \left(5 - \frac{8 \pi \sqrt{3}}{1800} t\right)^2\]

(a) \[12.5 = \left(5 - \frac{\pi \sqrt{3}}{225} t\right)^2\]

\[\sqrt{12.5} = 5 - \frac{\pi \sqrt{3}}{225} t\]

\[t = \frac{225}{\pi \sqrt{3}} \left(5 - \sqrt{12.5}\right) = 60.6\ \text{seconds}\]

(b) \[0 = \left(\sqrt{12.5} - \frac{\pi \sqrt{3}}{225} t\right)^2\]

\[t = \frac{225 \sqrt{12.5}}{\pi \sqrt{3}} = 146.2\ \text{seconds}\]

(c) The speed at which the water drains decreases as the amount of the water in the bathtub decreases.

7. (a) 5, 12, and 13; 8, 15, and 17

(b) 5 \cdot 12 \cdot 13 = 780 which is divisible by 3, 4, and 5

8 \cdot 15 \cdot 17 = 2040 which is divisible by 3, 4, and 5

7 \cdot 24 \cdot 25 = 4200 which is also divisible by 3, 4, and 5

(c) Conjecture: If \(a^2 + b^2 = c^2\) where \(a, b,\) and \(c\) are positive integers, then \(abc\) is divisible by 60.

9. \[ax^2 + bx + c = 0\]
\[x^2 + \frac{b}{a} x + \frac{c}{a} = 0\]
\[x_1 + x_2 = -\frac{b}{a}\]
\[x_1 \cdot x_2 = \frac{c}{a}\]
11. (a) \( z_m = \frac{1}{z} \)
\[
= \frac{1}{1 + i} = \frac{1}{1 + i} \cdot \frac{1 - i}{1 - i} \\
= \frac{1 - i}{2} = \frac{1}{2} - \frac{1}{2}i
\]
(b) \( z_m = \frac{1}{z} \)
\[
= \frac{1}{3 - i} = \frac{1}{3 - i} \cdot \frac{3 + i}{3 + i} \\
= \frac{3 + i}{10} = \frac{3}{10} + \frac{1}{10}i
\]
(c) \( z_m = \frac{1}{z} \)
\[
= \frac{1}{-2 + 8i} \\
= \frac{1}{-2 + 8i} \cdot \frac{-2 - 8i}{-2 - 8i} \\
= \frac{-2 - 8i}{68} = \frac{1}{34} - \frac{2}{17}i
\]

13. (a) \( c = i \)
The terms are: \( i, -1 + i, -i, -1 + i, -i, -1 + i, -i, \ldots \)
The sequence is bounded so \( c = i \) is in the Mandelbrot Set.

(b) \( c = 1 + i \)
The terms are: \( 1 + i, 1 + 3i, -7 + 7i, 1 - 97i, -9407 - 193i, \ldots \)
The sequence is unbounded so \( c = 1 + i \) is not in the Mandelbrot Set.

(c) \( c = -2 \)
The terms are: \( -2, 2, 2, 2, \ldots \)
The sequence is bounded so \( c = -2 \) is in the Mandelbrot Set.

15. \( y = x^4 - x^3 - 6x^2 + 4x + 8 \)
\[
= (x - 2)^2(x + 1)(x + 2)
\]
From the graph we see that \( x^4 - x^3 - 6x^2 + 4x + 8 > 0 \) on the intervals \((-\infty, -2) \cup (-1, 2) \cup (2, \infty)\).
Practice Test for Chapter 1

1. Graph \(3x - 5y = 15\).
2. Graph \(y = \sqrt{9 - x}\).

3. Solve \(5x + 4 = 7x - 8\).
4. Solve \(\frac{x}{3} - 5 = \frac{x}{5} + 1\).

5. Solve \(\frac{3x + 1}{6x - 7} = \frac{2}{5}\).
6. Solve \((x - 3)^2 + 4 = (x + 1)^2\).

7. Solve \(A = \frac{1}{2}(a + b)h\) for \(a\).
8. 301 is what percent of 4300?

9. Cindy has $6.05 in quarters and nickels. How many of each coin does she have if there are 53 coins in all?

10. Ed has $15,000 invested in two funds paying \(9\frac{1}{2}\%\) and 11\% simple interest, respectively. How much is invested in each if the yearly interest is $1582.50?

11. Solve \(28 + 5x - 3x^2 = 0\) by factoring.

12. Solve \((x - 2)^2 = 24\) by taking the square root of both sides.

13. Solve \(x^2 - 4x - 9 = 0\) by completing the square.

14. Solve \(x^2 + 5x - 1 = 0\) by the Quadratic Formula.

15. Solve \(3x^2 - 2x + 4 = 0\) by the Quadratic Formula.

16. The perimeter of a rectangle is 1100 feet. Find the dimensions so that the enclosed area will be 60,000 square feet.

17. Find two consecutive even positive integers whose product is 624.

18. Solve \(x^3 - 10x^2 + 24x = 0\) by factoring.
19. Solve \(\sqrt{6 - x} = 4\).

20. Solve \((x^2 - 8)^{2/3} = 4\).
21. Solve \(x^4 - x^2 - 12 = 0\).

22. Solve \(4 - 3x > 16\).
23. Solve \(\left|\frac{x - 3}{2}\right| < 5\).

24. Solve \(\frac{x + 1}{x - 3} < 2\).
25. Solve \(|3x - 4| \geq 9\).