E-1. **Order of operations when no parentheses are present:** We use the four rules for the order of operations.

(a) According to rule 2, to evaluate $2 + 3 \times 4 + 5$ we should perform the multiplication before we evaluate the sums:

$$2 + 3 \times 4 + 5 = 2 + 12 + 5 = 19.$$  

(b) According to rule 2, to evaluate $6 \div 3 \times 3 \wedge 2$ we should calculate the exponential first. This gives

$$6 \div 3 \times 3 \wedge 2 = 6 \div 3 \times 9.$$  
Then we should perform the multiplication and division, working from left to right. This gives

$$6 \div 3 \times 9 = 2 \times 9 = 18.$$  

(c) According to rule 2, to evaluate $3 \times 16 \div 2 \wedge 2$ we should calculate the exponential first. This gives

$$3 \times 16 \div 2 \wedge 2 = 3 \times 16 \div 4.$$  
Then we should perform the multiplication and division, working from left to right. This gives

$$3 \times 16 \div 4 = 48 \div 4 = 12.$$  

(d) According to rule 2, to evaluate $5 - 4 \times 3 + 8 \div 2 \times 3$ we should perform the multiplications and the division before performing the addition and the subtraction. We work from left to right. First we have

$$5 - 4 \times 3 + 8 \div 2 \times 3 = 5 - 12 + 8 \div 2 \times 3.$$  
Then we have

$$5 - 12 + 8 \div 2 \times 3 = 5 - 12 + 4 \times 3.$$  
Finally we have

$$5 - 12 + 4 \times 3 = 5 - 12 + 12 = 5.$$
S-1. **Basic calculations:** In typewriter notation, \( \frac{2.6 \times 5.9}{6.3} \) is \((2.6 \times 5.9) \div 6.3\), which equals 2.434... and so is rounded to two decimal places as 2.43.

S-3. **Basic calculations:** In typewriter notation, \( \frac{e}{\sqrt{\pi}} \) is \( e \div \left( \sqrt{\pi} \right) \), which equals 1.533... and so is rounded to two decimal places as 1.53.

S-5. **Parentheses and grouping:** When we add parentheses, \( \frac{7.3 - 6.8}{2.5 + 1.8} \) becomes \( (7.3 - 6.8) \div (2.5 + 1.8) \), which equals 0.116... and so is rounded to two decimal places as 0.12.

S-7. **Parentheses and grouping:** When we add parentheses, \( \frac{\sqrt{6 + e} + 1}{3} \) becomes \( \left( \sqrt{6 + e} + 1 \right) \div 3 \), which equals 1.317... and so is rounded to two decimal places as 1.32.

S-9. **Subtraction versus sign:** Noting which are negative signs and which are subtraction signs, we see that \(-\frac{3}{4 - 9}\) means \( \text{negative } \frac{3}{4} \text{ subtract } 9\). Adding parentheses and putting it into typewriter notation yields \( \text{negative } 3 \div (4 \text{ subtract } 9) \), which equals 0.6.

S-11. **Subtraction versus sign:** Noting which are negative signs and which are subtraction signs, we see that \(-\sqrt{8.6 - 3.9}\) means \( \text{negative } \sqrt{8.6 \text{ subtract } 3.9} \). In typewriter notation this is

\[ \text{negative } \sqrt{8.6 \text{ subtract } 3.9} , \]

which equals -2.167... and so is rounded to two decimal places as -2.17.

S-13. **Chain calculations:**

(a) To do this as a chain calculation, we first calculate \( \frac{3}{7.2 + 5.9} \) and then complete the calculation by adding the second fraction to this first answer. In typewriter notation \( \frac{3}{7.2 + 5.9} \) is \( 3 \div (7.2 + 5.9) \), which is calculated as 0.2290076336; this is used as Ans in the next part of the calculation. Turning to the full expression, we calculate it as \( \text{Ans} + \frac{7}{6.4 \times 2.8} \), which is, in typewriter notation, \( \text{Ans} + 7 \div (6.4 \times 2.8) \). This is 0.619..., which rounds to 0.62.

(b) To do this as a chain calculation, we first calculate the exponent, \( 1 - \frac{1}{36} \), and then the full expression becomes

\[ (1 + \frac{1}{36})^{\text{Ans}} . \]

In typewriter notation, the first calculation is \( 1 - 1 \div 36 \), and the second is \( (1 + 1 \div 36) \text{ } \wedge \text{ Ans} \). This equals 1.026... and so is rounded to two decimal places as 1.03.

S-15. **Evaluate expression:** In typewriter notation, \( e^{-3} - \pi^2 \) is \( e \text{ } \wedge \text{ (negative } 3) - \pi \text{ } \wedge \text{ 2} \), which equals -9.819... and so is rounded to two decimal places as -9.82.
1. **Arithmetic**: We will show each calculation in typewriter notation and give the numerical answer rounded to two decimal places. We will use \(-\) to denote subtraction and *negative* to denote a minus sign.

   (a) \((4.3 + 8.6)(8.4 - 3.5) = 63.21\)
   
   (b) \((2 \land 3.2 - 1) \div (\sqrt{3} + 4) = 1.43\)
   
   (c) \(\sqrt{(2 \land \text{negative } 3 + e)} = 1.69\)
   
   (d) \((2 \land \text{negative } 3 + \sqrt{7} + \pi)(e \land 2 + 7.6 \div 6.7) = 50.39\)
   
   (e) \((17 \times 3.6) \div (13 + 12 \div 3.2) = 3.65\)

3. **A bad investment**: The total value of your investment today is:

   \[
   \text{Original investment} - 7\% \text{ loss} = 720 - 0.07 \times 720 = \$669.60.
   \]

5. **Pay raise**: The percent pay raise is obtained from

   \[
   \text{Amount of raise} \div \text{Original hourly pay}.
   \]

   The raise was \(7.50 - 7.25 = 0.25\) dollar while the original hourly pay is \$7.25, so the fraction is \(0.25 \div 7.25 = 0.0345\). Thus we have received a raise of 3.45%.

7. **Trade discount**:

   (a) The cost price is \(9.99 - 40\% \times 9.99 = 5.99\) dollars.

   (b) The difference between the suggested retail price and the cost price is \(65.00 - 37.00 = 28.00\) dollars. We want to determine what percentage of \$65 this difference represents. We find the percentage by division: \(28.00 \div 65.00 = 0.4308\) or \(43.08\%\). This is the trade discount used.

9. **Present value**: We are given that the future value is \$5000 and that \(r = 0.12\). Thus the present value is

   \[
   \text{Present value} = \frac{\text{Future value}}{1 + r} = \frac{5000}{1 + 0.12} = 4464.29 \text{ dollars}.
   \]
11. **The Rule of 72:**

   (a) The Rule of 72 says our investment should double in

   \[
   \frac{72}{\text{interest rate}} = \frac{72}{13} = 5.54 \text{ years}.
   \]

   (b) Using Part (a), the future value interest factor is

   \[
   (1 + \text{interest rate})^{\text{years}} = (1 + 0.13)^{5.54} = 1.97.
   \]

   This is less than the doubling future value interest factor of 2.

   (c) Using our value from Part (c), the future value of a $5000 investment is

   \[
   \text{Original investment} \times \text{future value interest factor} = 5000 \times 1.97 = 9850.
   \]

   So our investment did not exactly double using the Rule of 72.

13. **The size of the Earth:**

   (a) The equator is a circle with a radius of approximately 4000 miles. The distance around the equator is its circumference, which is

   \[
   2\pi \times \text{radius} = 2\pi \times 4000 = 25,132.74 \text{ miles},
   \]

   or approximately 25,000 miles.

   (b) The volume of the Earth is

   \[
   \frac{4}{3}\pi \times \text{radius}^3 = \frac{4}{3}\pi \times 4000^3 = 268,082,573,100 \text{ cubic miles}.
   \]

   Note that the calculator gives 2.680825731E11, which is the way the calculator writes numbers in scientific notation. It means 2.680825731 \times 10^{11} and should be written as such. That is about 268 billion cubic miles or 2.68 \times 10^{11} cubic miles.

   (c) The surface area of the Earth is about

   \[
   4\pi \times \text{radius}^2 = 4\pi \times 4000^2 = 201,061,929.8 \text{ square miles},
   \]

   or approximately 201,000,000 square miles.
15. The length of the Earth’s orbit:

(a) If the orbit is a circle then its circumference is the distance traveled. That circumference is

\[ 2\pi \times \text{radius} = 2\pi \times 93 = 584.34 \text{ million miles}, \]

or about 584 million miles. This can also be calculated as

\[ 2\pi \times \text{radius} = 2\pi \times 93,000,000 = 584,336,233.6 \text{ miles}. \]

(b) Velocity is distance traveled divided by time elapsed. The velocity is given by

\[ \frac{\text{Distance traveled}}{\text{Time elapsed}} = \frac{584.34 \text{ million miles}}{1 \text{ year}} = 584.34 \text{ million miles per year}, \]

or about 584 million miles per year. This can also be calculated as

\[ \frac{584,336,233.6 \text{ miles}}{1 \text{ year}} = 584,336,233.6 \text{ miles per year}. \]

(c) There are 24 hours per day and 365 days per year. So there are \( 24 \times 365 = 8760 \) hours per year.

(d) The velocity in miles per hour is

\[ \frac{\text{Miles traveled}}{\text{Hours elapsed}} = \frac{584.34}{8760} = 0.0667 \text{ million miles per hour}. \]

This is approximately 67,000 miles per hour. This can also be calculated as

\[ \frac{584,336,233.6 \text{ miles}}{8760} = 66,705.05 \text{ miles per hour}. \]

17. Newton’s second law of motion: A man with a mass of 75 kilograms weighs \( 75 \times 9.8 = 735 \) newtons. In pounds this is \( 735 \times 0.225 \), or about 165.38.

19. Frequency of musical notes: The frequency of the next higher note than middle C is \( 261.63 \times 2^{1/12} \), or about 277.19 cycles per second. The D note is one note higher, so its frequency in cycles per second is

\( (261.63 \times 2^{1/12}) \times 2^{1/12} \),

or about 293.67.

21. Lean body weight in females: The lean body weight of a young adult female who weighs 132 pounds and has wrist diameter of 2 inches, abdominal circumference of 27 inches, hip circumference of 37 inches, and forearm circumference of 7 inches is

\[ 19.81 + 0.73 \times 132 + 21.2 \times 2 - 0.88 \times 27 - 1.39 \times 37 + 2.43 \times 7 = 100.39 \text{ pounds}. \]

It follows that her body fat weighs \( 132 - 100.39 = 31.61 \) pounds. To compute the body fat percent we calculate \( \frac{31.61}{132} \) and find 23.95%. 

1. **Parentheses and grouping**: In typewriter notation, \( \frac{5.7 + 8.3}{5.2 - 9.4} \) is \((5.7 + 8.3) \div (5.2 - 9.4)\), which equals \(-3.33 \ldots\) and so is rounded to two decimal places as \(-3.33\).

2. **Evaluate expression**: In typewriter notation, \( \frac{8.4}{3.5 + e^{-6.2}} \) is \(8.4 \div (3.5 + e^{(\text{negative} \ 6.2)})\), which equals 2.398... and so is rounded to two decimal places as 2.40.

3. **Evaluate expression**: In typewriter notation, \( \left(\frac{7 + 1}{e}\right)^{\frac{\pi}{2}} \) is \((7 + 1 \div e) \wedge (5 \div (2 + \pi))\), which equals 6.973... and so is rounded to two decimal places as 6.97. This can also be done as a chain calculation.

4. **Gas mileage**: The number of gallons required to travel 27 miles is
   \[ g = \frac{27}{15} = 1.8 \text{ gallons}. \]

   The number of gallons required to travel 250 miles is
   \[ g = \frac{250}{15} = 16.67 \text{ gallons}. \]

5. **Kepler’s third law**: The mean distance from Pluto to the sun is
   \[ D = 93 \times 249^{2/3} = 3680.86 \text{ million miles}, \]
   or about 3681 million miles. For Earth we have \( P = 1 \text{ year} \), and the mean distance is
   \[ D = 93 \times 1^{2/3} = 93 \text{ million miles}. \]

6. **Traffic signal**: If the approach speed is 80 feet per second then the length of the yellow light should be
   \[ n = 1 + \frac{80}{30} + \frac{100}{80} = 4.92 \text{ seconds}. \]
1.1 FUNCTIONS GIVEN BY FORMULAS

E-1. Determining when a correspondence is a function:

(a) The correspondence assigns to each element of the set \( D \) exactly one element of the set \( R \), so it defines a function \( f \) with domain \( D \) and range \( R \).

(b) The correspondence assigns to each element of the set \( D \) exactly one element of the set \( R \), so it defines a function \( f \) with domain \( D \) and range \( R \).

(c) The correspondence assigns to the element 1 both 8 and 5, so it does not define a function.

(d) The correspondence assigns to each element of the set \( D \) exactly one element of the set \( R \), so it defines a function \( f \) with domain \( D \) and range \( R \).

E-3. Functions on other sets:

(a) This defines a function because each president has exactly one last name.

(b) This does not define a function because there are some last names (such as Johnson) shared by different presidents. Another reason that this fails to be a function is that, at the time this text was written, not all last names were represented by presidents.

(c) This does not define a function because there are some automobiles for which more than one color appears on the body.

