Chapter 1: Linear Equations and Functions

Exercise 1.1

1. \[
4x - 7 = 8x + 2
\]
   \[
4x - 7 + 7 - 8x = 8x + 2 + 7 - 8x
\]
   \[
-4x = 9
\]
   \[
x = -\frac{9}{4}
\]

3. \[
x + 8 = 8(x + 1)
\]
   \[
x + 8 = 8x + 8
\]
   \[
x - 8x = 8 - 8
\]
   \[
-7x = 0
\]
   \[
x = 0
\]

5. \[
8(x - 2) = 6(3x - 4)
\]
   \[
8x - 16 = 18x - 24
\]
   \[
8x - 18x = -24 + 16
\]
   \[
-10x = -8
\]
   \[
x = \frac{-8}{-10} = \frac{4}{5}
\]

7. \[
-\frac{3x}{4} = 24
\]
   \[
-3x = 4(24) = 96
\]
   \[
x = -32
\]

9. \[
\frac{5x}{6} = 8
\]
   \[
6 \left( \frac{5}{6} x \right) = 6 \left( \frac{8}{3} \right)
\]
   \[
5x = 16
\]
   \[
x = \frac{16}{5}
\]

11. \[
2(x - 7) = 5(x + 3) - x
\]
    \[
2x - 14 = 5x + 15 - x
\]
    \[
2x - 5x + x = 15 + 14
\]
    \[
-2x = 29
\]
    \[
x = -\frac{29}{2}
\]

13. \[
\frac{5x}{2} - 4 = \frac{2x - 7}{6}
\]
    \[
6 \left( \frac{5}{2} x - 4 \right) = 6 \left( \frac{2x - 7}{6} \right)
\]
    \[
15x - 24 = 2x - 7
\]
    \[
15x - 2x = 24 - 7
\]
    \[
13x = 17
\]
    \[
x = \frac{17}{13}
\]

15. \[
\frac{5x - 1}{9} = \frac{5(x - 1)}{6}
\]
    \[
18 \left( \frac{5x - 1}{9} \right) = 18 \left( \frac{5x - 5}{6} \right)
\]
    \[
10x - 2 = 15x - 15
\]
    \[
10x - 15x = 2 - 15
\]
    \[
-5x = -13
\]
    \[
x = \frac{13}{5}
\]

17. \[
x + \frac{1}{3} = \frac{2 \left( x - \frac{2}{3} \right) - 6x}{3}
\]
    \[
x + \frac{1}{3} = \frac{2x - \frac{4}{3} - 6x}{3}
\]
    \[
x + \frac{1}{3} = \frac{6x - 4 - 18x}{3}
\]
    \[
x + 18x - 6x = -4 - 1
\]
    \[
15x = -5
\]
    \[
x = \frac{-5}{15} = \frac{1}{3}
\]

19. \[
\frac{33 - x}{5x} = 5x(2)
\]
    \[
33 - x = 10x
\]
    \[
x - 10x = -33
\]
    \[
-11x = -33
\]
    \[
x = 3
\]
    \[
\text{Check: } \frac{33 - 3}{5(3)} = 2
\]
    \[
\frac{30}{15} = 2
\]
    \[
2 = 2
\]
    \[
x = 3 \text{ is the solution.}
\]

21. \[
\frac{3}{x - 5} = \frac{3}{2x + 13}(x - 5)
\]
    \[
3 \left( \frac{3}{x - 5} \right) = 3 \left( \frac{2x + 13}{2(2x + 13)} \right)
\]
    \[
6x + 39 = 7(x - 5)
\]
    \[
6x - 7x = -35 - 39
\]
    \[
x = 74
\]
    \[
\text{Check: } \frac{3}{74 - 5} = \frac{3}{2(74) + 13}
\]
    \[
\frac{3}{69} = \frac{1}{161}
\]
    \[
\frac{3}{23} = \frac{1}{23}
\]
    \[
x = 74 \text{ is the solution.}
\]
23. \[ \frac{2x}{x-1} + \frac{1}{3} = \frac{5}{6} - \frac{1}{x-1} \]
Multiply each term by 6(x - 1).
12x + 2x - 6 = (5x - 5) - 6
12x + 2x - 5x = -5 - 6 + 2
9x = -9
x = -1
Check:
\[
\frac{2(-1)}{6(-1)} + \frac{1}{3} = \frac{5}{6} - \frac{1}{6(-1)}
\]
\[
\frac{2}{3} + \frac{1}{3} = \frac{5}{6} + \frac{1}{6}
\]
\[
\frac{2}{3} + \frac{1}{3} = \frac{3}{3} + \frac{1}{3}
\]
x = -1 is the solution.

25. \[ \frac{2x}{2x + 5} = \frac{2}{5} \]
Multiply each term by 6(2x + 5).
12x = 8x + 20 - 15
12x - 8x = 20 - 15
4x = 5 or x = \frac{5}{4}
Check:
\[
\frac{2(\frac{5}{4})}{2(\frac{5}{4}) + 5} = \frac{2}{5} - \frac{5}{4(\frac{5}{4}) + 10}
\]
\[
\frac{2}{\frac{5}{4} + 5} = \frac{2}{3} - \frac{5}{4(\frac{5}{4}) + 10}
\]
\[
\frac{2}{\frac{5}{4} + 5} = \frac{2}{3} - \frac{5}{15} + \frac{1}{3}
\]
x = \frac{5}{4} is the solution.

27. \[ -3.259x - 8.638 = -3.8(8.625x + 4.917) \]
3.259x - 8.638 = -32.775x - 18.6846
3.259x + 32.775x = 8.638 - 18.6846
36.034x = -10.0466
x = -\frac{10.0466}{36.034} = -0.279

29. \[ 0.000316x + 9.18 = 2.1(3.1 - 0.0029x) - 4.68 \]
0.000316x + 9.18 = 6.51 - 0.00609x - 4.68
0.000316x + 0.00609x = 6.51 - 4.68 - 9.18
0.006406x = -7.35
x = -\frac{7.35}{0.006406} \approx -1147.362

31. \[ 3x - 4y = 15 \]
\[ -4y = 3x + 15 \]
y = \frac{3x + 15}{-4}
y = \frac{-3x + 15}{-4}
y = \frac{3x - 15}{4}

33. \[ 2\left(9x + \frac{3}{2}\right) = 2(11) \]
18x + 3y = 22
3y = -18x + 22
y = -6x + \frac{22}{3}

35. \[ I = Prt \]
\[ \frac{I}{rt} = \frac{P}{rt} \]
\[ P = \frac{I}{rt} \]

37. \[ 93 + 69 + 89 + 97 + FE + FE = 90 \]
\[ 6 \]
\[ 2FE + 348 = 540 \]
\[ 2FE = 192 \]
\[ FE = 96 \]
A 96 is the lowest grade that can be earned on the final.

39. a. \[ 3w + 110 = 11(66 - 20) \]
3w + 110 = 506
3w = 396
w = 132
b. \[ 3(160) + 110 = 11(h - 20) \]
480 + 110 + 220 = 11h
11h = 810
h = 73.6 inches

41. \[ p = 75.4509 - 0.706948t \]
a. \[ p = 75.4509 - 0.706948(23) = 59.19\% \]
b. \[ 0 = 75.4509 - 0.706948t \]
\[ t = \frac{75.4509}{0.706948} \approx 106.7 \text{ years or in 2082.} \]

43. \$1 billion = $1000 million
1000 = 241.33 + 29t
29t = 1000 - 241.33
\[ t = 758.67 \]
t = 26 years or in 1988 + 26 = 2014

45. \[ x = \text{total served} \]
\[ x = \text{active + dropouts} \]
x = 6000 + \frac{1}{3}x
3x = 18000 + x
2x = 18000
x = 9000 youths served

47. a. \[ 7n - 12T = 52 \]
\[ -12T = -7n + 52 \]
\[ T = \frac{-7n + 52}{-12} \]
\[ T = \frac{7n - 52}{12} \]

b. 28 chirps in 15 seconds means 112 chirps in 1 minute (or 60 seconds).
\[ t = \frac{7(112) - 52}{12} \approx 61 \]
Temperature is 61°.

49. \[ A = P + Pr \]
6000 = P + P(0.1)(5)
6000 = 1.5P
\[ P = \frac{6000}{1.5} = 4000 \]
A = 6000
r = 0.1
\[ t = 5 \]
51. Let $x$ = amount in safe fund
   $120,000 - x$ = amount in risky fund
   Yield: $0.09x + 0.13(120,000 - x) = 12,000$
   $0.09x + 15,600 - 0.13x = 12,000$
   $-0.04x = -3600$
   $x = 90,000$
   The amount in the safe fund is $90,000.

53. Reduced salary: $2000 - 0.10(2000) = $1800
   Increased salary: $1800 + 0.20(1800) = $2160
   $160 = R\% \text{ of } 2000$
   $R = \frac{160}{2000} = 8\%$
   $160$ is an 8% increase.

55. $C + MU = SP$ ($MU = \text{Profit}$)
   $x_1 + 0.2x_1 = 480$
   $1.2x_1 = 480$
   $x_1 = 400$
   $x_2 - 0.2x_2 = 480$
   $0.8x_2 = 480$
   $x_2 = 600$
   Profit is $80 on $x_1$. Loss is $120 on $x_2$.
   The collector lost $40 on the transaction.

57. $\text{cost} + \text{markup} = \text{selling price}$
   $214.90 + 0.3x = x$
   $214.9 = x - 0.3x = 0.7x$
   $x = \frac{214.9}{0.7} = 307$
   The selling price is $307.

59. Wholesaler
   $154.98 + 0.1W = W$
   $0.9W = 154.98$
   $W = $172.20
   Retailer
   $172.20 + 0.3R = R$
   $0.7R = 172.20$
   $R = $246.00

Supplementary Exercises
1. Suppose we want to solve
   $2x - \frac{1}{3} = 3 + 1 \left( \frac{3x - 2}{5} \right)$
   a. Should we remove parentheses or clear the equation of fractions first?
   b. Answer True or False for each Step 1:
      I. $60x - 10 = 90 + 15(3x - 2)$
      II. $12x - 2 = 18 + 3 \left( \frac{3x - 2}{5} \right)$
      III. $2x - \frac{1}{3} = 3 + \frac{3}{2} x - \frac{1}{5}$

2. In a survey of 100 colleges it was found that enrollments fell into the following ranges:

<table>
<thead>
<tr>
<th>Number of Colleges</th>
<th>Range of Enrollment</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>10,000 to 15,000</td>
</tr>
<tr>
<td>60</td>
<td>15,001 to 25,000</td>
</tr>
<tr>
<td>25</td>
<td>25,001 to 30,000</td>
</tr>
</tbody>
</table>

Which of the following could be the average enrollment for these 100 colleges?
I. 18,000 II. 23,000 III. 25,000
   a. I only
   b. II only
   c. I and II only
   d. II and III only
   e. I, II, III

3. Item A costs the store $84 and the markup is 25% of the cost.
   Item B costs the store $60 and the markup is 25% of the selling price.
   What is the total selling price of A and B?
   a. $180
   b. $185
   c. $187
   d. $192
   e. none of these

Solutions to Supplementary Exercises
1. a. Remove parentheses first.
   b. $2x - \frac{1}{3} = 3 + \frac{3}{2} (3x - \frac{2}{5})$
      (We removed parentheses.)
      $60x - 10 = 90 + 45x - 6$
      (We multiplied both sides by 30.)
      Thus, I is False, II is True, and III is True.

