

## Section 7.3 Vectors in the Plane

**Objective:** In this lesson you learned how to write the component forms of vectors, perform basic vector operations, and find the direction angles of vectors.

Course Number

Instructor

Date

### Important Vocabulary

Define each term or concept.

Vector  $\mathbf{v}$  in the plane

Standard position

Zero vector

Unit vector

Standard unit vectors

Direction angle

### I. Introduction (Page 522)

A **directed line segment** has an \_\_\_\_\_ and a \_\_\_\_\_.

The magnitude of the directed line segment  $\overrightarrow{PQ}$ , denoted by \_\_\_\_\_, is its \_\_\_\_\_. The magnitude of a directed line segment can be found by . . .

#### *What you should learn*

How to represent vectors as directed line segments

### II. Component Form of a Vector (Page 523)

A vector whose initial point is at the origin  $(0, 0)$  can be uniquely represented by the coordinates of its terminal point  $(v_1, v_2)$ . This is the \_\_\_\_\_, written  $\mathbf{v} = \langle v_1, v_2 \rangle$ , where  $v_1$  and  $v_2$  are the \_\_\_\_\_ of  $\mathbf{v}$ .

The component form of the vector with initial point  $P = (p_1, p_2)$  and terminal point  $Q = (q_1, q_2)$  is

$$\overrightarrow{PQ} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}} = \mathbf{v}.$$

#### *What you should learn*

How to write the component forms of vectors

The **magnitude** (or length) of  $\mathbf{v}$  is:

$$\|\mathbf{v}\| = \sqrt{\quad} = \sqrt{\quad}$$

**Example 1:** Find the component form and magnitude of the vector  $\mathbf{v}$  that has (1, 7) as its initial point and (4, 3) as its terminal point.

### III. Vector Operations (Pages 524–526)

Geometrically, the product of a vector  $\mathbf{v}$  and a scalar  $k$  is . . .

If  $k$  is positive,  $k\mathbf{v}$  has the \_\_\_\_\_ direction as  $\mathbf{v}$ , and if  $k$  is negative,  $k\mathbf{v}$  has the \_\_\_\_\_ direction.

To add two vectors geometrically, . . .

This technique is called the \_\_\_\_\_ for vector addition because the vector  $\mathbf{u} + \mathbf{v}$ , often called the \_\_\_\_\_ of vector addition, is . . .

Let  $\mathbf{u} = \langle u_1, u_2 \rangle$  and  $\mathbf{v} = \langle v_1, v_2 \rangle$  be vectors and let  $k$  be a scalar (a real number). **Vector addition**, that is, the sum of  $\mathbf{u}$  and  $\mathbf{v}$ , is defined as the following vector:

$$\mathbf{u} + \mathbf{v} = \underline{\hspace{2cm}}$$

Scalar multiplication, that is, the scalar multiple of  $k$  times  $\mathbf{u}$ , is defined as the following vector:

$$k\mathbf{u} = \underline{\hspace{2cm}}$$

**Example 2:** Let  $\mathbf{u} = \langle 1, 6 \rangle$  and  $\mathbf{v} = \langle -4, 2 \rangle$ . Find:

- $3\mathbf{u}$
- $\mathbf{u} + \mathbf{v}$

***What you should learn***

How to perform basic vector operations and represent them graphically

Let  $\mathbf{u}$ ,  $\mathbf{v}$ , and  $\mathbf{w}$  be vectors and  $c$  and  $d$  be scalars. Complete the following properties of vector addition and scalar multiplication:

1.  $\mathbf{u} + \mathbf{v} =$  \_\_\_\_\_
2.  $(\mathbf{u} + \mathbf{v}) + \mathbf{w} =$  \_\_\_\_\_
3.  $\mathbf{u} + \mathbf{0} =$  \_\_\_\_\_
4.  $\mathbf{u} + (-\mathbf{u}) =$  \_\_\_\_\_
5.  $c(d\mathbf{u}) =$  \_\_\_\_\_
6.  $(c + d)\mathbf{u} =$  \_\_\_\_\_
7.  $c(\mathbf{u} + \mathbf{v}) =$  \_\_\_\_\_
8.  $1(\mathbf{u}) =$  \_\_\_\_\_
9.  $0(\mathbf{u}) =$  \_\_\_\_\_
10.  $\|c\mathbf{v}\| =$  \_\_\_\_\_

#### IV. Unit Vectors (Pages 526–527)

To find a unit vector  $\mathbf{u}$  that has the same direction as a given nonzero vector  $\mathbf{v}$ , . . .

***What you should learn***  
How to write vectors as linear combinations of unit vectors

In this case, the vector  $\mathbf{u}$  is called a \_\_\_\_\_  
\_\_\_\_\_.

**Example 3:** Find a unit vector in the direction of  $\mathbf{v} = \langle -8, 6 \rangle$ .

Let  $\mathbf{v} = \langle v_1, v_2 \rangle$ . Then the standard unit vectors can be used to represent  $\mathbf{v}$  as  $\mathbf{v} =$  \_\_\_\_\_, where the scalar  $v_1$  is called the \_\_\_\_\_ and the scalar  $v_2$  is called the \_\_\_\_\_. The vector sum  $v_1\mathbf{i} + v_2\mathbf{j}$  is called a \_\_\_\_\_ of the vectors  $\mathbf{i}$  and  $\mathbf{j}$ .

**Example 4:** Let  $\mathbf{v} = \langle -5, 3 \rangle$ . Write  $\mathbf{v}$  as a linear combination of the standard unit vectors  $\mathbf{i}$  and  $\mathbf{j}$ .

**Example 5:** Let  $\mathbf{v} = 3\mathbf{i} - 4\mathbf{j}$  and  $\mathbf{w} = 2\mathbf{i} + 9\mathbf{j}$ . Find  $\mathbf{v} + \mathbf{w}$ .

**V. Direction Angles** (Page 528)

If  $\mathbf{u}$  is a unit vector and  $\mathbf{q}$  is its direction angle, the terminal point of  $\mathbf{u}$  lies on the unit circle and

$$\mathbf{u} = \langle x, y \rangle = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

Now, if  $\mathbf{v}$  is any vector that makes an angle  $\mathbf{q}$  with the positive  $x$ -axis, it has the same direction as  $\mathbf{u}$  and

$$\mathbf{v} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

If  $\mathbf{v}$  can be written as  $\mathbf{v} = a\mathbf{i} + b\mathbf{j}$ , then the direction angle  $\mathbf{q}$  for  $\mathbf{v}$  can be determined from  $\tan \mathbf{q} = \underline{\hspace{2cm}}$ .

**Example 6:** Let  $\mathbf{v} = -4\mathbf{i} + 5\mathbf{j}$ . Find the direction angle for  $\mathbf{v}$ .

***What you should learn***  
How to find the direction angles of vectors

**VI. Applications of Vectors** (Pages 529–530)

Describe several real-life applications of vectors.

***What you should learn***  
How to use vectors to model and solve real-life problems

**Homework Assignment**

Page(s)

Exercises