E-5. Finding the range:

(a) The smallest range is the set of all numbers of the form \( x + 2 \) for some real number \( x \). Because every real number is of this form (just subtract 2 to find \( x \)), the smallest range is the set of all real numbers.

(b) The smallest range is the set of all numbers of the form \( x^2 \) for some real number \( x \). A number of that form is nonnegative, and every nonnegative number can be
written as the square of either of its square roots. Thus the smallest range is the set of all nonnegative real numbers.

(c) The smallest range is the set of all numbers of the form \(x^3\) for some real number \(x\). Every number can be written as the cube of its cube root, so the smallest range is the set of all real numbers.

(d) The smallest range is the set of all numbers of the form \(x^8 + 7\) for some real number \(x\). A number of that form is greater than or equal to 7. Furthermore, every number greater than or equal to 7 can be written in this form: Simply subtract 7 and take an eighth root to find \(x\). Thus the smallest range is the set of all real numbers greater than or equal to 7.

E-7. Onto functions:

(a) This function is onto: Given a number \(y\) in the range, \(y\) is a positive even integer, so \(\frac{y}{2}\) is a positive integer and \(f\left(\frac{y}{2}\right) = 2\frac{y}{2} = y\).

(b) This function is not onto: Because \(x^8\) is nonnegative when \(x\) is a real number, \(-1\) is not a function value. (In fact, no negative number is a function value.)

(c) This function is not onto: Because \(\frac{1}{x^2 + 1}\) is nonnegative when \(x\) is a real number, \(-1\) is not a function value. (In fact, all function values are greater than 0 and less than or equal to 1.)

(d) This function is onto because every element of the range is a function value.

(e) This function is onto because every element of the range is a function value.

E-9. Bijections:

(a) First, this function is one-to-one: Assume that \(f(x) = f(y)\). Then \(2x = 2y\), and thus \(x = y\) (divide both sides by 2). This shows that if \(x \neq y\) then \(f(x) \neq f(y)\). Second, we saw from Part (a) of Exercise E-7 that this function is onto. Thus it is a bijection.

(b) This function is not a bijection because (by Part (b) of Exercise E-7) it is not onto.

(c) This function is not a bijection because (by Part (c) of Exercise E-7) it is not onto.

(d) First, this function is one-to-one because no two distinct points in the domain are assigned to the same element of the range. Second, we saw from Part (d) of Exercise E-7 that this function is onto. Thus it is a bijection.

(e) This function is not a bijection because (by Part (e) of Exercise E-7) it is not onto. Another reason that this fails to be a bijection is that it is not one-to-one (because \(f(1) = f(3)\)).
S-1. **Evaluating formulas:** To evaluate \( f(x) = \frac{\sqrt{x + 1}}{x^2 + 1} \) at \( x = 2 \), simply substitute 2 for \( x \).

Thus the value of \( f \) at 2 is \( \frac{\sqrt{2 + 1}}{2^2 + 1} \), which equals 0.346... and so is rounded to 0.35.

S-3. **Evaluating formulas:** To evaluate \( g(x, y) = \frac{x^2 + y^3}{x^2 + y^2} \) at \( x = 4.1 \), \( y = 2.6 \), simply substitute 4.1 for \( x \) and 2.6 for \( y \). Thus the value of \( g \) when \( x = 4.1 \) and \( y = 2.6 \) is \( \frac{4.1^3 + 2.6^3}{4.1^2 + 2.6^2} \), which equals 3.669... and so is rounded to 3.67.

S-5. **Getting function values:** To get the function value \( f(6.1) \), substitute 6.1 for \( s \) in the formula \( f(s) = \frac{s^2 + 1}{s^2 - 1} \). Thus \( f(6.1) = \frac{6.1^2 + 1}{6.1^2 - 1} \), which equals 1.055... and so is rounded to 1.06.

S-7. **Evaluating functions of several variables:** To get the function value \( h(3, 2, 2, 9.7) \), substitute 3 for \( x \), 2 for \( y \), and 9.7 for \( z \) in the formula \( h(x, y, z) = \frac{x^y}{z} \). Thus \( h(3, 2, 2, 9.7) = \frac{3^2}{9.7} \), which equals 1.155... and so is rounded to 1.16.

S-9. **Using formulas:** To express the cost of buying 2 bags of potato chips, 3 sodas, and 5 hot dogs, note that these values correspond to \( p = 2 \), \( s = 3 \), and \( h = 5 \), and so the cost is expressed by \( c(2, 3, 5) \).

S-11. **Practicing calculations with formulas:** In each case, we want to find the value of \( f(3) \), given the formula for \( f(x) \), so we simply substitute 3 for \( x \):

(a) \( f(3) = 3 \times 3 + \frac{1}{3} \), which equals 9.333... and so is rounded to 9.33.

(b) \( f(3) = 3^{-3} - \frac{3^2}{3 + 1} \), which equals -2.212... and so is rounded to -2.21.

(c) \( f(3) = \sqrt{2} \times 3 + 5 \), which equals 3.316... and so is rounded to 3.32.

S-13. **Evaluating functions of several variables:** To evaluate \( M = P(e^r - 1)/(1 - e^{-rt}) \) at \( r = 0.1 \), \( P = 8300 \), and \( t = 24 \), substitute these values in the formula. The value is \( 8300(e^{0.1} - 1)/(1 - e^{-0.1 \times 24}) \), which equals 960.01.

1. **Tax owed:**

(a) In functional notation the tax owed on a taxable income of $13,000 is \( T(13,000) \).
   
   The value is
   
   \[ T(13,000) = 0.11 \times 13,000 - 500 = 930 \text{ dollars}. \]

(b) The tax owed on a taxable income of $14,000 is

\[ T(14,000) = 0.11 \times 14,000 - 500 = 1040 \text{ dollars}. \]

Using the answer to Part (a), we see that the tax increases by 1040 - 930 = 110 dollars.
(c) The tax owed on a taxable income of $15,000 is

\[ T(15,000) = 0.11 \times 15,000 - 500 = 1150 \text{ dollars.} \]

Thus the tax increases by $1150 - 1040 = 110$ dollars again.

3. Flying ball:

(a) In functional notation the velocity 1 second after the ball is thrown is \( V(1) \). The value is

\[ V(1) = 40 - 32 \times 1 = 8 \text{ feet per second.} \]

Because the upward velocity is positive, the ball is rising.

(b) The velocity 2 seconds after the ball is thrown is

\[ V(2) = 40 - 32 \times 2 = -24 \text{ feet per second.} \]

Because the upward velocity is negative, the ball is falling.

(c) The velocity 1.25 seconds after the ball is thrown is

\[ V(1.25) = 40 - 32 \times 1.25 = 0 \text{ feet per second.} \]

Because the velocity is 0, we surmise from Parts (a) and (b) that the ball is at the peak of its flight.

(d) Using the answers to Parts (a) and (b), we see that from 1 second to 2 seconds the velocity changes by

\[ V(2) - V(1) = -24 - 8 = -32 \text{ feet per second.} \]

Because

\[ V(3) = 40 - 32 \times 3 = -56 \text{ feet per second,} \]

from 2 seconds to 3 seconds the velocity changes by

\[ V(3) - V(2) = -56 - (-24) = -32 \text{ feet per second.} \]

Because

\[ V(4) = 40 - 32 \times 4 = -88 \text{ feet per second,} \]

from 3 seconds to 4 seconds the velocity changes by

\[ V(4) - V(3) = -88 - (-56) = -32 \text{ feet per second.} \]

Over each of these 1-second intervals the velocity changes by -32 feet per second. In practical terms, this means that the velocity decreases by 32 feet per second.
for each second that passes. This indicates that the downward acceleration of the ball is constant at 32 feet per second per second, which makes sense because the acceleration due to gravity is constant near the surface of Earth.

5. A population of deer:

(a) Now $N(0)$ represents the number of deer initially on the reserve and

$$N(0) = \frac{12.36}{0.03 + 0.55^0} = 12 \text{ deer}.$$ 

So there were 12 deer in the initial herd.

(b) We calculate using

$$N(10) = \frac{12.36}{0.03 + 0.55^{10}} = 379.92 \text{ deer}.$$ 

This says that after 10 years there should be about 380 deer in the reserve.

(c) The number of deer in the herd after 15 years is represented by $N(15)$, and this value is

$$N(15) = \frac{0.36}{0.03 + 0.55^{15}} = 410.26 \text{ deer}.$$ 

This says that there should be about 410 deer in the reserve after 15 years.

(d) The difference in the deer population from the tenth to the fifteenth year is given by $N(15) - N(10) = 410.26 - 379.92 = 30.34$. Thus the population increased by about 30 deer from the tenth to the fifteenth year.

7. Radioactive substances:

(a) The amount of carbon 14 left after 800 years is expressed in functional notation as $C(800)$. This is calculated as

$$C(800) = 5 \times 0.5^{800/5730} = 4.54 \text{ grams}.$$ 

(b) There are many ways to do this part of the exercise. The simplest is to note that half the amount is left when the exponent of 0.5 is 1 since then the 5 is multiplied by $0.5 = \frac{1}{2}$. The exponent in the formula is 1 when $t = 5730$ years.

Another way to do this part is to experiment with various values for $t$, increasing the value when the answer is less than 2.5 and decreasing it when the answer comes out more than 2.5. Students are in fact discovering and executing a crude version of the bisection method.
9. **What if interest is compounded more often than monthly?**

(a) We would expect our monthly payment to be higher if the interest is compounded daily since additional interest is charged on interest which has been compounded.

(b) Continuous compounding should result in a larger monthly payment since the interest is compounded at an even faster rate than with daily compounding.

(c) We are given that \( P = 7800 \) and \( t = 48 \). Because the APR is \( 8.04\% \) or 0.0804, we compute that

\[
 r = \frac{\text{APR}}{12} = \frac{0.0804}{12} = 0.0067.
\]

Thus the monthly payment is

\[
 M(7800, 0.0067, 48) = 7800\left(\frac{e^{0.0067} - 1}{1 - e^{-0.0067 \times 48}}\right) = 190.67 \text{ dollars}.
\]

Our monthly payment here is 10 cents higher than if interest is compounded monthly as in Example 1.2 (where the payment was $190.57).

11. **How much can I borrow?**

(a) Since we will be paying $350 per month for 4 years, then we will be making 48 payments, or \( t = 48 \). Also, \( r \) is the monthly interest rate of 0.75\%, or 0.0075 as a decimal. The amount of money we can afford to borrow in this case is given in functional notation by \( P(350, 0.0075, 48) \). It is calculated as

\[
P(350, 0.0075, 48) = 350 \times \frac{1}{0.0075} \times \left(1 - \frac{1}{(1 + 0.0075)^{48}}\right) = 14,064.67.
\]

(b) If the monthly interest rate is 0.25\% then we can afford to borrow

\[
P(350, 0.0025, 48) = 350 \times \frac{1}{0.0025} \times \left(1 - \frac{1}{(1 + 0.0025)^{48}}\right) = 15,812.54.
\]

(c) If we make monthly payments over 5 years then we will make 60 payments in all. So now we can afford to borrow

\[
P(350, 0.0025, 60) = 350 \times \frac{1}{0.0025} \times \left(1 - \frac{1}{(1 + 0.0025)^{60}}\right) = 19,478.33.
\]

13. **Brightness of stars**: Here we have \( m_1 = -1.45 \) and \( m_2 = 2.04 \). Thus

\[
t = 2.512^{m_2 - m_1} = 2.512^{2.04-(-1.45)} = 2.512^{3.49} = 24.89.
\]

Hence Sirius appears 24.89 times brighter than Polaris.

15. **Parallax**: We are given that \( p = 0.751 \). Thus the distance from Alpha Centauri to the sun is about

\[
d(0.751) = \frac{3.26}{0.751} = 4.34 \text{ light-years}.
\]
17. Mitscherlich’s equation:

(a) We are given that \( b = 1 \). Thus the percentage (as a decimal) of maximum yield is

\[
Y(1) = 1 - 0.5^1 = 0.5.
\]

Hence 50% of maximum yield is produced if 1 baule is applied.

(b) In functional notation the percentage (as a decimal) of maximum yield produced by 3 baules is \( Y(3) \). The value is

\[
Y(3) = 1 - 0.5^3 = 0.875,
\]

or about 0.88. This is 88% of maximum yield.

(c) Now 500 pounds of nitrogen per acre corresponds to \( \frac{500}{223} \) baules, so the percentage (as a decimal) of maximum yield is \( 1 - 0.5^{\frac{500}{223}} \), or about 0.79. This is 79% of maximum yield.

19. Thermal conductivity: We are given that \( k = 0.85 \) for glass and that \( t_1 = 24, t_2 = 5 \).

(a) Because \( d = 0.007 \), the heat flow is

\[
Q = \frac{0.85(24 - 5)}{0.007} = 2307.14 \text{ watts per square meter}.
\]

(b) The total heat loss is

\[
\text{Heat flow \times Area of window} = 2307.14 \times 2.5,
\]

or about 5767.85 watts.