2. Minimum average
   \[ \frac{15(10,000) + 60(15,001) + 25(25,001)}{15 + 60 + 25} = 16,751 \]
   Maximum average
   \[ \frac{15(15,000) + 60(25,000) + 25(30,000)}{15 + 60 + 25} = 24,750 \]
   I and II are the only possibilities.
   Answer: c

3. Cost + Markup = Selling price
   A: $84 + \frac{4}{3} (84) = 105$
   B: $60 + \frac{3}{4} x = x$
      $\frac{3}{4} x = 60$ or $x = 80$
      Total selling price = $105 + 80 = $185
      Answer: b
Exercise 1.2

1. \( y = 3x^3 \)
   \( y \) is a function of \( x \).

3. \( y^2 = 3x \)
   \( y \) is not a function of \( x \).
   If, for example, \( x = 3 \) there are two possible values for \( y \).

5. \( y \) is a function of \( x \) since for each \( x \) there is only one \( y \).
   \( D = \{1, 2, 3, 8, 9\} \),
   \( R = \{-4, 5, 16\} \)

7. \( R(x) = 8x - 10 \)
   a. \( R(0) = 8(0) - 10 = -10 \)
   b. \( R(2) = 8(2) - 10 = 6 \)
   c. \( R(-3) = 8(-3) - 10 = -34 \)
   d. \( R(1.6) = 8(1.6) - 10 = 2.8 \)

9. \( C(x) = 4x^2 - 3 \)
   a. \( C(0) = 4(0)^2 - 3 = -3 \)
   b. \( C(-1) = 4(-1)^2 - 3 = 1 \)
   c. \( C(-2) = 4(-2)^2 - 3 = 13 \)
   d. \( C\left(\frac{3}{2}\right) = 4\left(\frac{3}{2}\right)^2 - 3 = 6 \)

11. \( f(x) = x^3 - \frac{4}{x} \)
    a. \( f\left(\frac{1}{2}\right) = \left(-\frac{1}{2}\right)^3 - \frac{4}{\frac{1}{2}} = -\frac{1}{8} + 8 = \frac{63}{8} \)
    b. \( f(2) = 2^3 - 4 = 8 - 4 = 6 \)
    c. \( f(-2) = (-2)^3 - \frac{4}{-2} = -8 + 2 = -6 \)

13. \( f(x) = 1 + x + x^2 \)
    a. \( f(2+1) = f(3) = 1 + 3 + 3^2 = 13 \)
       \( f(2) + f(1) = 7 + 3 = 10 \)
       \( f(2) + f(1) \neq f(2+1) \)
    b. \( f(x + h) = 1 + (x + h) + (x + h)^2 \)
    c. No. This is equivalent to a.
    d. \( f(x) + h = 1 + x + x^2 + h \)
       No. \( f(x + h) \neq f(x) + h \)
    e. \( f(x + h) = 1 + (x + h) + (x + h)^2 \)
       \( = 1 + x + h + x^2 + 2xh + h^2 \)
       \( f(x) = 1 + x + x^2 \)
       \( f(x + h) - f(x) = h + 2xh + h^2 \)
       \( = h(1 + 2x + h) \)
       \( \frac{f(x + h) - f(x)}{h} = 1 + 2x + h \)

15. \( f(x) = x - 2x^2 \)
    a. \( f(x + h) = (x + h) - 2(x + h)^2 = -2x^2 - 4xh - 2h^2 + x + h \)
    b. \( f(x + h) - f(x) = (x + h) - 2(x + h)^2 - (x - 2x^2) \)
       \( = x + h - 2x^2 - 4xh - 2h^2 - x + 2x^2 \)
       \( = h - 4xh - 2h^2 \)
       \( \frac{f(x + h) - f(x)}{h} = \frac{h - 4xh - 2h^2}{h} = 1 - 4x - 2h \)

17. The vertical line test shows that graph (a) is a function of \( x \), and that graph (b) is not a function of \( x \).
19. Since (9, 10) and (5, 6) are points on the graph, (a) \( f(9) = 10 \) and (b) \( f(5) = 6 \).

21. a. The ordered pair \((a, b)\) satisfies the equation. Thus \( b = a^2 - 4a \).

b. The coordinates of \( Q = (1, -3) \). Since the point is on the curve, the coordinates satisfy the equation.

c. The coordinates of \( R = (3, -3) \). They satisfy the equation.

d. The \( x \) values are 0 and 4. These values are also solutions of \( x^2 - 4x = 0 \).

23. \( f(x) = 3x \) \( g(x) = x^3 \)

a. \((f + g)(x) = 3x + x^3\)

b. \((f - g)(x) = 3x - x^3\)

c. \((f \cdot g)(x) = 3x \cdot x^3 = 3x^4\)

d. \(\left(\frac{f}{g}\right)(x) = \frac{3x}{x^3} = \frac{3}{x^2}\)

25. \( f(x) = \sqrt{2x} \) \( g(x) = x^2 \)

a. \((f + g)(x) = \sqrt{2x} + x^2\)

b. \((f - g)(x) = \sqrt{2x} - x^2\)

c. \((f \cdot g)(x) = \sqrt{2x} \cdot x^2 = x^2 \sqrt{2x}\)

d. \(\left(\frac{f}{g}\right)(x) = \frac{\sqrt{2x}}{x^2}\)

27. \( f(x) = (x - 1)^3 \) \( g(x) = 1 - 2x \)

a. \((f \circ g)(x) = f(1 - 2x) = (1 - 2x - 1)^3 = -8x^3\)

b. \((g \circ f)(x) = g((x - 1)^3) = 1 - 2(x - 1)^3\)

c. \(f(f)(x) = f((x - 1)^3) = [(x - 1)^3 - 1]^3\)

d. \((f \cdot f)(x) = (x - 1)^3 \cdot (x - 1)^3 = (x - 1)^6\)

29. \( f(x) = 2\sqrt{x} \) \( g(x) = x^4 + 5 \)

a. \((f \circ g)(x) = f(x^4 + 5) = 2\sqrt{x^4 + 5}\)

b. \((g \circ f)(x) = g(2\sqrt{x}) = (2\sqrt{x})^4 + 5 = 16x^2 + 5\)

c. \(f(f)(x) = f(2\sqrt{x}) = 2\sqrt{2\sqrt{x}}\)

d. \((f \cdot f)(x) = 2\sqrt{2\sqrt{x}} = 4x\)

31. \( y = x^2 + 4 \)

There is no division by zero or square roots. Domain is all the reals, i.e., \( \{x : x \in \text{Reals}\} \).

Since \( x^2 \geq 0 \), \( x^2 + 4 \geq 4 \), the range is \( \geq 4 \) or \( \{y : y \geq 4\} \).

33. \( y = \sqrt{x} + 4 \)

There is no division by zero. To get a real number \( y \), we must have \( x + 4 \geq 0 \) or \( x \geq -4 \).

Domain: \( x \geq -4 \). The square root is always nonnegative. Thus, the range is \( \{y : y \in \text{Reals}, y \geq 0\} \).

35. \( D: \{x : x \geq 1, x \neq 2\} \)

37. \( D: \{x : 7 \leq x \leq 7\} \)

39. a. \( f(20) = 103,000 \) means it will take 20 years to pay off a debt of $103,000 (at $800 per month and 7.5% compounded monthly.)

b. \( f(5 + 5) = f(10) = 69,000; f(5) + f(5) = 80,000; \) No.

41. a. \( f(1950) = 16.5 \) means that in 1950 there were 16.5 workers supporting each person receiving Social Security benefits.

b. \( f(1990) = 3.4 \)

c. The points based on known data must be the same and those based on projections might be the same.

d. Domain: \( 1950 \leq t \leq 2050 \)

Range: \( 1.9 \leq n \leq 16.5 \)

43. a. Since the wind speed cannot be negative, \( s \geq 0 \).

b. \( f(10) = 45.694 + 1.75(10) = 29.26\sqrt{10} \)

At a temperature of \(-5^\circ F\) and a wind speed of 10 mph, the temperature feels like \(-29.33^\circ F\).

c. \( f(0) = 45.694, \) but \( f(0) \) should equal the air temperature, \(-5^\circ F\).

45. \( E = \frac{1 - 0.24t}{2 + t} \)

a. \( E \) is a function of \( t \).

b. The domain of the function is reals except \( t = -2 \).

c. Because negative times should not be considered, the domain is \( t \geq 0 \).

47. \( C = \frac{5}{9} F - \frac{160}{9} \)

a. \( C \) is a function of \( F \).

b. Mathematically, the domain is all reals.
c. Domain: \( \{ F : 32 \leq F \leq 212 \} \)
   Range: \( \{ C : 0 \leq C \leq 100 \} \)

d. \( C(40) = \frac{5}{9}(40) - \frac{160}{9} \)
   = \( \frac{200 - 160}{9} \)
   = \( \frac{40}{9} \)
   = 4.44°C

49. \( C(p) = \frac{7300p}{100 - p} \)

a. Domain: \( \{ p : 0 \leq p < 100 \} \)

b. \( C(45) = \frac{7300(45)}{100 - 45} = \frac{328,500}{55} = \$5972.73 \)

c. \( C(90) = \frac{7300(90)}{100 - 90} = \frac{657,000}{10} = \$65,700 \)

d. \( C(99) = \frac{7300(99)}{100 - 99} = \frac{722,700}{1} = \$722,700 \)

e. \( C(99.6) = \frac{7300(99.6)}{100 - 99.6} \)
   = \( \frac{727,080}{0.4} \)
   = \$1,817,700

In each case above, to remove \( p \% \) of the particulate pollution would cost \( C(p) \).

51. a. \( A \) is a function of \( x \).

b. \( A(2) = 2(50 - 2) = 96 \text{ sq ft} \)
   \( A(30) = 30(50 - 30) = 600 \text{ sq ft} \)

c. For the problem to have meaning we have \( 0 < x < 50 \).

