

Objective: Analyze the approximate error in the Trapezoidal Rule and Simpson's Rule.

Use the error formulas to find "n" such that the error in the approximation of the definite integral

$\int_0^1 \sin(x^2) dx$ is less than 0.00001 using the Trapezoidal Rule and Simpson's Rule.

ANSWER:

$$f(x) = \sin(x^2)$$

Trapezoidal Rule:

$$f''(x) = 2[-2x^2 \sin(x^2) + \cos(x^2)] \text{ in } [0,1]$$

$$|f''(x)| \text{ is maximum when } x = 1 \text{ and } |f''(1)| \approx 2.2853$$

$$\text{Error} \leq \frac{(1-0)^3}{12n^2} (2.2853) < 0.00001$$

$$n^2 > 19,044.17$$

$$n > 138.00$$

$$\text{Let } n = 139$$

Simpson's Rule:

$$f^{(4)} = (16x^4 - 12)\sin(x^2) - 48x^2 \cos(x^2) \text{ in } [0,1]$$

$$|f^{(4)}(x)| \text{ is maximum when } x \approx 0.852 \text{ and } |f^{(4)}(0.852)| \approx 28.4285$$

$$\text{Error} \leq \frac{(1-0)^5}{180n^4} (28.4285) < 0.00001$$

$$n^4 > 15,793.61$$

$$n > 11.21$$

$$\text{Let } n = 12$$