21. Fault rupture length: Here we have \( M = 6.5 \), so the expected length is

\[
L(6.5) = 0.0000017 \times 10.47^{6.5} = 7.25 \text{ kilometers}.
\]

23. Research project: Answers will vary.
1.2 FUNCTIONS GIVEN BY TABLES

E-1. Average rates of change using numbers:

(a) The average rate of change is

\[ \frac{f(3) - f(2)}{3 - 2} = \frac{27 - 8}{1} = 19. \]

(b) The average rate of change is

\[ \frac{f(5) - f(2)}{5 - 2} = \frac{19 - 10}{3} = 3. \]

(c) The average rate of change is

\[ \frac{f(4) - f(2)}{4 - 2} = \frac{\frac{1}{2} - \frac{1}{2}}{2} = -\frac{1}{8}. \]

(d) The average rate of change is

\[ \frac{f(3) - f(1)}{3 - 1} = \frac{12 - 2}{2} = 5. \]

E-3. Linear functions: If \( f(x) = mx + b \) then the average rate of change is

\[ \frac{f(q) - f(p)}{q - p} = \frac{(mq + b) - (mp + b)}{q - p} = \frac{mq - mp}{q - p} = \frac{m(q - p)}{q - p} = m. \]

Thus the average rate of change is \( m \), and this does not depend on either \( p \) or \( q \).

E-5. Sketching a graph: Because the average rate of change from \( x = 0 \) to \( x = 4 \) is 2, the change in \( f \) over this interval is \( 2 \times (4 - 0) = 8 \). Thus \( f(4) - f(0) = 8 \). Because \( f(0) = 1 \), we have \( f(4) = 8 + 1 = 9 \). Also, because the average rate of change from \( x = 4 \) to \( x = 6 \) is \( -1 \), the change in \( f \) over this interval is \( -1 \times (6 - 4) = -2 \). Thus \( f(6) - f(4) = -2 \). Because we saw that \( f(4) = 9 \), we have \( f(6) = -2 + 9 = 7 \). Thus the three function values determined are \( f(0) = 1 \), \( f(4) = 9 \), and \( f(6) = 7 \). Any graph exhibiting these three values is correct. Here is one possible graph:
E-7. **The effect of adding a linear function:** The average rate of change for $g$ is
\[ \frac{g(b) - g(a)}{b - a} = \frac{(f(b) + 3b + 5) - (f(a) + 3a + 5)}{b - a} \]
\[ = \frac{(f(b) - f(a)) + 3(b - a)}{b - a} = \frac{f(b) - f(a)}{b - a} + 3. \]
The first term on the far right is the average rate of change for $f$, so its value is 7; thus the average rate of change for $g$ is $7 + 3 = 10$.

E-9. **Central difference quotients:** The average rate of change from $x = 2$ to $x = 3$ is
\[ \frac{f(3) - f(2)}{3 - 2} = \frac{12 - 9}{1} = 3. \]
The average rate of change from $x = 3$ to $x = 5$ is
\[ \frac{f(5) - f(3)}{5 - 3} = \frac{16 - 12}{2} = 2. \]
The average of these two rates is $\frac{3 + 2}{2} = 2.5$. Thus the central difference quotient at $x = 3$ is $2.5$.

S-1. **A tabulated function:** Here $N(20)$ is 23.8 since that is the corresponding value in the table.

S-3. **Average rate of change:** The average rate of change in $N$ from $t = 20$ to $t = 30$ is given by the change in $N$ divided by the change in $t$:
\[ \frac{N(30) - N(20)}{30 - 20} = \frac{44.6 - 23.8}{10} = 2.08. \]
Thus the average rate of change in $N$ is 2.08.

S-5. **Averaging:** We can estimate the value of $N(35)$ by finding the average of $N(30)$ and $N(40)$ since 35 is the average of 30 and 40. The average is $\frac{N(30) + N(40)}{2} = \frac{44.6 + 51.3}{2}$, which equals 47.95, or about 48.0.

S-7. **Using average rates of change:** To estimate the value of $N(37)$, we calculate $N(37)$ as $N(30) + 7$ years of change at the average rate, that is, $N(37)$ is estimated to be $N(30) + 7 \times 0.67 = 44.6 + 7 \times 0.67$, which equals 49.29, or about 49.3.

S-9. **When limiting values occur:** We expect $c$ to have a limiting value of 0. This is because the average speed $c$ gets closer and closer to 0 as the time $t$ required to travel 100 miles increases.

S-11. **Average rate of change:** The average rate of change in $f$ from $x = 15$ to $x = 20$ is given by the change in $f$ divided by the change in $x$:
\[ \frac{f(20) - f(15)}{20 - 15} = \frac{-7.9 - (-3.6)}{5} = -0.86. \]
Thus the average rate of change in $f$ is $-0.86$. 
S-13. **A table:** Here \( N(2000) \) is 427.0 because that is the corresponding value in the table.

S-15. **Using average rate of change:** To estimate the value of \( N(2003) \), we calculate \( N(2000) \) plus 3 years of change at the average rate. That is, \( N(2003) \) is estimated to be

\[
N(2000) + 3 \times -1.67 = 427.0 - 5.01 = 421.99,
\]

or about 422.

1. **The American food dollar:**

   (a) Here \( P(1980) = 27\% \). This means that in 1980 Americans spent 27% of their food dollars eating out.

   (b) The expression \( P(1990) \) is the percent of the American food dollar spent eating away from home in 1990. Since 1990 falls halfway between 1980 and 2000, our best guess at \( P(1990) \) is the average of \( P(1980) \) and \( P(2000) \), or

   \[
   \frac{P(1980) + P(2000)}{2} = \frac{27 + 37}{2} = 32.
   \]

   Approximately 32% of the American food dollar in 1990 was spent eating out.

   (c) The average rate of change per year in percentage of the food dollar spent away from home from 1980 to 2000 is

   \[
   \frac{P(2000) - P(1980)}{2000 - 1980} = \frac{37 - 27}{20} = 0.5,
   \]

   or 0.5 percentage point per year.

   (d) The expression \( P(1997) \) is the percent of the American food dollar spent eating away from home in 1997. We estimate it as

   \[
   P(1997) = P(1980) + 17 \times \text{yearly change} = 27 + 17 \times 0.5 = 35.5,
   \]

   or about 36%.

   (e) Assuming the increase in \( P \) continues at the same rate of about 0.5 percentage point per year as we calculated in Part (c), then

   \[
   P(2003) = P(2000) + 3 \times \text{yearly change} = 37 + 3 \times 0.5 = 38.5,
   \]

   or about 39%.

3. **Cable TV:**

   (a) Here \( C(1995) = 60 \) million households. This means that 60 million American households had cable TV in 1995.
(b) The average rate of change per year from 1990 to 1995 is

\[
\frac{\text{Change in } C}{\text{Time elapsed}} = \frac{C(1995) - C(1990)}{5} = \frac{60 - 52}{5} = 1.6 \text{ million households per year.}
\]

(c) Since the average rate of change per year from 1990 to 1995 is 1.6 million households per year, we estimate

\[
C(1992) = C(1990) + 2 \times \text{yearly change} = 52 + 2 \times 1.6 = 55.2,
\]

or about 55 million households.

5. **A troublesome snowball**: Here \( W(t) \) is the volume of dirty water soaked into the carpet, so its limiting value is the total volume of water frozen in the snowball. The limiting value is reached when the snowball has completely melted.

7. **Carbon 14**:

(a) The average yearly rate of change for the first 5000 years is

\[
\frac{\text{Amount of change}}{\text{Years elapsed}} = \frac{C(5) - C(0)}{5000} = \frac{2.73 - 5}{5000} = -4.54 \times 10^{-4} \text{ gram per year.}
\]

That is \(-0.000454\) gram per year.

(b) We use the average yearly rate of change from Part (a):

\[
C(1.236) = C(0) + 1236 \times \text{yearly rate of change} = 5 + 1236 \times -0.000454 = 4.44 \text{ grams.}
\]

(c) The limiting value is zero since all of the carbon 14 will eventually decay.

9. **Effective percentage rate for various compounding periods**:

(a) We have that \( n = 1 \) represents compounding yearly, \( n = 2 \) represents compounding semiannually, \( n = 12 \) represents compounding monthly, \( n = 365 \) represents compounding daily, \( n = 8760 \) represents compounding hourly, and \( n = 525,600 \) represents compounding every minute.

(b) We have that \( E(12) \) is the EAR when compounding monthly, and \( E(12) = 12.683\% \).

(c) If interest is compounded daily then the EAR is \( E(365) \). So the interest accrued in one year is

\[
8000 \times E(365) = 8000 \times 0.12747 = \$1019.76.
\]

(d) If interest were compounded continuously then the EAR would probably be about 12.750\%. As the length of the compounding period decreases, the EAR given in the table appears to stabilize at this value.
11. **Growth in height:**

(a) In functional notation, the height of the man at age 13 is given by \( H(13) \).

From ages 10 to 15, the average yearly growth rate in height is

\[
\frac{\text{Inches increased}}{\text{Years elapsed}} = \frac{67.0 - 55.0}{5} = 2.4 \text{ inches per year.}
\]

Since age 13 is 3 years after age 10, we can estimate \( H(13) \) as

\[
H(10) + 3 \times \text{yearly growth} = 55.0 + 3 \times 2.4 = 62.2 \text{ inches.}
\]

(b) i. We calculate the average yearly growth rate for each 5-year period just as we calculated 2.4 inches per year as the average yearly growth rate from ages 10 to 15 in Part (a). The average yearly growth rate is measured in inches per year.

<table>
<thead>
<tr>
<th>Age change</th>
<th>0 to 5</th>
<th>5 to 10</th>
<th>10 to 15</th>
<th>15 to 20</th>
<th>20 to 25</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average yearly growth rate</td>
<td>4.2</td>
<td>2.5</td>
<td>2.4</td>
<td>1.3</td>
<td>0.1</td>
</tr>
</tbody>
</table>

ii. The man grew the most from age 0 to age 5.

iii. The trend is that as the man gets older, he grows more slowly.

(c) It is reasonable to guess that 74 or 75 inches is the limiting value for the height of this man. He grew only 0.5 inches from ages 20 to 25, so it is reasonable to expect little or no further growth from age 25 on.

13. **Tax owed:**

(a) The average rate of change over the first interval is

\[
\frac{T(16,200) - T(16,000)}{16,200 - 16,000} = \frac{888 - 870}{200} = 0.09 \text{ dollar per dollar.}
\]

Continuing in this way, we get the following table, where the rate of change is measured in dollars per dollar.

<table>
<thead>
<tr>
<th>Interval</th>
<th>Rate of change</th>
</tr>
</thead>
<tbody>
<tr>
<td>16,000 to 16,200</td>
<td>0.09</td>
</tr>
<tr>
<td>16,200 to 16,400</td>
<td>0.09</td>
</tr>
<tr>
<td>16,400 to 16,600</td>
<td>0.09</td>
</tr>
</tbody>
</table>

(b) The average rate of change has a constant value of 0.09 dollar per dollar. This suggests that, at every income level in the table, for every increase of $1 in taxable income the tax owed increases by $0.09, or 9 cents.

(c) Because the average rate of change is a nonzero constant and thus does not tend to 0, we would expect \( T \) not to have a limiting value but rather to increase at a constant rate as \( I \) increases.
15. **Yellowfin tuna:**

(a) The average rate of change in weight is

\[
\frac{W(110) - W(100)}{110 - 100} = \frac{56.8 - 42.5}{10} = 1.43 \text{ pounds per centimeter.}
\]

(b) The average rate of change in weight is

\[
\frac{W(180) - W(160)}{180 - 160} = \frac{256 - 179}{20} = 3.85 \text{ pounds per centimeter.}
\]

(c) Examining the table shows that the rate of change in weight is smaller for small tuna than it is for large tuna. Hence an extra centimeter of length makes more difference in weight for a large tuna.

(d) To estimate the weight of a yellowfin tuna that is 167 centimeters long we use the average rate of change we found in Part (b):

\[
W(160) + 7 \times 3.85 = 179 + 7 \times 3.85 = 205.95 \text{ pounds.}
\]

Hence the weight of a yellowfin tuna that is 167 centimeters long is 205.95, or about 206.0, pounds.

(e) Here we are thinking of the weight as the variable and the length as a function of the weight. The average rate of change in length is

\[
\frac{\text{Length at 256 pounds} - \text{Length at 179 pounds}}{256 - 179} = \frac{180 - 160}{256 - 179} = 0.26,
\]

so the average rate of change is 0.26 centimeter per pound. Note that this number is the reciprocal of the answer from Part (b).

(f) To estimate the length of a yellowfin tuna that weighs 225 pounds we use the average rate of change we found in Part (e):

\[
\text{Length at 179 pounds} + (225 - 179) \times 0.26 = 160 + 46 \times 0.26 = 171.96 \text{ centimeters.}
\]

Hence the length of a yellowfin tuna that weighs 225 pounds is 171.96, or about 172, centimeters.
17. **Widget production:**

(a) The average rate of change over the first interval is

\[
\frac{W(20) - W(10)}{10} = \frac{37.5 - 25.0}{10} = 1.25 \text{ thousand widgets per worker.}
\]

Continuing in this way, we get the following table, where the rate of change is measured in thousands of widgets per worker.

<table>
<thead>
<tr>
<th>Interval</th>
<th>Rate of change</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 to 20</td>
<td>1.25</td>
</tr>
<tr>
<td>20 to 30</td>
<td>0.63</td>
</tr>
<tr>
<td>30 to 40</td>
<td>0.31</td>
</tr>
<tr>
<td>40 to 50</td>
<td>0.15</td>
</tr>
</tbody>
</table>

(b) The average rate of change decreases and approaches 0 as we go across the table. This means that the increase in production gained from adding another worker gets smaller and smaller as the level of workers employed moves higher and higher. Eventually there is very little benefit in employing an extra worker.

