

## Section 8.4 Vectors and Dot Products

**Objective:** In this lesson you learned how to find the dot product of two vectors and find the angle between two vectors.

Course Number

Instructor

Date

**Important Vocabulary** Define each term or concept.

**Angle between two nonzero vectors**

**Orthogonal**

### I. The Dot Product of Two Vectors (Pages 612–613)

The **dot product** of  $\mathbf{u} = \langle u_1, u_2 \rangle$  and  $\mathbf{v} = \langle v_1, v_2 \rangle$  is \_\_\_\_\_ . This product yields a \_\_\_\_\_ .

Let  $\mathbf{u}$ ,  $\mathbf{v}$ , and  $\mathbf{w}$  be vectors in the plane or in space and let  $c$  be a scalar. Complete the following properties of the dot product:

- $\mathbf{u} \bullet \mathbf{v} =$  \_\_\_\_\_
- $\mathbf{0} \bullet \mathbf{v} =$  \_\_\_\_\_
- $\mathbf{u} \bullet (\mathbf{v} + \mathbf{w}) =$  \_\_\_\_\_
- $\mathbf{v} \bullet \mathbf{v} =$  \_\_\_\_\_
- $c(\mathbf{u} \bullet \mathbf{v}) =$  \_\_\_\_\_ = \_\_\_\_\_

**Example 1:** Find the dot product:  $\langle 5, -4 \rangle \bullet \langle 9, -2 \rangle$ .

#### *What you should learn*

How to find the dot product of two vectors and use the Properties of the Dot Product

### II. The Angle Between Two Vectors (Pages 613–615)

If  $\mathbf{q}$  is the angle between two nonzero vectors  $\mathbf{u}$  and  $\mathbf{v}$ , then  $\mathbf{q}$  can be determined from \_\_\_\_\_ .

**Example 2:** Find the angle between  $\mathbf{v} = \langle 5, -4 \rangle$  and  $\mathbf{w} = \langle 9, -2 \rangle$ .

#### *What you should learn*

How to find the angle between two vectors and how to determine whether two vectors are orthogonal

An alternative way to calculate the dot product between two vectors  $\mathbf{u}$  and  $\mathbf{v}$ , given the angle  $\theta$  between them, is

\_\_\_\_\_.

Two vectors  $\mathbf{u}$  and  $\mathbf{v}$  are orthogonal if \_\_\_\_\_.

**Example 3:** Are the vectors  $\mathbf{u} = \langle 1, -4 \rangle$  and  $\mathbf{v} = \langle 6, 2 \rangle$  orthogonal?

### III. Finding Vector Components (Pages 615–616)

Let  $\mathbf{u}$  and  $\mathbf{v}$  be nonzero vectors such that  $\mathbf{u} = \mathbf{w}_1 + \mathbf{w}_2$ , where  $\mathbf{w}_1$  and  $\mathbf{w}_2$  are orthogonal and  $\mathbf{w}_1$  is parallel to (or a scalar multiple of)  $\mathbf{v}$ . The vectors  $\mathbf{w}_1$  and  $\mathbf{w}_2$  are called \_\_\_\_\_. The vector  $\mathbf{w}_1$  is the **projection** of  $\mathbf{u}$  onto  $\mathbf{v}$  and is denoted by \_\_\_\_\_. The vector  $\mathbf{w}_2$  is given by \_\_\_\_\_.

Let  $\mathbf{u}$  and  $\mathbf{v}$  be nonzero vectors. The projection of  $\mathbf{u}$  onto  $\mathbf{v}$  is given by  $\text{proj}_{\mathbf{v}} \mathbf{u} =$  \_\_\_\_\_.

### IV. Work (Page 617)

The **work**  $W$  done by a constant force  $\mathbf{F}$  as its point of application moves along the vector  $\overrightarrow{PQ}$  is given by either of the following:

- 1.
- 2.

**What you should learn**  
How to write a vector as the sum of two vector components

**What you should learn**  
How to use vectors to find the work done by a force

#### Homework Assignment

Page(s)

Exercises