53. a. \( P(q(t)) = P(1000 + 10t) \)
   \( = 180(1000 + 10t) - \frac{(1000 + 10t)^2}{100} - 200 \)
   = \( 169,800 + 1600t - t^2 \)

b. \( q(15) = 1000 + 10(15) = 1150 \)
   \( P(q(15)) = \$193,575 \)

55. \[ \text{length} = x \]
   \[ \text{width} = y \]
   \[ L = 2x + 2y \]
   \[ 1600 = xy \text{ or } y = \frac{1600}{x} \]
   \[ L = 2x + 2\left(\frac{1600}{x}\right) = 2x + \frac{3200}{x} \]

57. Revenue = (no. of people)(price per person)
   Example: \( R = 30 \times 10 \)
   \( R = 31 \times 9.80 \)
   \( R = 32 \times 9.60 \)
   Solution: \( R = (30 + x)(10 - 0.20x) \)

Supplementary Exercises

1. Suppose that \( f(x) = \frac{1}{x-3} + \sqrt{x-2} \). What is the domain of \( f(x) \)?

2. If \( f(a, b) = a^2 + b^2 \), then \( f(3, 4) = ? \)

3. If \( f(a, b) = a^2 + b^2 \), then \( f(1, f(1, 2)) = ? \)

4. If \( f(x) = x^2 - 2 \), then \( \frac{f(2 + a)}{f(2) + f(a)} = ? \)
   a. \( \frac{a^2 + 4a + 4}{a^2} \)
   b. \( \frac{a^2 + 4a + 2}{a^2} \)
   c. \( \frac{a^2 + 4a + 4}{a^2 + 2} \)
   d. \( \frac{a^2 + 4a + 2}{a^2 + 2} \)
   e. 1

Solutions to Supplementary Exercises

1. \( f(x) = \frac{1}{x-3} + \sqrt{x-2} \)
   Rule out division by zero: \( x \neq 3 \)
   \( \sqrt{x-2} \) must be \( \geq 0 \): \( x \geq 2 \)
   Domain: \( \{ x : x \geq 2, x \neq 3 \} \)

2. \( f(a, b) = a^2 + b^2 \)
   \( f(3, 4) = 3^2 + 4^2 = 25 \)

4. \( f(2 + a) = (2 + a)^2 - 2 \)
   \( = 4 + 4a + a^2 - 2 \)
   \( = a^2 + 4a + 2; \)
   \( f(2) = 2^2 - 2 = 2; \]
   \( f(a) = a^2 - 2 \)
   \( \frac{f(2 + a)}{f(2) + f(a)} = \frac{a^2 + 4a + 2}{2 + (a^2 - 2)} \)
   \( = \frac{a^2 + 4a + 2}{a^2} \)

(Note: The purpose of this question is to show that \( f(2 + a) \neq f(2) + f(a) \).)
Answer: b
Exercise 1.3

1. \(3x + 4y = 12\)
   - x-intercept: \(y = 0\) then \(x = 4\).
   - y-intercept: \(x = 0\) then \(y = 3\).

   ![Graph of 3x + 4y = 12]

3. \(2x - 3y = 12\)
   - x-intercept: \(y = 0\) then \(x = 6\).
   - y-intercept: \(x = 0\) then \(y = -4\).

   ![Graph of 2x - 3y = 12]

5. \(3x + 2y = 0\)
   - x-intercept: \(y = 0\) then \(x = 0\).
   - Likewise, y-intercept is \(y = 0\).

   ![Graph of 3x + 2y = 0]

7. \((2, 1)\) and \((3, 4)\)
   \[m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 1}{3 - 2} = 3\]

9. \((3, 2)\) and \((-1, 2)\)
   \[m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 2}{3 - (-1)} = \frac{0}{4} = 0\]

For problems 11–17, use \(y = mx + b\) to obtain slope and y-intercept.

11. \(y = \frac{7}{3}x - \frac{1}{4}, \ m = \frac{7}{3}, \ b = -\frac{1}{4}\)

13. \(y = 3\) or \(0x + 3, \ m = 0, \ b = 3\)

15. \(x = -8\)
   - Slope is undefined.
   - There is no y-intercept.

17. \(2x + 3y = 6\) or \(y = -\frac{2}{3}x + 2, \ m = -\frac{2}{3}, \ b = 2\)

19. \(m = \frac{1}{2}, \ b = 3\)
   \[y = \frac{1}{2}x + 3\]

21. \(m = -2, \ b = \frac{1}{2}\)
   \[y = -2x + \frac{1}{2}\]

23. \(P(2, 0), \ m = \frac{1}{2}\)
   \[y - y_1 = m(x - x_1)\]
   \[y = \frac{1}{2}x - 1\]

25. \(P(-1, 3), \ m = -2\)
   \[y - y_3 = m(x - (-1))\]
   \[y = -2x + 1\]
27. \( P(-1, 1) \), \( m \) is undefined

\[ x = -1 \]

\[ y \]

\[ P(x, -1) \]

29. \( P_1 = (3, 2) \), \( P_2 = (-1, -6) \)

\[ m = \frac{2 - 2}{3 - (-1)} = \frac{-1}{1} = \frac{1}{13} \]

\[ y - 2 = \frac{1}{13} (x - 7) \]

\[ y = \frac{1}{13} x - \frac{7}{13} + 3 \]

\[ y = \frac{1}{13} x + \frac{32}{13} \] or \(-x + 13y = 32\)

31. \( P_1 = (7, 3) \), \( P_2 = (-6, 2) \)

\[ m = \frac{2 - 3}{-6 - 7} = \frac{-1}{-13} = \frac{1}{13} \]

\[ y - 3 = \frac{1}{13} (x - 7) \]

\[ y = \frac{1}{13} x - \frac{7}{13} + 3 \]

\[ y = \frac{1}{13} x + \frac{32}{13} \] or \(-x + 13y = 32\)

33. The \( y \)-coordinates are the same.

The line is horizontal. \( y = 0 \)

35. \( 3x + 2y = 6 \)

\[ 2x - 3y = 6 \]

\[ y = \frac{3}{2} x + 3 \]

\[ y = \frac{2}{3} x - 2 \]

Lines are perpendicular since \( \left( -\frac{3}{2} \right) \left( \frac{2}{3} \right) = -1 \).

37. \( 6x - 4y = 12 \)

\[ 3x - 2y = 6 \]

\[ y = \frac{6}{4} x - \frac{12}{4} \]

\[ y = \frac{3}{2} x - 3 \]

or \( y = \frac{3}{2} x - 3 \)

Parallel. Lines are the same.

39. If \( 3x + 5y = 11 \), then \( y = -\frac{3}{5} x + \frac{11}{5} \). So, \( m = -\frac{3}{5} \).

A line parallel will have the same slope. Thus, \( m = -\frac{3}{5} \) and \( P = (-2, -7) \) gives

\[ y - (-7) = -\frac{3}{5} (x - (-2)) \]

which simplifies to

\[ y = \frac{3}{5} x - \frac{41}{5} \]

41. If \( 5x - 6y = 4 \), then \( y = \frac{5}{6} x - \frac{4}{6} \). Slope of the perpendicular line is \( \frac{6}{5} \). Thus \( m = -\frac{6}{5} \) and \( P = (3, 1) \) gives \( y - 1 = -\frac{6}{5} (x - 3) \) which simplifies to \( y = -\frac{6}{5} x + \frac{23}{5} \).

43. a. 

b. \( 0 = 360,000 - 1500x \)

\[ x = \frac{360000}{1500} = 240 \] months

In 240 months, the building will be completely depreciated.

c. (60, 270,000) means that after 60 months the value of the building will be $270,000.
Chapter 1: Linear Equations and Functions

45. a. \[ R_{FS} = 85.714x + 88.381 \]

b.

\[ R_{SP} = 17.1714t + 104.238 \]

c. If \( t = 0 \), then \( R_{FS} = 88.38 \).
If \( t = 0 \), then \( R_{SP} = 104.24 \).
The tables are exact. The equations are a best fit for the data. The equations are not exact.
d. The equations are based on past performance. No one knows the results of future investments.

47. \( y = 0.1369x - 5.091255 \)

a. \( m = 0.1369 \)  \( b = -5.091255 \)
b. \( x = 0 \) means that there was a negative amount of transactions. Restrictions are \( x > 0 \) and \( y \geq 0 \).
c. With an increase of 1 (thousand) terminals, the amount of transactions increases by \$0.1369\) (billion).

49. \( y = 0.0838x + 4.95 \) Both units are in dollars.

51. a. \( (m, f) \) is the reference. Slope = \( \frac{838}{1000} \) = 0.838.
\[ f - 17.300 = 0.838(m - 22.300) \text{ or } f = 0.838m - 1387.4 \]
b. \( f = 0.838(30,000) - 1387.4 = \$23,752.60 \)

53. \( (x, p) \) is the reference. \( (0, 85,000) \) is one point.
\[ m = \frac{85 - 1700}{6} = -1700 \]
\[ p - 85,000 = -1700(x - 0) \text{ or } p = -1700x + 85,000 \]

55. \( (t, R) \) is the ordered pair.
\[ P_1 = \left( \frac{7}{2}, 11 \right), P_2 = (6, 19) \]
\[ m = \frac{19 - 11}{6 - \frac{7}{2}} = \frac{8}{\frac{5}{2}} = 3.2 \]
\[ R - 19 = 3.2t - 6 \text{ or } R = 3.2t - 19.2 + 19 \text{ or } R = 3.2t - 0.2 \]

57. \( P_1 = (200, 25), P_2 = (250, 49) \)
\[ m = \frac{49 - 25}{250 - 200} = \frac{24}{50} = 0.48 \]
\[ y = 0.48x - 25 \text{ or } y = 0.48x - 71 \]

Supplementary Exercises

1. What is the slope of the line with equation \( 4x + 3y = 12 \)?
   a. -4
   b. 4
   c. \( -\frac{4}{3} \)
   d. \( \frac{4}{3} \)
   e. \( -\frac{3}{4} \)

2. In problem 1, what is the x-intercept?
   a. 12
   b. 4
   c. 3
   d. 0
   e. \( -\frac{4}{3} \)

3. The cost of making widgets is \$40 for 4 widgets \$54 for 6 widgets . If the cost is a linear function, what are the fixed costs?

4. A firm determines that the cost and revenue functions for a product are \( C = 5x + 140 \) and \( R = 7x \), respectively. If the firm will make a profit on the sale of 100 units, how much profit will the firm make on the sale of the 101st unit?