(c) To estimate how many widgets will be produced if there are 55 full-time workers, we use the entry from the table for the average rate of change over the last interval:

\[
W(50) + 5 \times 0.15 = 48.4 + 5 \times 0.15 = 49.15 \text{ thousand widgets.}
\]

Hence the number of widgets produced by 55 full-time workers is about 49.2 thousand, or 49,200.

(d) Because the average rate of change is decreasing, the actual increase in production in going from 50 to 55 workers is likely to be less than what the average rate of change from 40 to 50 suggests. Thus our estimate is likely to be too high.

19. **The Margaria-Kalame test:**

(a) The average rate of change per year in excellence level from 25 years to 35 years old is

\[
\frac{168 - 210}{35 - 25} = -4.2 \text{ points per year.}
\]

(b) We estimate the power score needed for a 27-year-old man using the score for a 25-year-old man and the average rate of change from Part (a):

\[
210 + 2 \times -4.2 = 201.6 \text{ points.}
\]

Hence the power score that would merit an excellent rating for a 27-year-old man is 201.6, or about 202, points.

(c) The decrease in power score for excellent rating over these three periods is the greatest in the second period (35 years to 45 years), so we would expect to see the greatest decrease in leg power from 35 to 45 years old.
21. A **home experiment**: Answers will vary greatly. In general, there will be initially a small percentage of bread surface covered with mold. That percentage will quickly rise as the mold covers much of the bread surface. There are usually a few small patches which the mold covers more slowly. Ultimately all the bread surface is covered with mold. Here is a typical data table:

<table>
<thead>
<tr>
<th>Time</th>
<th>8 am</th>
<th>4 pm</th>
<th>12 am</th>
<th>8 am</th>
<th>4 pm</th>
<th>12 am</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mold</td>
<td>10%</td>
<td>25%</td>
<td>60%</td>
<td>98%</td>
<td>100%</td>
<td>100%</td>
</tr>
</tbody>
</table>

### 1.3 FUNCTIONS GIVEN BY GRAPHS

**E-1.** **Secant lines for graphs which are concave down**: One possible graph is the following.

(a) The secant line is below the graph between the two points and above the graph outside those points.

(b) Recall that using average rates of change to approximate function values is the same as approximating the graph by its secant line. Using Part (a), we see that the average rate of change will give an estimate that is too small between the two points used and too large outside them.

**S-1.** **A function given by a graph**: The value of \(f(1.8)\) is obtained from the graph by locating 1.8 on the horizontal axis, then moving vertically up to the point of the graph over 1.8, and then moving horizontally to locate the corresponding value on the vertical axis. Thus \(f(1.8)\) is a little below 3.5, so about 3.3.

**S-3.** **The maximum**: The graph reaches a maximum at its highest point, which is when \(x = 2.4\) and \(f\) is about 4.0.

**S-5.** **Decreasing functions**: The graph is decreasing when it is falling, so that is when \(x\) is between 2.4 and 4.8.
S-7. **Concavity again:** If a graph is increasing, but at a decreasing rate, then it will look like the portion of the graph in Exercise S-1 for \( x \) between 1.8 and 2.4; in particular the graph is concave down.

S-9. **Maximum and zero:** Answers will vary. One solution is

![Graph](image)

S-11. **Inflection points:** The points of inflection occur at about \( x = 1.2 \) and \( x = 3.6 \).

1. **Sketching a graph with given concavity:**
   (a) ![Sketch](image)
   (b) ![Sketch](image)

3. **A stock market investment**
   (a) According to the exercise our original investment made in 1970 was $10,000, so \( v(1970) = \$10,000 \). Our investment lost half its value in the 70’s, so \( v(1980) = \$5000 \). The investment was worth $35,000 in 1990, so \( v(1990) = \$35,000 \). Since the stock remained stable after 1990, the value of the investment stays at the same level of $35,000. So \( v(2000) = \$35,000 \).
The graph should reflect the function values from Part (a), and it should increase from 1980 to 1990 and remain flat after that. There remains room for interpretation, and many graphs similar to the one above are acceptable.

(c) The stock was most rapidly increasing in the 1980’s. For the graph above, 1985 is a good guess. (The answer will depend on what the graph in Part (b) looks like.)

5. **River flow:**

(a) Because the end of July is 7 months since the start of the year, the flow at that time is $F(7)$ in functional notation. According to the graph, the value is about 1500 cubic feet per second.

(b) The flow is at its greatest at $t = 6$, which corresponds to the end of June.

(c) The flow is increasing the fastest at $t = 5$, corresponding to an inflection point on the graph. The time is the end of May.

(d) The graph is practically level from $t = 0$ to $t = 2$, so the function is nearly constant there, and the average rate of change over this interval is about 0 cubic feet per second.

(e) Because the flow is measured near the river’s headwaters in the Rocky Mountains, we expect any change in flow to come from melting snow primarily. This is consistent with almost no change in flow during the first two months of the year (as seen in Part (d)), a maximum increase in flow at the end of May (as seen in Part (c)), and a peak flow one month later (as seen in Part (b)).
7. **Cutting trees:**

   (a) The graph shows the net stumpage value of a 60-year-old Douglas fir stand to be about $14,000 per acre.

   (b) The graph shows a 110-year-old Douglas fir stand has net stumpage value of $40,000 per acre.

   (c) When the costs involved in harvesting equal the commercial value, the net stumpage value is 0. Thus we need to know when $V$ is 0. The graph shows that a 30-year-old Douglas fir stand has a net stumpage value of $0 per acre.

   (d) The net stumpage value seems to be increasing the fastest in trees about 60 years old.

   (e) We would expect the trees to reach an age where they don’t grow as much. When this happens the net stumpage value should level out. Here is one possible extended graph for the stumpage value of the Douglas fir.

9. **Tornadoes in Oklahoma:**

   (a) The most tornadoes were reported in 1999. There were about 145 tornadoes reported that year.

   (b) The fewest tornadoes were reported in 2002. There were about 18 tornadoes reported then.

   (c) In 1995 there were about 79 tornadoes reported, and in 1996 there were about 46 tornadoes reported. Hence the average rate of decrease was

   \[
   \text{Decrease in } T \quad \text{Years elapsed} = \frac{79 - 46}{1} = 33 \text{ tornadoes per year.}
   \]

   (d) In 1997 the number of tornadoes reported was about 55, and in 1999 the number of tornadoes reported was about 145. Hence the average rate of increase was

   \[
   \text{Increase in } T \quad \text{Years elapsed} = \frac{145 - 55}{2} = 45 \text{ tornadoes per year.} \]
(e) The number of tornadoes reported in 1996 was about 46, and the number of tornadoes reported in 2000 was about 44. Hence the average rate of change was

\[
\frac{\text{Change in } T}{\text{Years elapsed}} = \frac{44 - 46}{4} = -0.5 \text{ tornado per year.}
\]

This is close to 0, and the answer can vary depending on how the graph is interpreted.

11. **Driving a car**: There are many possible stories. Common elements include that you start 6 miles from home and drive towards home, arriving there in about 12 minutes, staying at home for about 4 minutes, then driving about \(2\frac{1}{2}\) miles away from home and staying there.

Additional features might be the velocity (rate of change) towards home as about 30 mph and the velocity away from home as about 15 mph.

13. **Photosynthesis**: In Parts (a) and (b) answers may vary somewhat.

(a) We want to find where the graph corresponding to 80 degrees crosses the horizontal axis. The crossing point at about 0.7 thousand foot-candles, or about 700 foot-candles.

(b) We want to find where the graph corresponding to 80 degrees crosses the graph corresponding to 40 degrees. The crossing point is at about 0.8 thousand foot-candles, or about 800 foot-candles.

(c) Among the three graphs, the graph corresponding to 80 degrees meets the vertical axis at the lowest point, so a temperature of 80 degrees will result in the largest emission of carbon dioxide in the dark.

(d) The graph corresponding to 40 degrees is the most level, so that temperature gives the net exchange that is least sensitive to light.

15. **Carbon dioxide concentrations**:

(a) The minimum of the graph occurs at about 2 p.m.

(b) The maximum concentration is attained over the entire interval from about 6 a.m. to about 9 a.m.

(c) Net absorption is indicated by the interval where the graph is decreasing. This is the period from about 9 a.m. to about 2 p.m.

(d) From 6 a.m. to 9 a.m. the graph is level, so the net carbon dioxide exchange is zero during that period.
17. **Profit from fertilizer:**

(a) The maximum value of yield occurs at about 100 on the horizontal axis, so about 100 pounds of nitrogen per acre should be applied to produce maximum crop yield.

(b) The profit can be read by measuring the difference between the yield and cost graphs.

(c) The maximum difference between the graphs occurs at about 70 on the horizontal axis, so about 70 pounds of nitrogen per acre should be applied to produce maximum profit. (In this part answers may vary somewhat.)

19. **Laboratory experiment:** Answers will vary. Please see the website for further information.

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**1.4 FUNCTIONS GIVEN BY WORDS**

**E-1. A fence next to a river:** Assume that the width \( w \) is measured in yards.

(a) Let \( h \) be the height (in yards) of the rectangle. Three sides of the rectangle require fencing, with one having length \( w \) and two having length \( h \). Because we are to use 200 yards of fence, we know that \( w + 2h = 200 \). We solve this equation for \( h \):

\[
\begin{align*}
w + 2h &= 200 \\
2h &= 200 - w \\
h &= \frac{200 - w}{2}.
\end{align*}
\]

Thus the height in yards is \( h = \frac{200 - w}{2} \). (This can also be written as \( h = 100 - \frac{w}{2} \).)

(b) We find the area \( A \) of the rectangle (in square yards) in terms of \( w \) by using the formula for \( h \) from Part (a):

\[
A = wh = w \frac{200 - w}{2}.
\]

Thus the area in square yards is \( A = w \frac{200 - w}{2} \).
E-3. **Circumference in terms of area:** From the given formula we know that \( A = \frac{C^2}{4\pi} \). We solve this equation for \( C \):

\[
A = \frac{C^2}{4\pi} \\
4\pi A = C^2 \\
\sqrt{4\pi A} = C \\
2\sqrt{\pi A} = C.
\]

Thus the circumference can be written as \( C = 2\sqrt{\pi A} \).

E-5. **Making a box:**

(a) Two segments of size \( x \) are removed from the width of the original rectangle to obtain the width of the box. Thus the width of the box is \( 5 - 2x \).

(b) Two segments of size \( x \) are removed from the length of the original rectangle to obtain the length of the box. Thus the length of the box is \( 10 - 2x \).

(c) Because the tabs are folded up to form the box, the height of the box is \( x \).

(d) The volume of the box is

\[
\text{width} \times \text{length} \times \text{height} = (5 - 2x)(10 - 2x)x.
\]

Thus the volume is \( x(5 - 2x)(10 - 2x) \).

E-7. **A soda can with given volume:** Assume that the radius \( x \) and the height \( h \) are both measured in inches.

(a) From the statement of Part (c) of Exercise E-6 we know that the volume of the can is the area of the top times the height. Now the area of the top is \( \pi x^2 \) and the height is \( h \), so the formula for the volume is \( \pi x^2 h \). Since the volume is 25, we have the equation \( \pi x^2 h = 25 \). We solve this equation for \( h \):

\[
\pi x^2 h = 25 \\
h = \frac{25}{\pi x^2}.
\]

Thus the desired expression is \( h = \frac{25}{\pi x^2} \).

(b) From the statement of Part (a) of Exercise E-6 we know that the total surface area is the area of the top plus the area of the bottom plus the area of the cylindrical part. The top and bottom are formed by circles of radius \( x \), so each of these has area \( \pi x^2 \). We also know from the statement of Part (a) of Exercise E-6 that the area
of the cylindrical part of the can is the circumference of the can times the height. Because the radius of the can is \(x\), its circumference is \(2\pi x\), and thus the area of the cylindrical part is \(2\pi x h\). Hence the formula for the total surface area of the can is \(2\pi x^2 + 2\pi x h\). Using the expression for \(h\) in terms of \(x\) from Part (a) gives the formula \(2\pi x^2 + 2\pi x \times \frac{25}{\pi x^2}\). Simplifying gives the expression \(2\pi x^2 + \frac{50}{x}\) for the surface area of the can in terms of \(x\).

S-1. **A description:** If you have $5000 and spend half the balance each month, then the new balance will be

- New balance after 1 month = 5000 - \(\frac{1}{2}\) \times 5000 = 2500
- New balance after 2 months = 2500 - \(\frac{1}{2}\) \times 2500 = 1250
- New balance after 3 months = 1250 - \(\frac{1}{2}\) \times 1250 = 625
- New balance after 4 months = 625 - \(\frac{1}{2}\) \times 625 = 312.5

and so the balance left after 4 months is $312.50.

S-3. **A description:** We know that \(f(0) = 5\) and that each time \(x\) increases by 1, the value of \(f\) triples, that is, it is three times its previous value. Therefore

\[
egin{align*}
f(1) &= 3 \times f(0) = 3 \times 5 = 15 \\
f(2) &= 3 \times f(1) = 3 \times 15 = 45 \\
f(3) &= 3 \times f(2) = 3 \times 45 = 135 \\
f(4) &= 3 \times f(3) = 3 \times 135 = 405
\end{align*}
\]

so \(f(4) = 405\). On the other hand, \(5 \times 3^4\) is also 405.

