

Chapter 4 Rational Functions and Conics

Section 4.1

Rational function – A function that can be written in the form: $f(x) = \frac{N(x)}{D(x)}$, where $N(x)$ and $D(x)$ are

polynomials and $D(x)$ is not the zero polynomial

Vertical asymptote – The line $x = a$ is a vertical asymptote of the graph of f if $f(x) \rightarrow \infty$ or $f(x) \rightarrow -\infty$ as $x \rightarrow a$, either from the right or from the left.

Horizontal asymptote – The line $y = b$ is a horizontal asymptote of the graph of f if $f(x) \rightarrow b$ as $x \rightarrow \infty$ or $x \rightarrow -\infty$

Section 4.2

Slant (or oblique) asymptote – If the degree of the numerator of a rational function is exactly one more than the degree of the denominator, then the line determined by the quotient of the denominator into the numerator is a slant asymptote of the graph of the rational function

Section 4.3

Partial fraction – A rational expression can be written as the sum of two or more simpler fractions. Each simpler fraction is called a partial fraction.

Partial fraction decomposition – A rational expression written as the sum of two or more simpler rational expressions

Basic equation – The equation that results when both sides of the form of a decomposition is multiplied by the least common denominator

Section 4.4

Conic section or conic – The figure formed by the intersection of a plane and a double-napped cone

Degenerate conic – Figure formed when a plane passes through the vertex of a double-napped cone

Parabola – The set of all points (x, y) in a plane that are equidistant from a fixed line and a fixed point not on the line.

Directrix – A fixed line in the plane from which each point on a parabola is the same distance as the distance from the point to a fixed point in the plane

Focus – A fixed point in the plane from which each point on a parabola is the same distance as the distance from the point to a fixed line in the plane

Standard form of the equation of a parabola – The standard form of the equation of a parabola with vertex at $(0, 0)$ and directrix $y = -p$ is

$$x^2 = 4py, \quad p \neq 0$$

For directrix $x = -p$, the equation is

$$y^2 = 4px, p \neq 0$$

Ellipse – The set of all points (x, y) in a plane the sum of whose distances from two distinct fixed points (foci) is constant

Foci – Distinct fixed points in the plane such that the sum of the distances from each point on an ellipse is constant

Vertices – Points of intersection of the line through the foci and the ellipse

Major axis – The chord connecting the vertices of an ellipse

Center – The midpoint of the major axis

Minor axis – The chord perpendicular to the major axis at the center of an ellipse

Standard form of the equation of an ellipse – The standard form of the equation of an ellipse centered at the origin with major and minor axes of lengths $2a$ and $2b$ (where $0 < b < a$) is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{or} \quad \frac{x^2}{b^2} + \frac{y^2}{a^2} = 1.$$

Hyperbola – The set of all points (x, y) in a plane the difference of whose distances from two distinct fixed points is a positive constant

Branches – The two disconnected parts of the graph of a hyperbola

Transverse axis – The line segment connecting the vertices of a hyperbola

Standard form of the equation of a hyperbola – The standard form of the equation of a hyperbola with center at the origin (where $a \neq 0$ and $b \neq 0$) is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{or} \quad \frac{y^2}{a^2} - \frac{x^2}{b^2} = 1 .$$

Conjugate axis – The line segment of length $2b$ joining $(0, b)$ and $(0, -b)$ [or $(-b, 0)$ and $(b, 0)$]

Asymptotes of a hyperbola- The asymptotes of a hyperbola with center as $(0, 0)$ are $y = \pm \frac{b}{a} x$ for a

horizontal transverse axis or $y = \pm \frac{a}{b} x$ for a vertical transverse axis