Solutions to Supplementary Exercises

1. \( 4x + 3y = 12 \)
\[ 3y = -4x + 12 \]
\[ y = -\frac{4}{3}x + 4 \]
Answer: c

2. x-intercept means \( y = 0 \).
\[ 4x = 12 \]
\[ x = 3 \]
Answer: c

3. We need the equation for the cost.
\[ m = \frac{54 - 40}{6 - 4} = 7 \]
\[ C - 40 = 7(x - 4) \text{ or } C = 7x + 12 \]
Fixed costs are \$12.

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4. We need the marginal profit or the slope of the profit function.
\[ P = R - C = 7x - (5x + 140) = 2x - 140 \]
Thus, \( m = 2 \).
The firm will make a profit of $2 on the sale of the 101st unit.

Exercise 1.4

1. Not linear

3. Not linear

5. Not linear

7. Not linear

9. Not linear

11. \( y = x^3 - 12x - 1 \)

13. \( y = 0.01x^3 + 0.3x^2 - 72x + 150 \)
   a. \( y = 0.01x^3 + 0.3x^2 - 72x + 150 \)
   b. 

15. \( y = \frac{x + 15}{x^2 + 400} \)
   a. 
   b. Standard Window

17. a. The equation is linear, so the graph will be a line. Use the intercepts to determine a window.
   b. Window: \( x\text{-min} = -5 \quad y\text{-min} = -0.06 \)
   \( x\text{-max} = 35 \quad y\text{-max} = 0.02 \)
   c. \( y = 0.001x - 0.03 \)
19 and 21. Complete graphs can be seen with different windows. A hint is to look at the equation and try to determine the max and/or min of \( y \). Also, find the \( x \)-intercepts.

19. \( y = -0.15(x - 10.2)^2 + 10 \)

There is no min. Max value of \( y = 10 \).

\[ x \text{-intercepts: } x = 10.2 \pm 8 \text{ or } x = 2.2 \text{ or } 18.2 \]

Suggested: \( x \)-min = –5 \( x \)-max = 25 \( y \)-min = –15 \( y \)-max = 15 as one choice.

21. \( y = x^3 + 19x^2 - 62x - 840 \)

If \( x = 0 \), \( y = -42 \).

Starting point: \( x \)-min = –25 \( x \)-max = 15 \( y \)-min = –50 \( y \)-max = 50

Use text answers for graph.

27. \( x^2 + 2y = 6 \)

\[ 2y = -x^2 + 6 \]

\[ y = \frac{1}{2} x^2 + 3 \]

29. \( f(x) = x^3 - 3x^2 + 2 \)

\[ f(1) = 1^3 - 3(1)^2 + 2 = 0 \]

\[ f\left(-\frac{3}{2}\right) = \left(-\frac{3}{2}\right)^3 - 3\left(-\frac{3}{2}\right)^2 + 2 = -8.125 \]

Use your graphing calculator, and evaluate the function at these two points. If either of your answers differ, can you explain the difference?

31. As \( x \) gets large, \( y \) approaches 12.

When \( x = 0 \), \( y = -12 \). \( x \)-intercepts at \( \pm 1 \).

Suggested: \( x \)-min = –6 \( y \)-min = –15 \( x \)-max = 6 \( y \)-max = 15

33. \( y = \frac{x^2 - x - 6}{x^2 + 5x + 6} = \frac{(x - 3)(x + 2)}{(x + 3)(x + 2)} \)

What happens to \( y \) as \( x \) approaches –3? –2?

Note: For problems 35–37, find the zeros and find the \( x \)-intercepts are equivalent statements. In part (a) we use a graphing calculator’s TRACE OR ZERO function and in part (b) we find the exact solutions.

35. a. Graphing calculator approximation to 4 decimal places: \( x \approx -1.1098, 8.1098 \)

b. \( x = \frac{7}{2} \pm \frac{1}{2} \sqrt{13} \) by the quadratic formula.

37. a. The graphing calculator gives exact answers of \( x = 1, x = 4 \) using the ZERO function.

b. When this numerator is zero the function is zero or when \( x = 1 \) or \( x = 4 \).
39. \( x \)-intercept: When \( R = 0 \), \( x = \frac{28,000}{0.08} = 350,000 \)

\( R \)-intercept: When \( x = 0 \), \( R = 28,000 \)

b. The coordinates mean that when a male’s salary is $50 thousand, a female’s salary is $40,536 thousand.

41. a. \( F = 0.838M - 1.364 \)

b. \( E \geq 0 \) when \( 0 \leq p \leq 100 \).

c. Using the first equation as the better fit, it predicts $325 (nearest dollar) while the actual is $305.

43. a. \( E = 10,000p - 100p^2 \)

b. Near \( p = 0 \), cost grows without bound.

c. The coordinates of the point mean that the cost of obtaining stream water with 1% of the current pollution levels would cost $282,150.

d. The \( p \)-intercept means that the cost of stream water with 100% of the current pollution levels would cost $0.

45. a. \( R = -0.031t^2 + 0.776t + 0.179 \)

b. \( R = -0.031t^2 + 0.776t + 0.179 \)

c. The revenue reaches a maximum during 1992 but zeros out in 2005. The model could not be valid until 2020.

47. a. \( C = \frac{285,000}{P} - 2850 \)

b. Near \( p = 0 \), cost grows without bound.
5. \[4x - y = 3\] Multiply first row by 3.
\[2x + 3y = 19\]
Add the two equations.
\[14x = 28\]
Solve for the variable.
\[x = \frac{28}{14} = 2\]
Substitute for this variable in either original equation and solve for the other variable.
\[4(2) - y = 3\]
\[-y = 3 - 8 = -5\]
The solution of the system is \(x = 2\) and \(y = 5\).

7. \[2x - y = 2\] Multiply first row by 4.
\[3x + 4y = 6\]
Add the two equations.
\[11x = 14\]
Solve for the variable.
\[x = \frac{14}{11}\]
Substitute for this variable in either original equation and solve for the other variable.
\[2\left(\frac{14}{11}\right) - y = 2\]
\[-y = 2 - \frac{28}{11}\]
The solution of the system is \(x = \frac{14}{11}\) and \(y = \frac{6}{11}\).

9. \[3x - 2y = 6\] Solve for \(y\).
\[4y = 8\]
\[y = \frac{8}{4} = 2\]
Substitute for this variable in first equation and solve for the other variable.
\[3x = 6 + 4 = 10\]
\[x = \frac{10}{3}\]
The solution of the system is \(x = \frac{10}{3}\) and \(y = 2\).

11. \[-4x + 3y = -5\] Multiply first equation by 3.
\[3x - 2y = 4\] Multiply second equation by 4.
\[-12x + 9y = -15\]
\[12x - 8y = 16\]
Add the two equations.
\[y = 1\]
Substitute for this variable in either original equation and solve for the other variable.
\[-4x + 3(1) = -5\]
\[-4x = -8\]
\[x = 2\]
The solution of the system is \(x = 2\) and \(y = 1\).

13. \[\frac{3}{2}x - \frac{2}{3}y = -1\] Multiply first equation by 6.
\[8x + 3y = 11\] Multiply second equation by 7.
\[15x - 21y = -6\]
\[56x + 21y = 77\]
Add the two equations.
\[71x = 71\]
Substitute for this variable in either original equation and solve for the other variable.
\[8(1) + 3y = 11\]
\[3y = 3\]
\[y = 1\]
The solution of the system is \(x = 1\) and \(y = 1\) or (1, 1).

15. \[0.3u - 0.2v = 0.5\] Multiply first row by 3.
\[0.9u - 0.6v = 1.5\]
\[0.9u - 0.6v = 0.1\]
Subtract the two equations.
\[0 \neq 1.4\]
There is no solution. The system is inconsistent.
Note: If you have \(0 = \) a non zero number the system is always inconsistent.
17. \(0.2x - 0.3y = 4\)
\[2.3x - y = 1.2\]
Multiply 2nd row by 0.3.
\[0.69x - 0.3y = 0.36\]
Subtract the two equations.
\[-0.49x = 3.64\]
Solve for the variable.
\[x = -\frac{52}{7}\]
Substitute, solve for \(y\).
\[y = -\frac{128}{7}\]
The solution of the system is \(x = -\frac{52}{7}\) and \(y = -\frac{128}{7}\).

19. \(4x + 6y = 4\)
\[2x + 3y = 2\]
Multiply second row by \(-2\).
\[-4x - 6y = -4\]
Add the two equations.
\[0 = 0\]
There are infinitely many solutions. The system is dependent. Solve for one of the variables in terms of the remaining variable: \(y = \frac{2}{3} - \frac{2}{3}x\). Then a general solution is \(\begin{pmatrix} c \\ \frac{2}{3} - \frac{2}{3}c \end{pmatrix}\), where any value of \(c\) will give a particular solution.

21. \(\frac{x+y}{x} = 2\)
\(\frac{y-1}{x} = 6\)
Before we can solve this system, we must arrange each equation in \(ax + by = c\) form.
\[x + y = 8\]
\[6x - y = 7\]
Add and solve for \(x\).
\[x = 1\]
Substitute in an original equation and solve for remaining variable.
\[y = 7\]
The solution is \(x = 1\) and \(y = 7\) or \((1, 7)\).

23–25 Use the standard window and graph each equation. Use the TRACE or INTERSECT feature to find the solution.

23. \[
\begin{align*}
y &= 8 - \frac{3x}{2} \\
y &= \frac{3x}{4} - 1
\end{align*}
\]
Solution: \((4, 2)\)

25. \[
\begin{align*}
5x + 3y &= -2 \\
3x + 7y &= 4
\end{align*}
\]
Solution: \((-1, 1)\)
37. Eq. 1  \[ x + 2y + z = 2 \]  Steps 1, 2, and 3 of the systematic procedure are completed.
Eq. 2  \[ -y + 3z = 8 \]
Eq. 3  \[ 2z = 10 \]  Step 4: \( z = 5 \)
From Eq. 2  \( -y + 3(5) = 8 \) or \( y = 7 \)
From Eq. 1  \( x + 2(7) + 5 = 2 \) or \( x = -17 \)
The solution is \( x = -17 \), \( y = 7 \), \( z = 5 \).

29. Eq. 1  \[ x - y - 8z = 0 \]  Steps 1 and 2 of the systematic procedure are completed.
Eq. 2  \[ y + 4z = 8 \]
Eq. 3  \[ 3y + 14z = 22 \]
Step 3: \((-3)\times\) Eq. 2 added to Eq. 3 gives \( 2z = -2 \) or \( z = -1 \).
From Eq. 2  \( y + 4(-1) = 8 \) or \( y = 12 \)
From Eq. 1  \( x - 12 - 8(-1) = 0 \) or \( x = 4 \)
The solution is \( x = 4 \), \( y = 12 \), \( z = -1 \) or \((4, 12, -1)\).