S-5. **Getting a formula:** If the man loses 67 strands of hair each time he showers, then if he showers \(s\) times, he will lose \(67 \times s\) strands of hair. Thus \(N = 67s\).

S-7. **Getting a formula:** If you start with $500 and you add to that $37 each month, then the balance will be $500 plus $37 times the number of months. Thus the balance \(B\) is given by \(B = 500 + 37t\).

S-9. **Proportionality:** If \(f\) is proportional to \(x\) and the constant of proportionality is 8, then \(f = 8x\).

S-11. **Getting a formula for discounted items:** The cost per item decreases by $2 for each extra item rented. Because it costs $20 per item for 1 item, it costs \(20 - 2 \times 4 = 12\) dollars per item if 5 items are rented. The total cost for 5 items is \(5 \times 12 = 60\) dollars.
1. **United States population growth:**

(a) Since \( t \) is the number of years since 1960, the year 1963 is represented by \( t = 3 \).

In functional notation, the population of the U.S. in 1963 is given by \( N(3) \). To calculate its value, we use the fact that the population increases by 1.2% per year. Since \( N(0) = 180 \), in millions,

\[
N(1) = \text{Population in 1960} + 1.2\% \text{ growth} \\
= 180 + 0.012 \times 180 = 182.16 \text{ million}
\]

\[
N(2) = \text{Population in 1961} + 1.2\% \text{ increase} \\
= 182.16 + 0.012 \times 182.16 = 184.35 \text{ million}
\]

\[
N(3) = \text{Population in 1962} + 1.2\% \text{ growth} \\
= 184.35 + 0.012 \times 184.35 = 186.56 \text{ million}.
\]

(b) The table is given below, calculating \( N(4) \) and \( N(5) \) as in Part (a) above.

<table>
<thead>
<tr>
<th>Year</th>
<th>( t )</th>
<th>( N(t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1960</td>
<td>0</td>
<td>180.00</td>
</tr>
<tr>
<td>1961</td>
<td>1</td>
<td>182.16</td>
</tr>
<tr>
<td>1962</td>
<td>2</td>
<td>184.35</td>
</tr>
<tr>
<td>1963</td>
<td>3</td>
<td>186.56</td>
</tr>
<tr>
<td>1964</td>
<td>4</td>
<td>188.80</td>
</tr>
<tr>
<td>1965</td>
<td>5</td>
<td>191.06</td>
</tr>
</tbody>
</table>

(c) A graph is shown below.
(d) The formula $180 \times 1.012^t$ can be shown to give the same values as those found in Part (b) by substituting $t = 0$, $t = 1, \ldots$ into the formula:

<table>
<thead>
<tr>
<th>$t$</th>
<th>$180 \times 1.012^t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$180 \times 1.012^0 = 180.00$</td>
</tr>
<tr>
<td>1</td>
<td>$180 \times 1.012^1 = 182.16$</td>
</tr>
<tr>
<td>2</td>
<td>$180 \times 1.012^2 = 184.35$</td>
</tr>
<tr>
<td>3</td>
<td>$180 \times 1.012^3 = 186.56$</td>
</tr>
<tr>
<td>4</td>
<td>$180 \times 1.012^4 = 188.80$</td>
</tr>
<tr>
<td>5</td>
<td>$180 \times 1.012^5 = 191.06$</td>
</tr>
</tbody>
</table>

(e) Using the formula and the fact that 2000 corresponds to $t = 40$, we find that the population in 2000 from this prediction is

$$N(40) = 180 \times 1.012^{40} = 290.06 \text{ million people.}$$

This estimate is about 9 million higher than the actual population.

3. Altitude:

(a) Let $t$ be the time in minutes since takeoff and $A$ the altitude in feet. Then

$$A(0) = \text{Initial altitude} = 200 \text{ feet},$$

and

$$A(1) = \text{Initial altitude} + \text{Increase over 1 minute} = 200 + 150 \times 1 = 350 \text{ feet},$$

$$A(2) = \text{Initial altitude} + \text{Increase over 2 minutes} = 200 + 150 \times 2 = 500 \text{ feet},$$

$$A(3) = \text{Initial altitude} + \text{Increase over 3 minutes} = 200 + 150 \times 3 = 650 \text{ feet},$$

and so on. This suggests the formula

$$A = \text{Initial altitude} + \text{Increase over } t \text{ minutes} = 200 + 150 \times t,$$

or $A = 200 + 150t$.

(b) In functional notation the altitude 90 seconds after takeoff is $A(1.5)$ because 90 seconds corresponds to 1.5 minutes. The value is $200 + 150 \times 1.5 = 425 \text{ feet.}$
(c) Here is the graph.

![Graph of altitude变化](image)

The altitude changes at a constant rate, and this is reflected in the fact that the graph is a straight line.

5. A rental:

(a) If we rent the car for 2 days and drive 100 miles, it will cost us

$$2 \text{ days rental } + \text{ charge for 100 miles } = 29 \times 2 + 0.06 \times 100 = 64 \text{ dollars.}$$

(b) Let $d$ be the number of days we rent the car, $m$ the number of miles we drive the car, and $C$ the cost in dollars of renting the car. Then the formula that gives us the cost of renting a car is

$$C(d, m) = 29 \times \text{ days } + 0.06 \times \text{ miles } = 29d + 0.06m.$$  

(c) Since we drove from Dallas to Austin and back then we traveled a total of 500 miles. We kept the car for one week, or 7 days. Hence in functional notation the cost of the rental car is $C(7,500)$. This is calculated as

$$C(7,500) = 29 \times 7 + 0.06 \times 500 = 233 \text{ dollars.}$$

7. Preparing a letter, continued:

(a) There are 2 pages of regular stationery and 1 of fancy letterhead stationery, so the total cost is $2 \times 0.03 + 1 \times 0.16 = 0.22$ dollar, or 22 cents.

(b) We know from Part (a) that the cost for stationery is 0.22 dollar. The secretarial cost is $2 \times 6.25$ dollars, and the cost for the envelope is 0.38 dollar. In total, the letter costs $0.22 + 2 \times 6.25 + 0.38 = 13.10$ dollars.
(c) Let $c$ be the cost in dollars of the stationery and $p$ the number of pages. There are $p - 1$ pages of regular stationery and 1 of fancy letterhead stationery. Then
\[
c(p) = \text{Cost for fancy letterhead stationery} + \text{Cost for regular stationery} \\
= 0.16 + 0.03(p - 1).
\]

(d) Let $h$ be the number of hours spent typing the letter, let $p$ be the number of pages of the letter, and let $C$ be the cost of preparing and mailing the letter (measured in dollars). Then
\[
C(h, p) = \text{Secretarial cost} + \text{Cost for paper} + \text{Cost for envelope} \\
= 6.25h + 0.16 + 0.03(p - 1) + 0.38.
\]
(This can also be written as $C(h, p) = 6.25h + 0.03(p - 1) + 0.54$ or as $C(h, p) = 6.25h + 0.03p + 0.51$.)

(e) Here $h = \frac{25}{60}$ and $p = 2$, so the cost is
\[
C\left(\frac{25}{60}, 2\right) = 6.25 \times \frac{25}{60} + 0.16 + 0.03(2 - 1) + 0.38 = 3.17 \text{ dollars}.
\]

9. **Stock turnover rate:**

(a) If 350 shirts are sold annually then the number of orders of 50 shirts is $\frac{350}{50} = 7$.

(b) The annual stock turnover rate is 7, the number computed in Part (a), because this is the number of times that the average inventory of 50 shirts needs to be replaced if 350 shirts are sold in a year.

(c) If 500 shirts were sold in a year then the annual stock turnover rate would be $\frac{500}{50} = 10$.

(d) Let $T$ be the annual stock turnover rate and $S$ the number of shirts sold in a year. Then
\[
T = \frac{\text{Number of shirts sold}}{\text{Average inventory}} = \frac{S}{50}.
\]
Thus the formula is $T = \frac{S}{50}$.

11. **Total cost:**

(a) Let $N$ be the number of widgets produced in a month and $C$ the total cost in dollars. Because
\[
\text{Total cost} = \text{Variable cost} \times \text{Number of items} + \text{Fixed costs},
\]
we have $C = 15N + 9000$. 
(b) In functional notation the total cost is \( C(250) \), and the value is \( 15 \times 250 + 9000 = 12,750 \) dollars.

13. **More on revenue:**

(a) We compute

\[
\begin{align*}
p(100) &= 50 - 0.01 \times 100 = 49 \\
p(200) &= 50 - 0.01 \times 200 = 48 \\
p(300) &= 50 - 0.01 \times 300 = 47 \\
p(400) &= 50 - 0.01 \times 100 = 46 \\
p(500) &= 50 - 0.01 \times 500 = 45.
\end{align*}
\]

These values agree with those in the table.

(b) We have \( R = pN \), so \( R = (50 - 0.01N)N \) dollars.

(c) In functional notation the total revenue is \( R(450) \), and the value is

\[
(50 - 0.01 \times 450) \times 450 = 20,475 \text{ dollars}.
\]

15. **Renting motel rooms:**

(a) Since the group rents 2 extra rooms, we take off $4 from the base price of each room, which means that we charge $81 for each room. Since we rented 3 rooms altogether, we take in a total of \( 3 \times 81 = \$243 \).

(b) The formula that tells us how much to charge for each room is

\[
\text{Rate} = \text{Base price} - \$2 \text{ per extra room} \text{ dollars}.
\]

Now if \( n \) rooms are rented, then the number of extra rooms is \( n - 1 \). Since the base price is $85, the formula can be written as

\[
\text{Rate} = 85 - 2 \times \text{number of extra rooms} = 85 - 2(n - 1) \text{ dollars}.
\]

This can also be written as \( \text{Rate} = 87 - 2n \) dollars.

(c) The total revenue is the number of rooms times the rental per room:

\[
R(n) = \text{Rooms rented} \times \text{Price per room} = n \times (85 - 2(n - 1)) \text{ dollars}.
\]

This can also be written as \( R(n) = n(87 - 2n) \) dollars.
(d) The total cost of renting 9 rooms is expressed in functional notation as \( R(9) \). This is calculated as \( R(9) = 9(85 - 2 \times (9 - 1)) = 8621 \).

17. Catering a dinner:

(a) i. If 50 people attend, then your cost is the rental fee plus the caterer’s fee for each of the 50 people, or a total of \( 150 + 50 \times 10 = 650 \) dollars. If 50 people attend, then to break even, each ticket should cost \( \frac{650}{50} = 13 \) dollars.

ii. Let \( n \) be the number of people attending and \( C \) the amount in dollars you should charge per person. Your total cost is the rental fee plus the caterer’s fee for each of the \( n \) people, or a total of \( 150 + 10n \) dollars. Since \( n \) people attend, then to break even, each ticket should cost

\[
C = \frac{150 + 10n}{n} \text{ dollars.}
\]

This can also be written as \( C = \frac{150}{n} + 10 \), which can be thought of as each person’s share of the $150 rental fee plus the $10 caterer’s fee.

iii. If 65 people attend, then the amount to charge each is expressed in functional notation as \( C(65) \). This is calculated as

\[
C(65) = \frac{150}{65} + 10 = 12.31 \text{ per ticket.}
\]

(b) To make a profit of $100, you should think of your cost as being $100 more in determining your ticket price, so your new price would be

\[
P = \frac{100 + 150 + 10n}{n} = \frac{250 + 10n}{n} \text{ dollars.}
\]

This can also be written as \( P = \frac{250}{n} + 10 \) or as \( P = \frac{100}{n} + \frac{150}{n} + 10 \), which can be thought of as each person’s share of the $100 profit, plus each person’s share of the $150 rental fee, plus the $10 caterer’s fee.

19. Production rate:

(a) Let \( k \) be the constant of proportionality. Then \( t = kn \).

(b) Since \( t \) is total number of items produced and \( n \) is the number of employees, it follows that \( k \) is the number of items produced per employee.

21. Head and pressure:

(a) Because \( p \) is proportional to \( h \) with constant of proportionality 0.434, the equation is \( p = 0.434h \).
(b) The head of water at the mouth of the nozzle is \(8 \times 12 = 96\) feet. The back pressure is the value of \(p\) given by the equation in Part (a) with the head of \(h = 96\) feet: \(p = 0.434 \times 96 = 41.66\). Thus the back pressure is 41.66 pounds per square inch.

(c) The head is the height of the nozzle above the pumper, and in this case that value is \(185 - 40 = 145\) feet. The back pressure is \(p = 0.434 \times 145 = 62.93\) pounds per square inch.

23. Darcy’s law:

(a) Because \(V\) is proportional to \(S\) with constant of proportionality \(K\), the equation is \(V = KS\).

(b) The constant \(K\) equals the permeability of sandstone, 0.041 meter per day. The slope \(S\) is given as 0.03 meter per meter. We compute the velocity of the water flow using the equation in Part (a): \(V = 0.041 \times 0.03 = 0.00123\). The units are found by multiplying the units for \(K\) with those for \(S\), and we have that the velocity is 0.00123 meter per day.

(c) Now we take the constant \(K\) to be 41 meters per day. The slope \(S\) is still 0.03 meter per meter. The velocity of the water flow is \(V = 41 \times 0.03 = 1.23\) meters per day.

