Section 9.3 The Inverse of a Square Matrix

Objective: In this lesson you learned how to verify that two matrices are inverses of each other and find inverses of matrices and how to use inverse matrices to solve systems of linear equations.

I. The Inverse of a Matrix (Pages 660–661)

To verify that a matrix \( B \) is the inverse of the matrix \( A \), . . .

If a matrix \( A \) has an inverse, \( A \) is called \( \text{________________________} \) or \( \text{nonsingular} \). Otherwise, \( A \) is called \( \text{________________________} \).

To have an inverse, a matrix must be \( \text{________________________} \). Not all square matrices have inverses. However, if a matrix does have an inverse, that inverse is \( \text{________________________} \).

II. Finding Inverse Matrices (Pages 662–663)

To find the inverse of a square matrix \( A \) of order \( n \), . . .

Example 1: Find the inverse of the matrix \( A = \begin{bmatrix} 1 & 2 & 4 \\ 1 & 0 & 2 \\ 2 & 3 & 6 \end{bmatrix} \).
III. The Inverse of a $2 \times 2$ Matrix  (Page 664)

If $A$ is a $2 \times 2$ matrix given by $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $A$ is invertible if and only if $ad - bc \neq 0$. Moreover, if this condition is true, the inverse of $A$ is given by:

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

The denominator is called the determinant of the $2 \times 2$ matrix $A$.

Example 2: Find the inverse of the matrix $B = \begin{bmatrix} 3 & 9 \\ -2 & -7 \end{bmatrix}$.

IV. Systems of Linear Equations  (Page 665)

If $A$ is an invertible matrix, the system of linear equations represented by $AX = B$ has a unique solution given by

$$X = A^{-1}B$$

Example 3: Use an inverse matrix to solve (if possible) the system of linear equations:

$$\begin{cases} 12x + 8y = 416 \\ 3x + 5y = 152 \end{cases}$$