31. Eq. 1  \[ x + 4y - 2z = 9 \]  Step 1 is completed.
Eq. 2  \[ x + 5y + 2z = -2 \]
Eq. 3  \[ x + 4y - 2z = 22 \]
Step 2:
\[ x + 4y - 2z = 9 \]  Eq. 1
\[ y + 4z = -11 \]  \((-1)\times\) Eq. 1 added to Eq. 2
\[ -26z = 13 \]  \((-1)\times\) Eq. 1 added to Eq. 3
Step 3 is also completed.
Step 4: \( z = -\frac{1}{2} \) from Eq. 5.
From Eq. 4  \( y + 4\left(-\frac{1}{2}\right) = -11 \) or \( y = -9 \)
From Eq. 1  \( x + 4(-9) - 2\left(-\frac{1}{2}\right) = 9 \) or \( x = 44 \)
The solution is \( x = 44 \), \( y = -9 \), \( z = -\frac{1}{2} \) or \((44, -9, -\frac{1}{2})\).

33. \( x \) = amount of safe investment.
\( y \) = amount of risky investment.
\[ x + y = 145,600 \]  Total amount invested
\[ 0.1x + 0.18y = 20,000 \]  Income from investments
The solution is the solution of the above system of equations.
\[ x + 1.8y = 200,000 \]  \( (10) \times \) second equation
\[ 0.8y = 54,400 \]  Subtract equations
\[ y = 68,000 \]  Solve for \( y \) or amount of risky investment.
Substituting \( y = 68,000 \) into one of the original equations we have \( x + 68,000 = 145,600 \) or \( x = 77,600 \).
Solution: Put $77,600 in a safe investment and $68,000 in a risky investment.

35. \( x \) = amount invested at 10%.
\( y \) = amount invested at 12%.
\[ x + y = 23,500 \]  Total amount invested
\[ 0.10x + 0.12y = 2550 \]  Investment income
Solve the system of equation:
\[ x + y = 23,500 \]
\[ x + 1.2y = 25,500 \]  \( (10) \times \) second equation
\[ 0.2y = 2000 \]  Subtract equations
\[ y = 10,000 \]  Solve for \( y \)
Substituting into the first equation we have \( x + 10,000 = 23,500 \) or \( x = 13,500 \).
Thus, $13,500 is invested at 10% and $10,000 is invested at 12%.
37. \( A = \) ounces of substance A.
\( B = \) ounces of substance B.

Required ratio \( \frac{A}{B} = \frac{3}{5} \) gives \( 5A - 3B = 0 \).

Required nutrition is \( 5\% A + 12\% B = 100\% \). This gives \( 5A + 12B = 100 \).

The % notation can be trouble. Be careful! Now we can solve the system.
\[
\begin{align*}
5A - 3B &= 0 \\
5A + 12B &= 100
\end{align*}
\]

Subtract first equation from second.
\( B = \frac{100}{15} = \frac{20}{3} \)

Substituting into the original equation gives \( 5A - 3\left(\frac{20}{3}\right) = 0 \) or \( A = 4 \).

The solution is 4 ounces of substance A and \( \frac{2}{3} \) ounces of substance B.

39. \( x = \) population of species A.
\( y = \) population of species B.

\[
\begin{align*}
2x + y &= 10,600 & \text{units of first nutrient} \\
3x + 4y &= 19,650 & \text{units of second nutrient}
\end{align*}
\]

\[
\begin{align*}
8x + 4y &= 42,400 \quad (4) \times \text{first equation} \\
3x + 4y &= 19,650
\end{align*}
\]

\( 5x = 22,750 \) Subtract
\( x = 4550 \) Solve for x

Substituting \( x = 4550 \) into an original equation we have \( 2(4550) + y = 10,600 \).
So, \( y = 1500 \). Solution is 4550 of species A and 1500 of species B.

41. \( x = \) amount of 20% concentration.
\( y = \) amount of 5% concentration.

\[
\begin{align*}
x + y &= 10 \quad \text{amount of solution} \\
0.20x + 0.05y &= 0.155(10) \quad \text{concentration of medicine}
\end{align*}
\]

Solving this system of equations:
\[
\begin{align*}
x + y &= 10 \quad (1) \\
x + 0.25y &= 7.75 \quad (5) \times \text{second equation} \\
0.75y &= 2.25 \quad \text{Subtract equations} \\
y &= 3 \quad \text{Solve for y}
\end{align*}
\]

Substituting into the first equation we have \( x + 3 = 10 \) or \( x = 7 \).
The solution is 3 cc of 5% concentration and 7 cc of 20% concentration.

43. \( x = \) number of $20 tickets.
\( y = \) number of $30 tickets.

\[
\begin{align*}
x + y &= 16,000 \quad \text{total number of tickets} \\
20x + 30y &= 380,000 \quad \text{total revenue}
\end{align*}
\]

To solve the system of equations multiply Eq. 1 by 30.
\[
\begin{align*}
30x + 30y &= 480,000 \\
20x + 30y &= 380,000
\end{align*}
\]

\( 10x = 100,000 \) or \( x = 10,000 \)

Substituting into the first equation gives \( y = 6000 \).
Sell 10,000 tickets for $20 each and 6000 tickets for $30 each.

45. \( x = \) amount of 20% solution to be added.
\( 0.20x = \) concentration of nutrient in 20% solution.
\( 0.02(100) = 2 \) is the concentration of nutrient in 2% solution.
\( 0.20x + 2 = 0.10(x + 100) \)
\( 0.20x + 2 = 0.10x + 10 \)
\( 0.1x = 8 \) or \( x = 80 \) cc of 20% solution is needed.
47. \( x = \) ounces of substance A, 
\( y = \) ounces of substance B, and 
\( z = \) ounces of substance C.
\[
5x + 15y + 12z = 100 \quad \text{Nutrition requirements}
\]
\[
x = z \quad \text{Digestive restrictions}
\]
\[
y = \frac{1}{2}z \quad \text{Digestive restrictions}
\]
Since both \( x \) and \( y \) are in terms of \( z \), we can substitute in the first equation and solve for \( z \).

So, \( 5z + 3z + 12z = 100 \) or \( 20z = 100 \). So, \( z = 5 \). Now, since \( x = z \), we have \( x = 5 \).

Since \( y = \frac{1}{5}z \), we have \( y = 1 \). The solution is 5 ounces of substance A, 1 ounce of substance B, and 5 ounces of substance C.

49. \( A = \) number of A type clients. 
\( B = \) number of B type clients. 
\( C = \) number of C type clients.
\[
A + B + C = 500 \quad \text{Total clients}
\]
\[
200A + 500B + 300C = 150,000 \quad \text{Counseling costs}
\]
\[
300A + 200B + 100C = 100,000 \quad \text{Food and shelter}
\]
To find the solution we must solve the system of equations.

\[
\begin{align*}
\text{Eq. 1} & : A + B + C = 500 \\
\text{Eq. 2} & : 2A + 5B + 3C = 1500 \\
\text{Eq. 3} & : 3A + 2B + C = 1000
\end{align*}
\]

\[
\begin{align*}
\text{Eq. 4} & : 3B + C = 500 \\
\text{Eq. 5} & : -B - 2C = -500
\end{align*}
\]

Substituting \( C = 200 \) into Eq. 4 gives \( 3B + 200 = 500 \) or \( 3B = 300 \). So, \( B = 100 \).

Substituting \( C = 200 \) and \( B = 100 \) into Eq. 1 gives \( A + 100 + 200 = 500 \). So, \( A = 200 \).

Thus, the solution is 200 type A clients, 100 type B clients, and 200 type C clients.

Supplementary Exercises

1. If \( \begin{cases} 2x + 3y = 0 \\ 4x - y = 7 \end{cases} \), then \( x = ? \)
   
   a. \( -\frac{2}{3} \)
   
   b. 0
   
   c. 1
   
   d. \( \frac{3}{2} \)
   
   e. \( \frac{7}{4} \)

2. If \((a, b)\) is the solution to the system of equations
   \[
   \begin{cases}
   0.5x + y = 20 \\
   0.3x + 0.2y = 28
   \end{cases}
   \]
   then \( a + b = ? \)
   
   a. -160
   
   b. 48
   
   c. 80
   
   d. 160
   
   e. none of these
3. A bank loaned $128,000 for the development of two products, A and B. The amount loaned for product B was $12,000 less than twice the amount loaned for product A. $A + B = 128,000$ is one equation needed to find the amounts loaned for A and B. The other equation is

- A. $2B - A = 12,000$
- B. $A - 2B = 12,000$
- C. $2A - B = 12,000$
- D. $A = 12,000 - 2B$

Solutions to Supplementary Exercises

1. $\begin{cases} 2x + 3y = 0 \\ 4x - y = 7 \end{cases}$
   
   $2x + 3y = 0$

   $12x - 3y = 21$

   $14x = 21$

   $x = \frac{3}{2}$

   Answer: d

2. $\begin{cases} 0.5x + y = 20 \\ 0.3x + 0.2y = 28 \end{cases}$

   $0.5x + y = 20$

   $1.5x + y = 40$

   $0.5(120) + y = 20$

   $y = -40$

   $a + b = 80$

   Answer: c

3. amount loaned for B was 12,000 less than twice the amount loaned for A

   $$ B = 2A - 12,000 $$

   This simplifies to $2A - B = 12,000$.

Exercise 1.6

1. a. Total cost = Variable cost + Fixed cost

   $$ C(x) = 17x + 3400 $$

   b. $C(200) = 17(200) + 3400 = $6800$

3. a. $R(x) = 34x$

   b. $R(300) = 34(300) = $10,200$

5. a. $P(x) = R(x) - C(x) = 34x - (17x + 3400) = 17x - 3400$

   b. $P(300) = 17(300) - 3400 = $1700$

7. a. $P(x) = R(x) - C(x) = 80x - (43x + 1850) = 37x - 1850$

   b. $P(30) = 37(30) - 1850 = -$740$

   The total costs are more than the revenue.

   c. $P(x) = 0$ or $37x - 1850 = 0$

   So, $x = \frac{1850}{37} = 50$ units is the break-even point.

9. $C(x) = 5x + 250$

   a. $m = 5$, C-intercept: 250

   b. $MC = 5$ means that each additional unit produced costs $5$.

   c. Slope = marginal cost. C-intercept = fixed costs.

   d. $5$, $5$ ($MC = 5$ at every point)

11. $R = 27x$

   a. $m = 27$

   b. 27; each additional unit sold yields $27 in revenue.

   c. In each case, one more unit yields $27.

