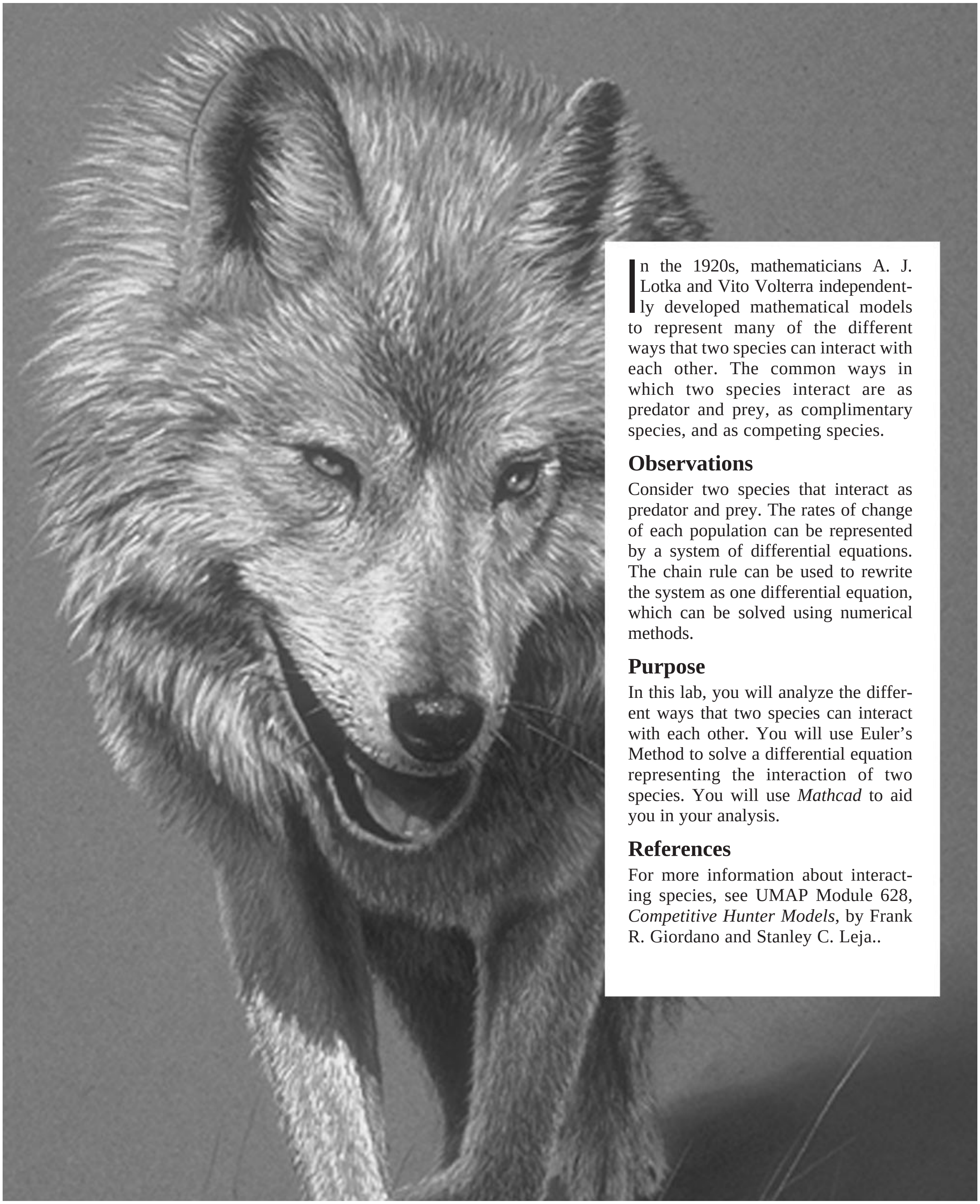
LAB 20

INTERACTING POPULATIONS

Euler's Method



In the 1920s, mathematicians A. J. Lotka and Vito Volterra independently developed mathematical models to represent many of the different ways that two species can interact with each other. The common ways in which two species interact are as predator and prey, as complimentary species, and as competing species.