25. The \(3x + 1\) problem:

(a) We have

\[
\begin{align*}
f(1) &= 3(1) + 1 = 4 \\
f(4) &= \frac{4}{2} = 2 \\
f(2) &= \frac{2}{2} = 1.
\end{align*}
\]

The procedure repeats this cycle over and over.

(b) We have

\[
\begin{align*}
f(5) &= 3(5) + 1 = 16 \\
f(16) &= \frac{16}{2} = 8 \\
f(8) &= \frac{8}{2} = 4 \\
f(4) &= 2 \\
f(2) &= 1.
\end{align*}
\]

It took 5 steps to get to 1.
(c) We have

\[
\begin{align*}
  f(7) &= 3(7) + 1 = 22 \\
  f(22) &= 11 \\
  f(11) &= 34 \\
  f(34) &= 17 \\
  f(17) &= 52 \\
  f(52) &= 26 \\
  f(26) &= 13 \\
  f(13) &= 40 \\
  f(40) &= 20 \\
  f(20) &= 10 \\
  f(10) &= 5.
\end{align*}
\]

We followed the rest of this trail in Part (b). So it takes 16 steps to get to 1 starting with 7.

(d) These answers will vary. If a number is found that does not lead back to 1, the computation should be checked very carefully.

Chapter 1 Review Exercises

1. **Evaluating formulas**: To get the function value \(M(9500, 0.01, 24)\), substitute \(P = 9500\), \(r = 0.01\), and \(t = 24\) in the formula

\[
M(P, r, t) = \frac{Pr(1 + r)^t}{(1 + r)^t - 1}.
\]

The result is

\[
\frac{9500 \times 0.01 \times (1 + 0.01)^{24}}{(1 + 0.01)^{24} - 1},
\]

which equals 447.20.

2. **U.S. population**:

   (a) Because 1790 corresponds to \(t = 0\), the population in 1790 was \(3.93 \times 1.03^0 = 3.93\) million.
(b) Because 1810 is 20 years after 1790, we take \( t = 20 \) and get that \( N(20) \) is functional notation for the population in 1790.

(c) To find the population in 1810 we put \( t = 20 \) in the formula. The result is \( 3.93 \times 1.03^{20} = 7.10 \), so the population in 1810 was 7.10 million according to the formula.

3. **Averages and average rate of change:**

(a) Because 5 is halfway between 4 and 6, we estimate \( f(5) \) by

\[
\frac{f(4) + f(6)}{2} = \frac{40.1 + 43.7}{2} = 41.9.
\]

(b) The average rate of change is the change in \( f \) divided by the change in \( x \), and that is

\[
\frac{f(6) - f(4)}{2} = \frac{43.7 - 40.1}{2} = 1.8.
\]

4. **High school graduates:**

(a) Here \( N(1989) \) represents the number, in millions, graduating from high school in 1989. According to the table, its value is 2.47 million.

(b) In functional notation the number of graduates in 1988 is \( N(1988) \). We estimate its value by averaging:

\[
\frac{N(1987) + N(1989)}{2} = \frac{2.65 + 2.47}{2} = 2.56.
\]

Thus there were about 2.56 million graduating in 1988.

(c) The average rate of change is the change in \( N \) divided by the change in \( t \), and that is

\[
\frac{N(1991) - N(1989)}{2} = \frac{2.29 - 2.47}{2} = -0.09 \text{ million per year.}
\]

(d) To estimate the value of \( N(1994) \), we calculate \( N(1991) \) plus 3 years of change at the average rate found in the previous part. So, \( N(1994) \) is estimated to be

\[
N(1991) + 3 \times -0.09 = 2.29 - 0.27 = 2.02.
\]

Our estimate for \( N(1994) \) is 2.02 million.

5. **Increasing, decreasing, and concavity:**

(a) The function is increasing from 1990 to 2000.

(b) The function is concave down from 1996 to 2000 and concave up from 1990 to 1996.
(c) There is an inflection point at $d = 1996$.

6. **Logistic population growth:**

(a) The population grows rapidly at first and then the growth slows. Eventually it levels off.

(b) The population reaches 300 in mid-1997.

(c) The population is increasing most rapidly in 1996.

(d) The point of most rapid population increase is an inflection point.

7. **Getting a formula:** The balance (in dollars) is the initial balance of $780$ minus $39$ times the number of withdrawals. Thus $B = 780 - 39t$.

8. **Cell phone charges:**

(a) Let $t$ denote the number of text messages and $C$ the charge, in dollars.

(b) The charge (in dollars) is the flat monthly rate of $39.95$ plus $0.10$ times the number of messages in excess of $100$. That excess is $t - 100$, so the formula is $C = 39.95 + 0.1(t - 100)$.

(c) In functional notation the cost if you have 450 messages is $C(450)$. The value is $39.95 + 0.1(450 - 100) = 74.95$ dollars.

(d) If the number of text messages is less than $100$ then the only charge is the flat monthly rate of $39.95$. So the formula is $C = 39.95$.

9. **Cell phone charges again:**

(a) Let $t$ denote the number of text messages, $m$ the number of minutes, and $C$ the charge, in dollars.

(b) The charge (in dollars) is the flat monthly rate of $34.95$, plus $0.35$ times the number of minutes in excess of $4000$, plus $0.10$ times the number of messages in excess of $100$. Thus the formula is $C = 34.95 + 0.35(m - 4000) + 0.1(t - 100)$.

(c) The charges are $34.95 + 0.35(6000 - 4000) + 0.1(450 - 100) = 769.95$ dollars.

(d) If the number of text messages is less than $100$ then the only charges are the flat monthly rate of $34.95$ plus $0.35$ times the number of minutes in excess of $4000$. So the formula is $C = 34.95 + 0.35(m - 4000)$.

(e) We use the formula from Part (d). The charges are $34.95 + 0.35(4200 - 4000) = 104.95$ dollars.
10. **Practicing calculations:**

(a) We calculate that \( C(0) = 0.2 + 2.77e^{-0.37 \times 0} = 2.97 \).

(b) We calculate that \( C(0) = \frac{12.36}{0.03 + 0.55^0} = 12 \).

(c) We calculate that \( C(0) = \frac{0 - 1}{\sqrt{0 + 1}} = -1 \).

(d) We calculate that \( C(0) = 5 \times \frac{0.5^{0/5730}}{} = 5 \).

11. **Amortization:**

(a) We calculate that \( M(5500, 0.01, 24) = 5500 \times 0.01 \times (1 + 0.01)^{24} - 1 = 258.90 \) dollars.

Your monthly payment if you borrow $5500 at a monthly rate of 1% for 24 months is $258.90.

(b) In functional notation the payment is \( M(8000, 0.006, 36) \). The value is \( \frac{8000 \times 0.006 \times (1 + 0.006)^{36}}{(1 + 0.006)^{36} - 1} = 247.75 \) dollars.

12. **Using average rate of change:**

(a) The average rate of change is the change in \( f \) divided by the change in \( x \), and that is \( \frac{f(3) - f(0)}{3} = \frac{55 - 50}{3} = 1.67 \).

(b) Again, the average rate of change is the change in \( f \) divided by the change in \( x \), and in this case that is \( \frac{f(6) - f(3)}{3} = \frac{61 - 55}{3} = 2 \).

(c) Because 4 is 1 unit more than 3 and the average rate of change is 2, we estimate \( f(4) \) by \( f(3) + 1 \times 2 = 55 + 2 = 57 \).

13. **Timber values under Scribner scale:**

(a) The average rate of change from $20 to $24 is \( \frac{81.60 - 68.00}{24 - 20} = 3.4 \). The units here are dollar value per MBF Scribner divided by dollar value per cord. Continuing in this way, we get the following table, where the rate of change has the units just given. Note that the change in the variable for the last interval is 8, not 4.

<table>
<thead>
<tr>
<th>Interval</th>
<th>Rate of change</th>
</tr>
</thead>
<tbody>
<tr>
<td>20 to 24</td>
<td>3.4</td>
</tr>
<tr>
<td>24 to 28</td>
<td>3.4</td>
</tr>
<tr>
<td>28 to 36</td>
<td>3.4</td>
</tr>
</tbody>
</table>
(b) No, the value per MBF Scribner should not have a limiting value: Its rate of change is a nonzero constant, so we expect it to increase at a constant rate.

(c) We use the average rate of change from $24 to $28 to estimate the value per MBF Scribner when the value per cord is $25. That estimate is $81.60 + 1 \times 3.4 = 85$ dollars per MBF Scribner. Because $85$ is greater than $71$, if you are selling then $25$ per cord is a better value, but if you are buying then $71$ per MBF Scribner is a better value. Another way to do this is to note by inspecting the table that the value of $25$ per cord is greater than the value of $71$ per MBF Scribner: The value of $25$ per cord is greater than the value of $24$ per cord listed in the table, so it is higher than the equivalent value of $81.60$ per MBF Scribner listed in the table.

14. **Concavity**: If a graph is decreasing at an increasing rate then it is concave down. If it is decreasing at a decreasing rate then it is concave up.

15. **Longleaf pines**:

(a) The height of the tree increases quickly at first, but the growth rate decreases as the tree ages. It makes sense for a young tree to grow more quickly than an older tree.

(b) According to the graph the tree height for a 60-year-old tree is about 132 feet.

(c) Yes, there is a limiting value, since the graph eventually levels off.

(d) The graph is concave down. This means that the height is increasing at a decreasing rate, so each year the amount of growth decreases.

16. **Getting a formula**:

(a) If we rent 3 rooms we get a discount of $2 \times 2 = 4$ dollars, so each room will cost $56 - 4 = 52$ dollars.

(b) Because each room will cost $52$ dollars, we will pay $3 \times 52 = 156$ dollars altogether.

(c) If we rent $n$ rooms we get a discount of $2(n - 1) = 2n - 2$ dollars. If we let $R$ denote the rental cost in dollars per room then $R = 56 - 2(n - 1)$ or $R = 58 - 2n$ dollars.

(d) Let $C$ denote the total cost in dollars. Then $C = n \times (56 - 2(n - 1))$ or $C = n \times (58 - 2n)$.

17. **A wedding reception**:

(a) If you invite 100 guests then the cost is $3200$ for the venue plus $31$ times $50$ (because 100 guests makes an excess of 50 over the number included). Thus the cost is $3200 + 31 \times 50 = 4750$ dollars.
(b) If you invite \( n \) guests then the cost is $3200 for the venue plus $31 times the number in excess of 50. That excess is \( n - 100 \), so if we let \( C \) denote the cost in dollars then
\[
C = 3200 + 31(n - 50) \text{ or } C = 1650 + 31n.
\]

(c) We want to find \( n \) so that \( C = 5500 \). By the formula from Part (b) this says that
\[
1650 + 31n = 5500.
\]
By trial and error (or by solving for \( n \) using algebra) we find that we can invite 124 guests.

18. **Limiting values:**

(a) No, not all tables show limiting values.

(b) We can identify a limiting value from a table by checking whether the last few entries in the table show little change. If so, the limiting value is approximated by the trend established by the last few entries.

(c) No, not all graphs show limiting values.

(d) We can identify a limiting value from a graph by checking whether the last portion of the graph levels off. If so, the limiting value is approximated by that value where the graph is level.
2.1 TABLES AND TRENDS

E-1. Verifying the basic exponential limit: We show tables for two values of \( a \). The table below on the left corresponds to \( a = 0.5 \), so the function is \( 0.5^t \). The table on the right corresponds to \( a = 0.3 \), so the function is \( 0.3^t \). In both cases the function values are approaching 0, in support of the statement of the basic exponential limit.

<table>
<thead>
<tr>
<th>( X )</th>
<th>( Y_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>0.25</td>
</tr>
<tr>
<td>4</td>
<td>0.0825</td>
</tr>
<tr>
<td>6</td>
<td>0.01563</td>
</tr>
<tr>
<td>8</td>
<td>0.00391</td>
</tr>
<tr>
<td>10</td>
<td>9.8E-4</td>
</tr>
<tr>
<td>12</td>
<td>2.4E-4</td>
</tr>
</tbody>
</table>

\( X=0 \)

\( X=0 \)

E-3. Calculating with the basic exponential limit:

(a) Because \( \lim_{t \to \infty} 0.5^t = 0 \) by the basic exponential limit, we have

\[
\lim_{t \to \infty} 15(1 - 3 \times 0.5^t) = 15(1 - 3 \times 0) = 15.
\]

(b) Because \( \lim_{t \to \infty} 0.4^t = 0 \) and \( \lim_{t \to \infty} 0.7^t = 0 \), we have

\[
\lim_{t \to \infty} \frac{6 + 0.4^t}{3 - 0.7^t} = \frac{6 + 0}{3 - 0} = 2.
\]

(c) Because \( \lim_{t \to \infty} 0.5^t = 0 \), we have

\[
\lim_{t \to \infty} \sqrt{7} + 0.5^t = \sqrt{7} + 0 = \sqrt{7}.
\]

(d) Recall from the basic properties of exponents that \( \frac{2^t}{3^t} = \left( \frac{2}{3} \right)^t \). Thus

\[
\lim_{t \to \infty} \frac{2^t}{3^t} = \lim_{t \to \infty} \left( \frac{2}{3} \right)^t = 0;
\]
here we have applied the basic exponential limit with \(a = \frac{2}{3}\).