13. $R(x) = 27x$, $C(x) = 5x + 250$

   a. $P(x) = 27x - (5x + 250) = 22x - 250$

   b. $m = 22$

   c. Marginal profit is 22.

   d. Each additional unit sold gives a profit of $22.

   To maximize profit sell all that you can produce. Note that this is not always true.

15. $(x, P)$ is the correct form.

   $P_1 = (200, 3100)$

   $P_2 = (250, 6000)$

   $m = \frac{6000 - 3100}{250 - 200} = 58$

   $P - 3100 = 58(x - 200)$ or $P = 58x - 8500$

   The marginal profit is 58.

17. a. The revenue function is the graph that passes through the origin.

   b. At a production of zero the fixed costs are $2000.

   c. From the graph, the break-even point is 400 units and $3000 in revenue or costs.

   d. Marginal cost $= \frac{3000 - 2000}{400 - 0} = 2.5$

   Marginal revenue $= \frac{3000 - 0}{400 - 0} = 7.5$
19. \( R(x) = C(x) = 85x = 35x + 1650 \) or \( 50x = 1650 \) or \( x = 33 \).

Thus, 33 necklaces must be sold to break even.

21. a. \( R(x) = 12x, C(x) = 8x + 1600 \)

b. \( R(x) = C(x) \) if \( 12x = 8x + 1600 \) or \( 4x = 1600 \) or \( x = 400 \).

It takes 400 units to break even.

23. a. \( P(x) = R(x) - C(x) = 12x - (8x + 1600) = 4x - 1600 \)

By setting \( P(x) = 0 \) we get \( x = 400 \).

Same as 21(b).

25. a. \( R(x) = 54.90x \)

b. \( P_1 = (2000, 50000) \)

\( P_2 = (800, 32120) \)

\[ m = \frac{32120 - 50000}{800 - 2000} = \frac{-17880}{-1200} = 14.90 \]

\( y = 5000 = 14.90(x = 2000) \) or \( y = 14.90x + 20200 = C(x) \)

c. From \( 54.90x = 14.90x + 20200 \) we have \( x = 505 \) units to break even.

27. If price increases, then the demand for the product decreases.

29. a. If \( p = $100 \), then \( q = 600 \) (approximately).

b. If \( p = $100 \), then \( q = 300 \).

c. There is a shortage since more is demanded.

31. Demand: \( 2p + 5q = 200 \)

\[ 2(60) + 5q = 200 \]

\[ 5q = 80 \]

\[ q = 16 \]

Supply: \( p - 2q = 10 \)

\[ 60 - 2q = 10 \]

\[ 2q = 50 \]

\[ q = 25 \]

There will be a surplus of 9 units at a price of $60.00.

33. Remember that \((q, p)\) is the correct form.

\( P_1 = (240, 900) \)

\( P_2 = (315, 850) \)

\[ m = \frac{850 - 900}{315 - 240} = \frac{50}{75} = \frac{2}{3} \]

Note: \( m < 0 \) for demand equations.

\[ p - 900 = -\frac{2}{3}(q - 240) \] or \[ p = -\frac{2}{3}q + 1060 \]

35. \((q, p)\) is the correct form.

\( P_1 = (10000, 1.50) \)

\( P_2 = (5000, 1.00) \)

\[ m = \frac{1 - 1.50}{5000 - 10000} = \frac{-0.50}{-5000} = 0.0001 \]

Note: \( m > 0 \) for supply equations.

\[ p - 1 = 0.0001(q - 5000) \] or \[ p = 0.0001q + 0.5 \]

37. a. The decreasing function is the demand curve.

The increasing function is the supply curve.

b. Reading the graph, we have equilibrium at \( q = 30 \) and \( p = 25 \).

39. a. Reading the graph, at \( p = 20 \) we have 20 units supplied.

b. Reading the graph, at \( p = 20 \) we have 40 units demanded.

c. At \( p = 20 \) there is a shortage of 20 units.

41. By observing the graph in the figure, we see that a price below the equilibrium price results in a shortage.
Exercise 1.6

47. Demand: (80, 350) and (120, 300) are two points. \( m = \frac{350 - 300}{80 - 120} = -\frac{5}{4} \)  
\[ p - p_1 = m(q - q_1) \]  \[ p - 300 = -\frac{5}{4}(q - 120) \] or \[ p = -\frac{5}{4}q + 450 \]  
Supply: (60, 280) and (140, 370) are two points. \( m = \frac{280 - 370}{60 - 140} = -\frac{9}{8} \)  
\[ p - p_1 = m(q - q_1) \]  \[ p - 280 = -\frac{9}{8}(q - 60) \] or \[ p = -\frac{9}{8}q + 212.5 \]  
Now, set these two equations for \( p \) equal to each other and solve for \( q \).  
\[ -\frac{5}{4}q + 450 = -\frac{9}{8}q + 212.5 \]  
Required for equilibrium.  
\[ 9q + 1700 = -10q + 3600 \]  
Multiply both sides by 8 to simplify.  
\[ 19q = 1900 \]  
\[ q = 100 \]  
Substituting \( q = 100 \) into one of the original equations gives \( p = 325 \).  
Thus, the equilibrium point is \( (q, p) = (100, 325) \).

49. a. Reading the graph, we have that the tax is $15.  
b. From the graph, the original equilibrium was \( (100, 100) \).  
c. From the graph, the new equilibrium is \( (50, 110) \).  
d. The supplier suffers because the increased price reduces the demand.

51. New supply price: \( p = 15q + 30 + 38 = 15q + 68 \)  
\[ 15q + 68 = -4q + 220 \]  
Required condition  
\[ 19q = 152 \]  
\[ q = 8 \]  
Substituting \( q = 8 \) into one of the original equations gives \( p = 188 \).  
Thus, the new equilibrium point is \( (q, p) = (8, 188) \).

53. New supply price: \[ p = \frac{q}{20} + 10 + 5 = \frac{q}{20} + 15 \]  
\[ \frac{q}{20} + 15 = -\frac{q}{20} + 65 \]  
Required condition  
\[ q + 300 = -q + 1300 \]  
\[ 2q = 1000 \]  
\[ q = 500 \]  
Thus, \( p = \frac{500}{20} + 15 = 40 \).  
The new equilibrium point is \( (500, 40) \).

55. Demand: \( p = \frac{-q + 2100}{60} \)  
Supply: \( p = \frac{q + 540}{120} \)  
New supply: \[ p = \frac{q + 540}{120} + \frac{1}{2} = \frac{q + 540}{120} + \frac{60}{120} = \frac{q + 600}{120} \]  
\[ \frac{q + 600}{120} = -\frac{q + 2100}{60} \]  
Required condition  
\[ q + 600 = -2q + 4200 \]  
Multiply both sides by 120  
\[ 3q = 3600 \]  
\[ q = 1200 \]  
Thus, \( p = \frac{1200 + 600}{120} = 15 \).  
The new equilibrium quantity is 1200.  
The new equilibrium price is $15.
Supplementary Exercises

1. A portable TV sells for $48. The production of TVs has a fixed cost of $576 and a variable cost of $12 for each TV produced. How many TVs must be sold to break even?
   a. 12
   b. 16
   c. 48
   d. 576
   e. 1152

2. For the graph given below, what quantity is demanded at a price of $30?

   ![Graph with points (50, 25)]

   a. 20
   b. 30
   c. 40
   d. 50
   e. 70

3. For the graph in problem 2, what quantity is supplied at a price of $30?

   a. 20
   b. 30
   c. 40
   d. 50
   e. 70

Solutions to Supplementary Exercises

1. Break even means profit is zero. \( P = R - C \)
   \( P = 48x - (12x + 576) = 36x - 576 \)
   \( P = 0 \) at \( x = 16 \).
   Answer: b

2. The demand curve is a falling curve.
   At \( p = 30 \), \( q = 40 \).
   Answer: c

3. The supply curve is a rising curve.
   At \( p = 30 \), \( q = 70 \).
   Answer: e

Review Exercises

For this set of exercises we will not give reasons for any steps or list any formulas.

1. \( x + 7 = 14 \)
   \( x = 14 - 7 \)
   \( x = 7 \)

2. \( 3x - 8 = 23 \)
   \( 3x = 31 \)
   \( x = \frac{31}{3} \)

3. \( 2x - 8 = 3x + 5 \)
   \( -x = 13 \)
   \( x = -13 \)

4. \( \frac{6x + 3}{6} = \frac{5(x - 2)}{9} \)
   \( \frac{6x + 3}{6} = \frac{5(x - 2)}{9} \)
   \( 3(6x + 3) = 10(x - 2) \)
   \( 18x + 9 = 10x - 20 \)
   \( 8x = -29 \)
   \( x = \frac{-29}{8} \)

5. \( 2x + \frac{1}{2} = \frac{x}{2} + \frac{1}{3} \)
   \( 12x + 3 = 3x + 2 \)
   \( 9x = -1 \)
   \( x = \frac{-1}{9} \)

6. \( 0.6x + 4 = x - 0.02 \)
   \( 4.02 = 0.4x \)
   \( 10.5 = x \)

7. \( \frac{6}{3x - 5} = \frac{6}{2x + 3} \)
   \( 6(2x + 3) = 6(3x - 5) \)
   \( 2x + 3 = 3x - 5 \)
   \( 3 + 5 = 3x - 2x \)
   \( x = 8 \)

8. \( 3y - 6 = -2x - 10 \)
   \( 3y = -2x - 4 \)
   \( y = -\frac{2x - 4}{3} \)

9. \( \frac{2x + 5}{x + 7} = \frac{1}{3} + \frac{x - 11}{2(x + 7)} \)
   \( 6(2x + 5) = 2(x + 7) + 3(x - 11) \)
   \( 12x + 30 = 2x + 14 + 3x - 33 \)
   \( 12x - 2x - 3x = 14 - 33 - 30 \)
   \( 7x = -49 \)
   \( x = -7 \)