Observations

Consider two species that interact as predator and prey. The rates of change of each population can be represented by a system of differential equations. The chain rule can be used to rewrite the system as one differential equation, which can be solved using numerical methods.

Purpose

In this lab, you will analyze the different ways that two species can interact with each other. You will use Euler's Method to solve a differential equation representing the interaction of two species. You will use *Mathcad* to aid you in your analysis.

References

For more information about interacting species, see UMAP Module 628, *Competitive Hunter Models*, by Frank R. Giordano and Stanley C. Leja..

Data

Consider a predator and prey relationship involving foxes and rabbits. The rabbits are a food source for the foxes, so the rabbits are the prey and the foxes are the predator in this relationship. Let x represent the number of rabbits, let y represent the number of foxes, and let t represent the time (in months). Then the rates of change of each population can be represented by the following system of differential equations. (In this system, a, b, m, and n are positive constants.)

$$\frac{dx}{dt} = ax - bxy \quad \text{and} \quad \frac{dy}{dt} = -my + nxy$$

These equations have many possible solutions that can be obtained by solving the following differential equation.

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-my + nxy}{ax - bxy}$$

The particular solutions depend on the initial values of x and y and on the values of a, b, m, and n.

Exercises

Name ______________________________

Date ____________________ Class ______________

Instructor ______________________________

1. ***Euler's Method.*** Consider the differential equation $y' = F(x, y)$ with the initial condition $y(x_0) = y_0$. At any point in the domain of F, $F(x_k, y_k)$ yields the slope of the solution at that point. Euler's Method gives a discrete set of estimates of the y-values of a solution of the differential equation using the iterative formula

$$y_{k+1} = y_k + F(x_k, y_k)\Delta x$$

where $\Delta x = x_{k+1} - x_k$. Given that a certain predator and prey relationship can be modeled by

$$y' = \frac{-0.3y + 0.006xy}{0.8x - 0.04xy}$$

with $(x_0, y_0) = (55, 10)$ where the prey population x and predator population y are measured in hundreds, use Euler's Method to approximate the value of y and complete the table below. Describe the change in the predator population y as the prey population x grows. Which population appears to be changing at a faster rate?

k	***x***	***y***
0	55	10
1	80	
2	105	

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2. ***The Magnitude of Δx.*** In Exercise 1, the y-values were estimated using $\Delta x = 25$. Describe how decreasing the magnitude of Δx affects the accuracy of Euler's Method.

3. ***Using Mathcad to do Euler's Method.*** Use *Mathcad* to approximate the value of y for the predator-prey model given in Exercise 1 for $55 \le x \le 105$ and $\Delta x = 5$. Compare the results of each table and explain any differences. Did decreasing the magnitude of Δx affect the accuracy of Euler's Method? If so, what was the effect?

4. ***Graphing the Results.*** Use *Mathcad* to graph the results of Exercises 1 and 3 together. Which graph do you think better models the interacting populations? *Mathcad* can be used to solve differential equations. Use *Mathcad* to graph a solution of the model given in Exercise 1 and compare it to the graphs of the other results.

5. ***Reasonable Assumptions?*** Some of the assumptions made about the predator and prey populations described in this lab's Data are listed below. Discuss the reasonableness of each assumption.

- The prey has an abundant food supply.

- The predator feeds exclusively on the prey.

- The environment can support unlimited quantities of the prey.

- There is no need to consider females and males separately.

6. ***Considering the Limits.*** What if the environment can't support unlimited quantities of the prey? Does this limit the size of the predator population? Explain.

7. ***Do They Die Out?*** Consider the set of differential equations

$$\frac{dx}{dt} = ax - bxy \text{ and } \frac{dy}{dt} = -my + nxy$$

modeling a prey population and a predator population, respectively. Is there a point where the prey or predator populations die out? If so, list a point where this situation occurs. What do you think happens to the prey population if the predator population is eliminated? Is there a point where both populations die out?

8. ***Using Different Initial Values.*** Use *Mathcad* to repeat Exercise 1 for different initial values. Explain what happens to the quantities of both populations when the initial values are the same.