E-5. Calculating using the division trick:

(a) The highest power of \(t\) that appears is \(t^1 = t\), so we divide top and bottom by \(t\). We have

\[
\lim_{t \to \infty} \frac{4t + 5}{t + 1} = \lim_{t \to \infty} \left( \frac{4 + \frac{5}{t}}{1 + \frac{1}{t}} \right) = \lim_{t \to \infty} \frac{4 + 5 \times 0}{1 + 0} = 4.
\]

Here we have applied the basic power limit to \(\frac{1}{t}\).

(b) The highest power of \(t\) that appears is \(t^3 = t^3\), so we divide top and bottom by \(t^3\). We have

\[
\lim_{t \to \infty} \frac{6t^3 + 5}{2t^3 + 3t + 1} = \lim_{t \to \infty} \left( \frac{6 + \frac{5}{t}}{2 + \frac{3}{t^2} + \frac{1}{t}} \right) = \lim_{t \to \infty} \frac{6 + 5 \times 0}{2 + 3 \times 0 + 0} = 3.
\]

Here we have applied the basic power limit to \(\frac{1}{t^2}\) and \(\frac{1}{t^3}\).

(c) The highest power of \(t\) that appears is \(t^3 = t^3\), so we divide top and bottom by \(t^3\). We have

\[
\lim_{t \to \infty} \frac{t^2 + 1}{t^3 + 1} = \lim_{t \to \infty} \left( \frac{\frac{1}{t} + \frac{1}{t^2}}{1 + \frac{1}{t^2}} \right) = \lim_{t \to \infty} \frac{\frac{1}{t} + \frac{1}{t^2}}{1 + \frac{1}{t^2}} = \frac{0 + 0}{1 + 0} = 0.
\]

Here we have applied the basic power limit to \(\frac{1}{t}\) and \(\frac{1}{t^3}\).

E-7. Calculating using both exponential and power limits:

(a) We have

\[
\lim_{t \to \infty} \frac{a + \frac{1}{t}}{b + 0.2^t} = \lim_{t \to \infty} \frac{a}{b + 0} = \frac{a}{b}.
\]

Here we have used the basic power limit for \(\frac{1}{t}\) and the basic exponential limit for \(0.2^t\).

(b) We have

\[
\lim_{t \to \infty} \left( b \times 0.8^t - \frac{a}{t^2} \right) = b \times 0 - a \times 0 = 0.
\]

Here we have used the basic exponential limit for \(0.8^t\) and the basic power limit for \(\frac{1}{t^2}\).

(c) The highest power of \(t\) that appears is \(t^2 = t^2\), so we divide top and bottom by \(t^2\). We have

\[
\lim_{t \to \infty} \frac{at^2 + 0.3^t}{bt^2 + 1} = \lim_{t \to \infty} \frac{at^2 + 0.3^t \times \left( \frac{1}{t^2} \right)}{bt^2 + 1 \times \left( \frac{1}{t^2} \right)} = \lim_{t \to \infty} \frac{a + 0.3^t \times \frac{1}{t^2}}{b + \frac{1}{t^2}} = \frac{a + 0 \times 0}{b + 0} = \frac{a}{b}.
\]

Here we have used the basic exponential limit for \(0.3^t\) and the basic power limit for \(\frac{1}{t^2}\).
E-9. **Concentration of salt**: The eventual concentration of salt is given by the following limit:

\[
\lim_{t \to \infty} \left( a + \frac{b}{t} \right) = a + b \times 0 = a.
\]

(Here we have used the basic power limit for \(\frac{1}{t}\).) Thus the eventual concentration of salt in the solution is \(a\) pounds per gallon.

S-1. **Making a table**: To make a table for \(f(x) = x^2 - 1\) showing function values for \(x = 4, 6, 8, \ldots\), we enter the function as \(Y1 = X^2 - 1\) and use a table starting value of 4 and a table increment value of 2, resulting in the following table.

<table>
<thead>
<tr>
<th>(X)</th>
<th>(Y1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>15</td>
</tr>
<tr>
<td>6</td>
<td>35</td>
</tr>
<tr>
<td>8</td>
<td>63</td>
</tr>
<tr>
<td>10</td>
<td>99</td>
</tr>
<tr>
<td>12</td>
<td>143</td>
</tr>
<tr>
<td>14</td>
<td>195</td>
</tr>
<tr>
<td>16</td>
<td>255</td>
</tr>
</tbody>
</table>

\(X=4\)

S-3. **Making a table**: To make a table for \(f(x) = 16 - x^3\) showing function values for \(x = 3, 7, 11, \ldots\), we enter the function as \(Y1 = 16 - X^3\) and use a table starting value of 3 and a table increment value of 4, resulting in the following table.

<table>
<thead>
<tr>
<th>(X)</th>
<th>(Y1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>11</td>
</tr>
<tr>
<td>7</td>
<td>-327</td>
</tr>
<tr>
<td>11</td>
<td>-1215</td>
</tr>
<tr>
<td>15</td>
<td>-3359</td>
</tr>
<tr>
<td>19</td>
<td>-6883</td>
</tr>
<tr>
<td>23</td>
<td>-12151</td>
</tr>
<tr>
<td>27</td>
<td>-19667</td>
</tr>
</tbody>
</table>

\(X=3\)

S-5. **Finding a limiting value**: We enter the function as \(Y1 = (4X^2 - 1)/(7X^2 + 1)\) and use a table starting value of 0 and a table increment value of 20, resulting in the following table. The table suggests a limiting value of about 0.5714.

<table>
<thead>
<tr>
<th>(X)</th>
<th>(Y1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>20</td>
<td>0.57087</td>
</tr>
<tr>
<td>40</td>
<td>0.57320</td>
</tr>
<tr>
<td>60</td>
<td>0.57357</td>
</tr>
<tr>
<td>80</td>
<td>0.57439</td>
</tr>
<tr>
<td>100</td>
<td>0.57141</td>
</tr>
<tr>
<td>120</td>
<td>0.57141</td>
</tr>
</tbody>
</table>

\(X=0\)
S-7. **Finding a minimum**: To find the minimum value of \( f \), we make a table. We enter the function as \( Y1 = X^2 - 8X + 21 \) and use a table starting value of 0 and a table increment value of 1, resulting in the following table.

![Table](image)

The table shows that \( f \) has a minimum value of 5 at \( x = 4 \), at least for values of \( x \) up to 6. Scrolling down the table through \( x = 20 \), we see that the function increases, so indeed the minimum value of \( f \) is 5 at \( x = 4 \).

S-9. **Finding a maximum**: To find the maximum value of \( f \), we make a table. We enter the function as \( Y1 = 9X^2 - 2^X + 1 \) and use a table starting value of 0 and a table increment value of 1. Scrolling farther down the table yields the following table.

![Table](image)

The table shows that \( f \) reaches a maximum of 321 at \( x = 8 \).
S-11. Making a table: To make tables for \( f(x) = \frac{13}{0.93^x + 0.05} \), we enter the function as \( Y1 = \frac{13}{0.93^x + 0.05} \). First we use a table starting value of 1 and a table increment value of 1, resulting in the table on the left. Then we use a table starting value of 10 and a table increment value of 10, resulting in the table on the right.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( Y1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>13.285</td>
</tr>
<tr>
<td>2</td>
<td>14.209</td>
</tr>
<tr>
<td>3</td>
<td>15.216</td>
</tr>
<tr>
<td>4</td>
<td>16.229</td>
</tr>
<tr>
<td>5</td>
<td>17.242</td>
</tr>
<tr>
<td>6</td>
<td>18.255</td>
</tr>
<tr>
<td>7</td>
<td>19.268</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( x )</th>
<th>( Y1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>24.345</td>
</tr>
<tr>
<td>20</td>
<td>45.726</td>
</tr>
<tr>
<td>30</td>
<td>79.525</td>
</tr>
<tr>
<td>40</td>
<td>123.97</td>
</tr>
<tr>
<td>50</td>
<td>169.81</td>
</tr>
<tr>
<td>60</td>
<td>206.83</td>
</tr>
<tr>
<td>70</td>
<td>231.23</td>
</tr>
</tbody>
</table>

1. Harvard Step Test: Here is a table of values for \( E \) as a function of \( P \):

<table>
<thead>
<tr>
<th>( x )</th>
<th>( Y1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>150</td>
<td>100</td>
</tr>
<tr>
<td>160</td>
<td>93.75</td>
</tr>
<tr>
<td>170</td>
<td>88.235</td>
</tr>
<tr>
<td>180</td>
<td>83.333</td>
</tr>
<tr>
<td>190</td>
<td>78.947</td>
</tr>
<tr>
<td>200</td>
<td>75</td>
</tr>
<tr>
<td>210</td>
<td>71.429</td>
</tr>
</tbody>
</table>

(a) From the table above, we see that the index \( E \) decreases with increasing values of the variable \( P \). This should hold for all values of \( P \) because increasing the denominator in the fraction defining \( E \) makes the fraction smaller. This means that a person having a larger total pulse count after the exercise than another person has a lower physical efficiency index and hence is not as physically fit.

(b) The index of someone with a total pulse count of 200 is expressed in functional notation by \( E(200) \). From the table we generated we find the value to be 75.

(c) We saw in Part (b) that the index of someone with a total pulse count of 200 is 75. This is in the interval from 65 to 79 of the table in the text, and that table indicates that the physical condition of such a person is high average.

(d) According to the table in the text, for an excellent rating the index must be 90 or above. From the table we generated we see that this transition occurs between \( P = 160 \) and \( P = 170 \). Here is a table examining the function values more closely:
We see that the index is 90 or above when the total pulse count is 166 or lower.

3. **Earlier public high school enrollment**: Here are two tables of values for $N$ as a function of $t$:

(a) From the second table we generated we find the value of $N(7)$ to be 13.75 million students. This is the number (in millions) of students enrolled in U.S. public high schools in the year 1972 (because that is 7 years after 1965).

(b) Examining the tables shown above suggests that the maximum value of the function occurs when $t = 11$. Extending the table to the end of the period (when $t = 20$) shows that the function values continue to decrease after $t = 11$. Thus the enrollment was the largest in 1976 (11 years after 1965). From the second table we see that the largest enrollment was 14.07 million students.

(c) Here 1965 corresponds to $t = 0$, and 1985 corresponds to $t = 20$. From the first table above we find that $N(0) = 11.65$, and examining further the table on our calculator yields the value $N(20) = 12.45$. The average yearly rate of change is then

$$\frac{N(20) - N(0)}{20} = \frac{12.45 - 11.65}{20} = 0.04 \text{ million students per year}.$$ 

Thus the average yearly rate of change from 1965 to 1985 is 0.04 million students per year, or 40,000 students per year. This relatively small result is misleading because, as we saw in Part (b), the enrollment actually increased significantly over this period (reaching 14.07 million students in 1972) before it decreased to a level near the original value.
5. **Competition**: This question can be answered by trial and error, but we will solve it by finding two formulas and making a table. Let \( n \) be the number of races run, \( F = F(n) \) the time in seconds it takes the first friend to run a mile, and \( S = S(n) \) the time in seconds it takes the second friend. We have

\[
\text{Time taken} = \text{Initial time taken} - \text{Decrease in time} \times \text{Number of races}.
\]

Now for the first friend the initial time taken is 7 minutes, or 420 seconds, and the decrease in time is 13 seconds. This gives the formula \( F = 420 - 13n \). Similar reasoning gives the formula \( S = 440 - 16n \) for the second friend. Here is a partial table of values of these two functions:

<table>
<thead>
<tr>
<th>( X )</th>
<th>( Y_1 )</th>
<th>( Y_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>407</td>
<td>424</td>
</tr>
<tr>
<td>2</td>
<td>394</td>
<td>408</td>
</tr>
<tr>
<td>4</td>
<td>381</td>
<td>392</td>
</tr>
<tr>
<td>8</td>
<td>368</td>
<td>386</td>
</tr>
<tr>
<td>16</td>
<td>350</td>
<td>360</td>
</tr>
<tr>
<td>32</td>
<td>342</td>
<td>344</td>
</tr>
<tr>
<td>64</td>
<td>329</td>
<td>328</td>
</tr>
<tr>
<td>( X=1 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

From the table we see that the first race in which the second friend beats the first is the seventh race.

7. **Counting when order matters**: Here is a table of values for the factorial function:

<table>
<thead>
<tr>
<th>( X )</th>
<th>( Y_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>24</td>
</tr>
<tr>
<td>5</td>
<td>120</td>
</tr>
<tr>
<td>6</td>
<td>720</td>
</tr>
<tr>
<td>7</td>
<td>5040</td>
</tr>
<tr>
<td>( X=7 )</td>
<td></td>
</tr>
</tbody>
</table>

(a) The number of ways you can arrange 5 people in a line is 5!. From the table we see that the value is 120. (This can also be computed directly: \( 5! = 5 \times 4 \times 3 \times 2 \times 1 = 120 \).) Thus there are 120 ways to arrange 5 people in a line.

(b) From the table we see that the factorial function first exceeds 1000 when the variable is 7, so 7 (or more) people will result in more than 1000 possible arrangements for a line.
(c) The number of guesses is the same as the number of ways to arrange 4 people in a line because there are 4 different digits given. Thus the number is $4!$, which is 24. Hence we would need at most 24 guesses.

(d) The number of shufflings is the same as the number of ways to arrange 52 people in a line. Thus the number is $52!$. The calculator gives that $52! = 8.07 \times 10^{67}$. Thus there are $8.07 \times 10^{67}$ possible shufflings of a deck of cards.