There is no solution since we have division by zero when \( x = -7 \).
10. Yes.
11. $y^2 = 9x$, is not a function of $x$. If $x = 1$, then $y = \pm 3$.
12. Yes.
13. $y = \sqrt{9-x}$
   Domain: $9 - x \geq 0$ or $9 \geq x$ or $x \leq 9$.
   Range: Positive square root means $y \geq 0$.
14. $f(x) = x^2 + 4x + 5$
   a. $f(-3) = (-3)^2 + 4(-3) + 5 = 9 - 12 + 5 = 2$
   b. $f(4) = (4)^2 + 4(4) + 5 = 16 + 16 + 5 = 37$
   c. $f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right) + 5 = \frac{1}{4} + 2 + 5 = \frac{29}{4}$
15. $g(x) = x^2 + \frac{1}{x}$
   a. $g(-1) = -(-1)^2 + \frac{1}{-1} = 1 - 1 = 0$
   b. $g\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 + \frac{1}{\frac{1}{2}} = \frac{1}{4} + 2 = \frac{9}{4}$
   c. $g(0.1) = (0.1)^2 + \frac{1}{0.1} = 0.01 + 10 = 10.01$
16. $f(x) = 9x - x^2$
   $f(x + h) = 9(x + h) - (x + h)^2$
   $= 9x + 9h - x^2 - 2xh - h^2$
   $f(x) = 9x - x^2$
   $f(x + h) - f(x) = 9h - 2xh - h^2 = h(9 - 2x - h)$
   $\frac{f(x + h) - f(x)}{h} = 9 - 2x - h$
17. $y$ is a function of $x$. (Use vertical rule test.)
18. No, fails vertical line test.
19. $f(2) = 4$
20. $x = 0, x = 4$
21. a. $f(4) = 7$
   b. $f(x) = 2$ if $x = -1, 3$
   c. $f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right) + 5 = \frac{1}{4} + 2 + 5 = \frac{29}{4}$
22. $f(x) = 3x + 5, g(x) = x^2$
   a. $(f + g)x = (3x + 5) + x^2 = x^2 + 3x + 5$
   b. $\frac{f}{g} x = \frac{3x + 5}{x^2}$ or $3x + \frac{5}{x} = \frac{3}{x} + \frac{5}{x^2}$
   c. $f(g(x)) = f(x^2) = 3x^2 + 5$
   d. $(f \circ f)x = f((3x + 5) = 3(3x + 5) + 5 = 9x + 20$
23. $5x + 2y = 10$
   $x$-intercept: If $y = 0$, $x = 2$
   $y$-intercept: If $x = 0$, $y = 5$
24. $6x + 5y = 9$
   $x$-intercept: If $y = 0$, $x = \frac{9}{6} = \frac{3}{2}$
   $y$-intercept: If $x = 0$ or $y = \frac{9}{5}$
25. $x = -2$
   $x$-intercept: $x = -2$
   There is no $y$-intercept.
26. $P_1(2, -1); P_2(-1, -4)$
   $m = \frac{-4 - (-1)}{-1 - 2} = \frac{-3}{-3} = 1$
27. $(-3.8, -7.16)$ and $(-3.8, 1.16)$
   $m = \frac{-7.16 - 1.16}{-3.8 - (-3.8)} = \frac{-8.32}{0}$
   Slope is undefined.
28. \(2x + 5y = 10\)
   \(y = -\frac{2}{5}x + 2\)
   \(m = -\frac{2}{5}, \ b = 2\)

29. \(x = -\frac{3}{4}y + \frac{3}{2}\)
   \(m = -\frac{3}{4}, \ b = \frac{3}{2}\)

30. \(m = 4, \ b = 2, \ y = 4x + 2\)

31. \(m = -\frac{1}{2}, \ b = 3, \ y = -\frac{1}{2}x + 3\)

32. \(P = (-2, 1), \ m = \frac{2}{5}\)
   \(y - 1 = \frac{2}{5}(x + 2)\) or \(y = \frac{2}{5}x + \frac{9}{5}\)

33. \((-2, 7)\) and \((6, -4)\)
   \(m = \frac{-4 - 7}{6 - (-2)} = -\frac{11}{8}\)
   \(y - 7 = -\frac{11}{8}(x - (-2))\) or \(y = -\frac{11}{8}x + 17\)
   \(y = \frac{11}{8}x + 4\)

34. \(P_1(-1, 8); \ P_2(-1, -1)\)
   The line is vertical since the \(x\)-coordinates are the same.
   Equation: \(x = -1\)

35. Parallel to \(y = 4x - 6\) means \(m = 4\).
   \(y - 6 = 4(x - 1)\) or \(y = 4x + 2\)

36. \(P(-1, 2); \ \perp\ \text{to} \ 3x + 4y = 12\)
   or \(y = -\frac{3}{4}x + 3\)
   \(m = -\frac{4}{3}\)
   \(y - 2 = -\frac{4}{3}(x + 1)\) or \(y = \frac{4}{3}x + 10\)
   \(y = \frac{4}{3}x + 10\)

37. \(x^2 + y - 2x - 3 = 0\)
   \(y = -x^2 + 2x + 3\)

38. \(y = x^3 - 27x + 54\)

39. a. \(y = (x + 6)(x - 3)(x - 15)\)
   b. \(y = (x + 6)(x - 3)(x - 5)\)
   c. The graph in (a) shows the complete graph. The graph in (b) shows a piece that rises toward the high point and a piece between the high and low points.

40. \(y = x^2 - x - 42\) is a parabola opening upward.
   a. \(y = x^2 - x - 42\)
   b. \(y = x^2 - x - 42\)
   c. (a) shows the complete graph. The \(y\)-min is too large in (b) to get a complete graph.

41. \(y = \sqrt{\frac{x + 3}{x}}\)
   \(x \neq 0; \ x + 3 \geq 0\) or \(x \geq -3\); Domain: \(x \neq 0, \ x \geq -3\)

42. \(4x - 2y = 6\)
   \(3x + 3y = 9\)
   Then, \(12x - 6y = 18\)
   \(6x + 6y = 18\)
   \(18x = 36\)
   \(x = 2\)
   \(4(2) - 2y = 6\)
   \(-2y = -2\)
   \(y = 1\)
   Solution: \((2, 1)\)
43. \[2x + y = 19\]
\[x - 2y = 12\]
Then, \[4x + 2y = 38\]
\[x - 2y = 12\]
\[5x = 50\]
\[x = 10\]
\[2(10) + y = 19\]
\[y = -1\]
Solution: (10, -1)

44. \[3x + 2y = 5\]
\[2x - 3y = 12\]
Then, \[9x + 6y = 15\]
\[4x - 6y = 24\]
\[13x = 39\]
\[x = 3\]
\[3(3) + 2y = 5\]
\[2y = -4\]
\[y = -2\]
Solution: (3, -2)

45. \[6x + 3y = 1\]
\[y = -2x + 1\]
\[6x + 3(-2x + 1) = 1\]
\[6x - 6x + 3 = 1\]
\[3 = 1\]
No solution.

46. \[4x - 3y = 253\]
\[8x - 6y = 506\]
\[4(10) - 3y = 253\]
\[13x + 2y = -12\]
\[39x + 6y = -36\]
\[-3y = 213\]
\[47x = 470\]
\[y = -71\]
\[x = 10\]
Solution: (10, -71)

47. \[x + 2y + 3z = 5\] Steps 1 and 2: Nothing to be done.
\[y + 11z = 21\] Step 3: \[x + 2y + 3z = 5\]
\[5y + 9z = 13\]
\[y + 11z = 21\]
\[-46z = -92\]
Step 4: \[z = 2\]
\[y + 11(2) = 21\]
\[x + 2(-1) + 3(2) = 5\]
\[y = -1\]
\[x = 1\]
Solution is \(x = 1, y = -1, z = 2\).

48. \[x + y - z = 12\] Thus \(z = 9\)
\[2y - 3z = -7\]
\[2y - 27 = -7\]
\[3x + 3y - 7z = 0\]
\[2y = 20\] or \(y = 10\)
\[x + y - z = 12\]
\[x + 10 - 9 = 12\]
\[2y - 3z = -7\]
\[x = 11\]
\[-4z = -36\] Solution: (11, 10, 9)

49. Student has total points of 91 + 82 + 88 + 50 + 42 + 42 = 395.
Total of possible points is 300 + 150 + 200 = 650.
To earn an A students need at least 0.9(650) = 585 points.
Student must earn 585 - 395 = 190 points on the final. This is the same as 95%.

50. Diesel: \[C = 0.16x + 18,000\]
\[0.21x + 16,000 = 0.16x + 18,000\]
Gas: \[C = 0.21x + 16,000\]
\[0.05x = 2000\]
\[x = 40,000\]
Costs are equal at 40,000 miles. A truck is used more than 40,000 miles in 5 years.
Buy the diesel.
51. a. Yes
   b. No
   c. \( f(300) = 4 \)

52. a. \( f(80) = 565.44 \)
   b. The monthly payment on a $70,000 loan is $494.75.
   c. \( f(120) = 2 \cdot f(60) = 2(424.07) = $848.14 \)
      Two $60,000 loans are the same as a $120,000 loan.

53. \( P(x) = 180x - \frac{x^2}{100} - 200 \)
   \( x = q(t) = 1000 + 10t \)
   a. \( (P \circ q)(t) = P(1000 + 10t) \)
      \[ = 180(1000 + 10t) - \frac{(1000 + 10t)^2}{100} - 200 \]
   b. \( x = q(15) = 1000 + 10(15) = 1150 \) units produced
      \( P(1150) = 180(1150) - \frac{(1150)^2}{100} - 200 = $193,575 \)

54. \( W(L) = kL^3 \), \( L(t) = 50 - \frac{(t - 20)^2}{10}, 0 \leq t \leq 20 \)
   \( (W \circ L)(t) = W\left(50 - \frac{(t - 20)^2}{10}\right) = 0.02\left(50 - \frac{(t - 20)^2}{10}\right)^3 \)

55. \( d = \frac{t}{4.8} \)
   a. 
   b. \((9.6, 2)\) means that the thunderstorm is two miles away if flashes are 9.6 seconds apart.

56. \( H_c = 90 - T_a \)

57. \((x, P)\) is the required form.
   \( P_1 = (200, 3100), P_2 = (250, 6000) \)
   \( m = \frac{6000 - 3100}{250 - 200} = \frac{2900}{50} = 58 \)
   \( P - 3100 = 58(x - 200) \) or
   \( P(x) = 58x - 8500 \)
Use \((C, F)\) to get equation.
\[ P_{1}(0, 32); P_{2}(100, 212) \]
\[
m = \frac{212 - 32}{100 - 0} = \frac{180}{100} = 1.8
\]
\[ F - 32 = \frac{9}{5}(C - 0) \]
\[
F = \frac{9}{5}C + 32.
\]
Also \(C = \frac{5}{9}(F - 32).\)

59. a. \[
y = 120x^2 - 20x^3
\]

b. Algebraically, \(y \geq 0\) if
\[
120x^2 - 20x^3 = 20x^2(x - 6) \geq 0.
\]
Answer: \(0 \leq x \leq 6\)

60. \[
v^2 = 1960(h + 10)
\]
\[
h + 10 = \frac{v^2}{1960}
\]
\[
h = \frac{v^2 - 10}{1960}
\]

61. \(x = \) amount of safer investment and \(y = \) amount of other investment.
\[
x + y = 150000
\]
\[
0.095x + 0.11y = 15000
\]
Solving the system:
\[
0.11x + 0.11y = 16500
\]
\[
0.095x + 0.11y = 15000
\]
\[
0.015x = 1500
\]
\[
x = 100000
\]
Then \(y = 50000.\) Thus, invest \$100,000 at 9.5\% and \$50,000 at 11\%.

