

Section 10.5 Applications of Matrices and Determinants

Objective: In this lesson you learned how to use Cramer's Rule to solve systems of linear equations and how to use determinants and matrices to model and solve problems.

Course Number

Instructor

Date

I. Cramer's Rule (Pages 754–756)

Cramer's Rule states that if a system of n linear equations in n variables has a coefficient matrix A with a nonzero determinant $|A|$, the solution of the system is

$$x_1 = \frac{|A_1|}{|A|}, \quad x_2 = \frac{|A_2|}{|A|}, \dots, x_n = \frac{|A_n|}{|A|}$$

where the i th column of A_i is _____

_____.

Cramer's Rule does not apply if the determinant of the coefficient matrix is _____, in which case the system has either no solution or _____.

Example 1: Use Cramer's Rule to solve the system of linear equations.

$$\begin{cases} 2x + y + z = 6 \\ -x - y + 3z = 1 \\ y - 2z = -3 \end{cases}$$

II. Area of a Triangle (Page 757)

The area of a triangle with vertices (x_1, y_1) , (x_2, y_2) , and (x_3, y_3) is

$$\text{Area} = \pm \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

where the symbol \pm indicates that the appropriate sign should be chosen to yield a positive area.

Example 2: Find the area of a triangle whose vertices are $(-3, 1)$, $(2, 4)$, and $(5, -3)$.

What you should learn

How to use Cramer's Rule to solve systems of linear equations

What you should learn

How to use determinants to find the areas of triangles

III. Lines in a Plane (Pages 758–759)

Three points (x_1, y_1) , (x_2, y_2) , and (x_3, y_3) are collinear (lie on the same line) if and only if

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0.$$

What you should learn
How to use a determinant to find an equation of a line passing through two points

Example 3: Determine whether the points $(-2, 4)$, $(0, 3)$, and $(8, -1)$ are collinear.

An equation of the line passing through the distinct points (x_1, y_1) and (x_2, y_2) is given by

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0.$$

Example 4: Find an equation of the line passing through the points $(-2, 9)$ and $(3, -1)$.

IV. Cryptography (Pages 760–762)

A cryptogram is . . .

What you should learn
How to use matrices to code and decode messages

To use matrix multiplication to encode and decode messages, . . .

Homework Assignment

Page(s)

Exercises