Chapter 8 Sequences, Series, and Probability

Section 8.1

Infinite sequence – A function whose domain is the set of positive integers
Terms of a sequence – The function values $a_1, a_2, a_3, a_4, \ldots, a_n, \ldots$ of an infinite sequence
Finite sequence – A sequence whose domain consists of the first n positive integers only
Recursive – A sequence is recursive if one or more of the first few terms are given and all other terms are defined using previous terms
Factorial – If n is a positive integer, n factorial is defined by $n! = 1 \cdot 2 \cdot 3 \cdot 4 \cdots (n-1) \cdot n$
Summation or sigma notation – The sum of the first n terms of a sequence is represented by

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + a_4 + \cdots + a_n$$

where i is called the index of summation, n is the upper limit of summation, and 1 is the lower limit of summation
Infinite series – The sum of all the terms of an infinite sequence denoted by

$$a_1 + a_2 + a_3 + \ldots + a_i + \ldots = \sum_{i=1}^{\infty} a_i$$

Finite series or *n*th partial sum – The sum of the first n terms of an infinite sequence denoted by

$$a_1 + a_2 + a_3 + a_4 + \ldots + a_n = \sum_{i=1}^{n} a_i$$

Section 8.2

Arithmetic sequence – A sequence in which the differences between consecutive terms are the same
Common difference – The difference d between consecutive terms of an arithmetic sequence. That is,
$d = a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \ldots$

Section 8.3

Geometric sequence – A sequence in which the ratios of consecutive terms are the same
Common ratio – The ratio r between consecutive terms of a geometric sequence. That is,

$$r = \frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_4}{a_3} = \cdots, \qquad r \neq 0$$

Infinite geometric series or geometric series – The summation of the terms of an infinite geometric sequence

Section 8.4

Mathematical induction – A form of mathematical proof in which it must be shown for a statement, P_n, involving the positive integer n that P_1 is true and that the truth of P_k implies the truth of P_{k+1} for every positive k
First differences – The differences found by subtracting consecutive terms of a sequence
Second differences – The differences found by subtracting consecutive first differences

Section 8.5

Binomial coefficients – The coefficients of a binomial expansion
Binomial Theorem – In the expansion of $(x + y)^n = x^n + nx^{n-1}y + \ldots + {}_nC_r x^{n-r}y^r + \ldots + nxy^{n-1} + y^n$ the coefficient of $x^{n-r}y^r$ is

$${}_nC_r = \frac{n!}{(n-r)!r!}$$

Pascal's Triangle – A triangular pattern, named for the French mathematician Blaise Pascal, in which the first and last numbers in each row are 1and every other number in each row is formed by adding the two numbers immediately above the number. These numbers are precisely the same as the coefficients of binomial expansions.

Section 8.6
Fundamental Counting Principle – Let E_1 and E_2 be two events. The first event E_1 can occur in m_1 different ways. After E_1 has occurred, E_2 can occur in m_2 different ways. The number of ways that the two events can occur is $m_1 \cdot m_2$
Permutation – An ordering of n different elements such that one element is first, one is second, one is third, and so on
Distinguishable permutations – Suppose a set of n objects has n_1 of one kind of object, n_2 of a second kind, n_3 of a third kind, and so on, with $n = n_1 + n_2 + n_3 + \ldots + n_k$. Then the number of distinguishable permutations of the n objects is

$$\frac{n!}{n_1! \cdot n_2! \cdot n_3! \cdot \cdots \cdot n_k!}$$

Combination – A subset of a set of n elements in which the order is not important

Section 8.7
Experiment – Any happening for which the result is uncertain
Outcomes – The possible results of an experiment
Sample space – The set of all possible outcomes of an experiment
Event – Any sub-collection of a sample space
Probability – A measure of the likelihood that an event will occur based on chance
Mutually exclusive – Two events A and B (from the same sample space) are mutually exclusive if A and B have no outcomes in common
Independent events – Two events are independent if the occurrence of one has no effect on the occurrence of the other
Complement of an event – The collection of all outcomes in the sample space that are not in the event