62. \(x = \) liters of 20\% solution
\(y = \) liters of 70\% solution
\[
x + y = 4
\]
\[
0.2x + 0.7y = 1.4
\]
\[
x + y = 4
\]
\[
x + 3.5y = 7
\]
\[
2.5y = 3
\]
\[
y = 1.2
\]
\[
x + 1.2 = 4
\]
\[
x = 2.8
\]
Answer: 2.8 liters of 20\%, 1.2 of 70\%.

63. \(S: p = 4q + 5, D: p = -2q + 81\)
   a. \(S: 53 = 4q + 5 \quad D: 53 = -2q + 81\)
   \[
   4q = 48 \quad q = 28
   \]
   \[
   q = 12
   \]
   b. Demand is greater.
   There is a shortfall.
   c. Price is likely to increase.

64. \[\text{Market Equilibrium}\]

65. \(C(x) = 38.80x + 4500, R(x) = 61.30x\)
   a. Marginal cost is \$38.80.
   b. Marginal revenue is \$61.30.
   c. Marginal profit is \$61.30 - 38.80 = \$22.50.
   d. \(61.30x = 38.80x + 4500\)
   \[
   22.50x = 4500
   \]
   \[
   x = 200 \text{ units to break even.}
   \]

66. \(FC = \$1500, \ VC - \$22 \text{ per unit, } R = \$52 \text{ per unit}\)
   a. \(C(x) = 22x + 1500\)
   b. \(R(x) = 52x\)
   c. \(P = R - C = 30x - 1500\)
   d. \(MC = 22\)
   e. \(MR = 52\)
   f. \(MP = 30\)
   g. Break even means \(30x - 1500 = 0\)
   or \(x = 50.\)
67. Supply: \( m = \frac{100 - 200}{200 - 400} = \frac{1}{2} \)
Demand: \( m = \frac{200 - 0}{200 - 600} = -\frac{1}{2} \)
\( p - 100 = \frac{1}{2} (q - 200) \)
\( p = \frac{1}{2} q + 300 \)
\( p - 0 = -\frac{1}{2} (q - 600) \)
So, \( \frac{1}{2} q = -\frac{1}{2} q + 300 \) or \( q = 300 \).
The equilibrium price is \( p = \frac{1}{2} (300) = 150 \).

68. New supply equation: \( p = \frac{q}{10} + 8 + 2 = \frac{q}{10} + 10 \)
Demand: \( p = -\frac{q}{10} + 150 \)
\( \frac{q}{10} + 10 = -\frac{q}{10} + 150 \)
\( 2q = 140 \) or \( q = 700 \)
\( p = 700 + 10 = 80 \)
Solution: \((700, 80)\)

Chapter Test

1. \( 4x - 3 = \frac{x}{2} + 6 \)
\( 8x - 6 = x + 12 \)
\( 7x = 18 \)
\( x = \frac{18}{7} \)

2. \( \frac{3}{x} + 4 = \frac{4x}{x + 1} \)
\( 3(x + 1) + 4x(x + 1) = 4x(x) \)
\( 3x + 3 + 4x^2 + 4x = 4x^2 \)
\( 7x = -3 \)
\( x = -\frac{3}{7} \)

3. \( \frac{3x - 1}{4x - 9} = \frac{5}{7} \)
\( 7(3x - 1) = 5(4x - 9) \)
\( 21x - 7 = 20x - 45 \)
\( x = -38 \)

4. \( f(x) = 7 + 5x - 2x^2 \)
\( f(x + h) = 7 + 5(x + h) - 2(x + h)^2 \)
\( = 7 + 5x + 5h - 2x^2 - 4xh - 2h^2 \)
\( f(x) = 7 + 5x - 2x^2 \)
\( f(x + h) - f(x) = 5h - 4xh - 2h^2 \)
\( \frac{f(x + h) - f(x)}{h} = 5 - 4x - 2h \)

5. \( 5x - 6y = 30 \)
x-intercept: 6
y-intercept: -5

6. \( 7x + 5y = 21 \)
x-intercept: 3
y-intercept: \( \frac{21}{5} \)

7. \( f(x) = \sqrt{4x + 16} \)
a. \( 4x + 16 \geq 0 \)
\( 4x \geq -16 \)
Domain: \( x \geq -4 \); Range: \( y \geq 0 \)
For range, note square root is positive.
b. \( f(3) = \sqrt{12 + 16} = 2\sqrt{7} \)
c. \( f(5) = \sqrt{20 + 16} = 6 \)
8. \((-1, 2)\) and \((3, -4)\)
\[ m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - 2}{3 - (-1)} = \frac{-6}{4} = -\frac{3}{2} \]
\[ y - 2 = -\frac{3}{2}(x - (-1)) \]
\[ y = -\frac{3}{2}x + \frac{1}{2} \]
9. \(5x + 4y = 15\)
\[ y = -\frac{5}{4}x + \frac{15}{4} \]
\[ m = -\frac{5}{4}, \ b = \frac{15}{4} \]
10. Point \((-3, -1)\)
   a. Undefined slope means vertical line. \(x = -3\)
   b. \(\perp\) to \(y = \frac{1}{4}x + 2\) means \(m = -4\).
   Thus, \(y + 1 = -4(x + 3)\) or \(y = -4x - 13\).
11. a. is not a function since for each \(x\) there are two \(y\)'s.
   b. is a function since for each \(x\) there is only one \(y\).
   c. is not a function for same reason as (a).
12. \(3x + 2y = -2\)
\[ 4x + 5y = 2 \]
\[ 12x + 8y = -8 \]
\[ 12x + 15y = 6 \]
\[ -7y = -14 \]
\[ y = 2 \]
\[ 3x + 2(2) = -2 \]
\[ 3x = -6 \]
\[ x = -2 \]
Solution: \((-2, 2)\)
13. \(f(x) = 5x^2 - 3x, \ g(x) = x + 1\)
   a. \((fg)(x) = (5x^2 - 3x)(x + 1)\)
   b. \(g(g(x)) = g(x + 1) = (x + 1) + 1 = x + 2\)
   c. \((f \circ g)(x) = f(x + 1)\)
   \[ = 5(x + 1)^2 - 3(x + 1) \]
   \[ = 5x^2 + 10x + 5 - 3x - 3 \]
   \[ = 5x^2 + 7x + 2 \]
14. \(R(x) = 38x, \ C(x) = 30x + 1200\)
   a. \(\overline{MC} = 30\)
   b. \(P(x) = 38x - (30x + 1200)\)
   \[ = 8x - 1200 \]
   c. Break-even means \(P(x) = 0\).
   \[ 8x = 1200 \text{ or } x = 150 \text{ units} \]
   d. \(\overline{MP} = 8\). Each unit sold makes a profit of $8.
15. a. \(R(x) = 50x\)
   b. \(C(100) = 10(100) + 18000\)
   \[ = 19,000 \]
   It costs $19,000 to make 100 units.
   c. \(50x = 10x + 18000\)
   \[ 40x = 18000 \]
   \[ x = 450 \text{ units} \]
16. \(S : p = 5q + 1500, \ D : p = -3q + 3100\)
\[ 5q + 1500 = -3q + 3100 \]
\[ 8q = 1600 \text{ or } q = 200 \]
\[ p(200) = 5(200) + 1500 = 2500 \]
17. \(y = 360000 - 1500x\)
   a. \(b = 360,000\)
   The original value is $360,000.
   b. \(m = -1500\).
   The building is depreciating $1500 each month.
18. \(x = \text{number of reservations}\)
\[ 0.90x = 360 \]
\[ x = 400 \]
Accept 400 reservations.
19. \(x = \text{amount invested at 9\%}\)
\(y = \text{amount invested at 6\%}\)
\[ x + y = 20000 \]
\[ 0.09x + 0.06y = 1560 \]
\[ 0.09x + 0.09y = 1800 \]
\[ 0.09x + 0.06y = 1560 \]
\[ 0.03y = 240 \]
\[ y = 8000 \]
Invest $8000 at 6\% and $12000 at 9\%.

Extended Applications

I. Hospital Administration

1. Revenue per case = $1000
Annual fixed costs
\[ = 180,000 + 270,000 = 450,000 \]
Annual variable costs
\[ = (380 + 15 + 20 \cdot \frac{1}{4}x) = 400x \], where \(x\) is the number of operations per year.
2. Break-even occurs when
   \[ \text{Revenue} = \text{Total Costs} \]
   \[ 1000x + 450000 + 400x = 600x + 450000 \]
   \[ x = 750 \]
   The hospital must perform 750 operations per year to break even.

3. We have \((70 \text{ operations/month})(12 \text{ months/year})\) gives 840 operations/year with a savings of \((840 \text{ operations})(\$50 \text{ savings}) = \$42,000\) on supplies. However, leasing the machine would cost \$50,000. Thus adding the machine would reduce the hospital’s profits by \$8000\ a year at the current level of operations.
   (Note that 1000 operations must be performed each year to cover the cost of the machine: \([(\$50)(1000) = \$50,000]\).)

4. Profit = Revenue – Cost
   \[ P(x) = 1000x - (450000 + 400x) \]
   \[ = 600x - 450000 \]
   At current level of operations, the annual profit is:
   \[ P(840) = 600(840) - 450000 \]
   \[ = 504000 - 450000 \]
   \[ = 54000 \]
   With \((40 \text{ new operations/month})(12 \text{ months/year}) = 480 \text{ new operations/year},\) the new level of operations is \(840 + 480 = 1320\).
   The advertising costs are \((\$10000/\text{month})(12 \text{ months/year}) = \$120000\) per year.
   At the new level of operations, the profit would be: \[ P(1320) = 600(1320) - 450000 - 120000 \]
   \[ = 792000 - 570000 \]
   \[ = 222000 \]
   The increase in profit is \$222,000 – 54,000 = \$168,000\.

5. Each extra operation adds \$1000 – 400 = \$600\ of profit. If the ad campaign costs \$10,000 per month it must generate \$10,000 per month \[ \frac{10000}{600} = \frac{16}{3} \text{ operations/month to cover its cost.} \]

6. Recall that the break-even point for leasing the machine is 1000 operations per year. If the ad campaign meets its projections, 1320 operations per year will be performed, with a savings of \((320)(\$50) = \$16,000\) on medical supplies by leasing the machine. They should reconsider their decision. (Note that this example illustrates that if the assumptions on which a decision was made change, it may be time to take another look at the decision.)

II. Fund Raising

Answers will vary.