9. APR and EAR:

(a) We would expect the EAR to be larger if interest is compounded more often because once interest is compounded it accrues additional interest.

(b) We proceed by making a table of values for

$$\text{EAR} = \left(1 + \frac{\text{APR}}{n}\right)^n - 1 = \left(1 + \frac{0.1}{n}\right)^n - 1$$

The table below on the left shows that when $n = 1$ (yearly compounding), the EAR is exactly 10%. The table on the right has been extended to include $n = 12$, monthly compounding, and it shows the EAR to be 10.471%.

<table>
<thead>
<tr>
<th>$X$</th>
<th>$Y_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1025</td>
</tr>
<tr>
<td>2</td>
<td>0.10337</td>
</tr>
<tr>
<td>3</td>
<td>0.10408</td>
</tr>
<tr>
<td>4</td>
<td>0.10426</td>
</tr>
<tr>
<td>5</td>
<td>0.10439</td>
</tr>
</tbody>
</table>

The table below includes $n = 365$ (daily compounding), and it shows an EAR of 10.516%.

<table>
<thead>
<tr>
<th>$X$</th>
<th>$Y_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>360</td>
<td>0.10516</td>
</tr>
<tr>
<td>361</td>
<td>0.10516</td>
</tr>
<tr>
<td>362</td>
<td>0.10516</td>
</tr>
<tr>
<td>363</td>
<td>0.10516</td>
</tr>
<tr>
<td>364</td>
<td>0.10516</td>
</tr>
<tr>
<td>365</td>
<td>0.10516</td>
</tr>
<tr>
<td>366</td>
<td>0.10516</td>
</tr>
</tbody>
</table>
(c) On a loan of $5000 compounded monthly, after one year the interest owed would be

\[ 5000 \times \text{monthly EAR} = 5000 \times 0.10471 = 523.55 \text{ dollars}. \]

The total amount owed is the loan amount plus the interest, which is $5000 + 523.55 = 5523.55 \text{ dollars}.

If we compound daily, after one year the interest owed would be

\[ 5000 \times \text{daily EAR} = 5000 \times 0.10516 = 525.80 \text{ dollars}. \]

That gives a total amount owed of $5525.80.

(d) The EAR when we compound continuously is

\[ r = e^{\text{APR}} - 1 = e^{0.10} - 1 = 0.10517. \]

This is 10.517%, which is only 0.0046 percentage point (less than 0.05 percentage point) higher than monthly compounding.

11. An amortization table for continuous compounding:

(a) The monthly interest rate for this exercise is \( r = \frac{\text{APR}}{12} = \frac{0.09}{12} = 0.0075 \), we borrowed \( P = 3500 \), and \( t = 24 \) months is the life of the loan. Our monthly payment for compounding continuously would be

\[ M = \frac{3500(e^{0.0075 \times 24} - 1)}{e^{0.0075 \times 24} - 1} = 159.95. \]

This is only 5 cents more than the payment for compounding monthly.

(b) The amortization table is a table of values for

\[ B = \frac{P(e^{rt} - e^{rk})}{e^{rt} - 1} = \frac{3500 \times (e^{0.0075 \times 24} - e^{0.0075 \times k})}{e^{0.0075 \times 24} - 1}. \]

The amortization table is below. Comparing the entries here with those in Part (c) of Exercise 10 above, we see that the current entries are larger, but the difference is less than a dollar.
13. **Inventory:**

(a) The exercise tells us that \( N = 36, \ c = 850, \) and \( f = 230 \). So inventory expense is given by

\[
E(Q) = \left( \frac{Q}{2} \right) \times 850 + \left( \frac{36}{Q} \right) \times 230.
\]

(b) For 3 cars, the yearly inventory expense is

\[
E(3) = \left( \frac{3}{2} \right) \times 850 + \left( \frac{36}{3} \right) \times 230 = $4035.
\]

(c) We can find this by generating a table of values for the inventory cost function given in Part (a). As we scroll down the table we see that the cost decreases until we get to $3770, which corresponds to ordering 4 cars at a time. The table is shown below for \( Q = 1 \) to \( Q = 7 \) cars per order.

<table>
<thead>
<tr>
<th>Number of payments made</th>
<th>Amount still owed</th>
<th>Number of payments made</th>
<th>Amount still owed</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3500.00</td>
<td>13</td>
<td>1682.51</td>
</tr>
<tr>
<td>1</td>
<td>3366.40</td>
<td>14</td>
<td>1535.23</td>
</tr>
<tr>
<td>2</td>
<td>3231.79</td>
<td>15</td>
<td>1386.83</td>
</tr>
<tr>
<td>3</td>
<td>3096.17</td>
<td>16</td>
<td>1237.32</td>
</tr>
<tr>
<td>4</td>
<td>2959.53</td>
<td>17</td>
<td>1086.69</td>
</tr>
<tr>
<td>5</td>
<td>2821.85</td>
<td>18</td>
<td>934.92</td>
</tr>
<tr>
<td>6</td>
<td>2683.15</td>
<td>19</td>
<td>782.01</td>
</tr>
<tr>
<td>7</td>
<td>2543.40</td>
<td>20</td>
<td>627.94</td>
</tr>
<tr>
<td>8</td>
<td>2402.59</td>
<td>21</td>
<td>472.72</td>
</tr>
<tr>
<td>9</td>
<td>2260.73</td>
<td>22</td>
<td>316.33</td>
</tr>
<tr>
<td>10</td>
<td>2117.80</td>
<td>23</td>
<td>158.76</td>
</tr>
<tr>
<td>11</td>
<td>1973.79</td>
<td>24</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>1828.70</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(d) Since we expect to sell 36 cars this year and we order 4 cars at a time to minimize our inventory cost, then we will place \( \frac{36}{4} = 9 \) orders to Detroit this year.

(e) The average rate of increase in yearly inventory cost from ordering four cars to ordering six cars is

\[
\text{Change in cost } \div \text{Change in cars} = \frac{E(6) - E(4)}{6 - 4} = \frac{3930 - 3770}{2} = 880 \text{ per year per additional car.}
\]
15. **Falling with a parachute:**

(a) The velocity 2 seconds into the fall is expressed in functional notation as \(v(2)\). Its value is

\[
v(2) = 20(1 - 0.2^2) = 19.2 \text{ feet per second.}
\]

(b) The average change in velocity during the first second of the fall is

\[
\frac{\text{Change in velocity}}{\text{Seconds}} = \frac{v(1) - v(0)}{1} = \frac{16 - 0}{1} = 16 \text{ feet per second per second.}
\]

The average velocity from the fifth to sixth second is

\[
\frac{\text{Change in velocity}}{\text{Seconds}} = \frac{v(6) - v(5)}{1} = \frac{19.999 - 19.994}{1} = 0.005 \text{ foot per second per second.}
\]

The velocity is increasing as the seconds go by (as can be seen from a table of values), but the average rate of increase is decreasing.

(c) We make a table of values for velocity. The table below, which shows velocity from the \(t = 2\) through \(t = 8\), indicates that velocity levels off at about 20 feet per second.

(d) Now 99% of terminal velocity is 99% of 20 feet per second, which is \(0.99 \times 20 = 19.8\) feet per second. The table above shows that this occurs approximately 3 seconds into the fall.

In Example 2.1 it took 25 seconds to reach 99% of terminal velocity, while in this exercise it took only 3 seconds. This indicates that objects (such as parachutes) for which air resistance is large will reach terminal velocity faster. Thus, we would expect the feather to reach terminal velocity more quickly than a cannonball.

17. **Profit:**

(a) Assume that the total cost \(C\) is measured in dollars. Because

\[
\text{Total cost} = \text{Variable cost} \times \text{Number of items} + \text{Fixed costs},
\]

we have \(C = 50N + 150\).
(b) Assume that the total revenue $R$ is measured in dollars. Because

$$\text{Total revenue} = \text{Selling price} \times \text{Number of items},$$

we have $R = 65N$.

(c) Assume that the profit $P$ is measured in dollars. Because

$$\text{Profit} = \text{Total revenue} - \text{Total cost},$$

we have $P = R - C$. Using the formulas from Parts (a) and (b), we obtain

$$P = 65N - (50N + 150).$$

(This can also be written as $P = 15N - 150$.)

(d) We want to find for what value of $N$ we have $P(N) = 0$. Examining a table of values for $P$ shows that this occurs when $N = 10$. Thus a break-even point occurs at a production level of 10 widgets per month.

19. A precocious child and her blocks:

(a) If we think of going around the rectangle and adding up the sides as we encounter them, we see that the perimeter is $h + w + h + w$. That is two $h$’s and two $w$’s, so $P = 2h + 2w$ inches.

(b) We substitute $w = \frac{64}{h}$ in the above formula and get

$$P = 2h + 2 \times \frac{64}{h} \text{ inches}.$$ 

(c) To solve this exercise we first generate a table for $P(h)$. This is shown below for heights of 5 through 11 blocks. The smallest value is 32, and this happens when the height is 8. If the height is 8 then the width is $w = \frac{64}{8} = 8$ inches. So the blocks should be arranged in an 8 by 8 square to get a minimum perimeter.

(d) In this case the perimeter would still be $P = 2h + 2w$, and then $P = 2h + 2 \times \frac{60}{h}$ (measuring $P$ in inches). We make a table of values as before. The table below
shows a minimum perimeter of 31 inches, and this occurs when the height is 8 inches. However, a height of 8 inches gives a width of \( w = \frac{60}{8} = 7.5 \) inches, which cannot be accomplished without cutting the blocks into pieces.

We know that \( hw = 60 \) and so \( h \) must divide evenly into 60. This means that \( h \) can only be one of 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, or 60. If we look at these values in the table we see that a height of 6 or 10 leads us to a perimeter of 32, which is the minimum perimeter among these values. If the height is 6, then the width must be 10, and so the child should lay the blocks in a 6 by 10 pattern if she wants to get a minimum perimeter.

21. Growth in length of haddock:

(a) The value of \( L(4) \) is

\[
L(4) = 53 - 42.82 \times 0.82^4 = 33.64 \text{ centimeters.}
\]

(This can also be found from a table of values for \( L \).) This means that a haddock that is 4 years old is approximately 33.64 centimeters long.

(b) Using a table for the function \( L \) gives the values \( L(5) = 37.12, L(10) = 47.11, \)
\( L(15) = 50.82, \) and \( L(20) = 52.19. \) Now the average yearly rate of growth in
length from age 5 years to age 10 years is

\[
\frac{L(10) - L(5)}{10 - 5} = \frac{47.11 - 37.12}{5} = 2.00 \text{ centimeters per year.}
\]

The average yearly rate of growth in length from age 15 years to age 20 years is

\[
\frac{L(20) - L(15)}{20 - 15} = \frac{52.19 - 50.82}{5} = 0.27 \text{ centimeter per year.}
\]

The growth rate is lower over the later period, and this suggests that haddock grow more rapidly when they are young than when they are older.

(c) Scanning down a table on the calculator indicates that the function \( L \) increases to a limiting value of 53 as \( t \) gets larger and larger. Thus the longest haddock is 53 centimeters long.
23. **California earthquakes**: Here is a table of values for the function $p$:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.379</td>
</tr>
<tr>
<td>1</td>
<td>0.38744</td>
</tr>
<tr>
<td>2</td>
<td>0.17812</td>
</tr>
<tr>
<td>3</td>
<td>0.05758</td>
</tr>
<tr>
<td>4</td>
<td>0.01395</td>
</tr>
<tr>
<td>5</td>
<td>0.00271</td>
</tr>
<tr>
<td>6</td>
<td>4.4E-4</td>
</tr>
</tbody>
</table>

(a) We are given that $n = 3$, and from the table we see that the function value is $p(3) = 0.058$. Thus the probability is 0.058 or 5.8%.

(b) Scanning down the table suggests that the function values approach 0 as the variable $n$ increases. Thus the limiting value is 0. This means that it is very unlikely that a California home will be affected by a large number of earthquakes over a 10-year period.

(c) We are given that $n = 0$, and from the table we see that the function value is $p(0) = 0.379$. Thus the probability of a California home being affected by no major earthquakes over a 10-year period is 0.379 or 37.9%.

(d) According to the hint, we have

\[
\text{Probability of no major earthquake occurring} + \text{Probability of at least one major earthquake occurring} = 1,
\]

which can be rearranged to give

\[
\text{Probability of at least one major earthquake occurring} = 1 - \text{Probability of no major earthquake occurring}.
\]

We saw in Part (c) that the probability on the right has the value 0.379. Thus the probability of at least one major earthquake occurring is $1 - 0.379 = 0.621$ or 62.1%.

25. **Research project**: Answers will vary.
E-1. **Shifting:**

(a) We shift the graph 2 units to the left:

![Graph shifted left]

(b) We shift the original graph up 2 units:

![Graph shifted up]

(c) We shift the original graph 2 units to the right:

![Graph shifted right]
(d) We shift the original graph down 2 units:

E-3. **Combinations of shifting and stretching:**

(a) We stretch the graph horizontally by a factor of 2:

(b) We stretch the graph from Part (a) vertically by a factor of 2:
(c) We shift the graph from Part (b) up 1 unit:

E-5. **Adding functions**: We show the graph with a horizontal span of $-5$ to $5$ and a vertical span of $-5$ to $20$. The darker graph is the sum.

The heights of points on the graphs of $f$ and $g$ are added to get the heights of points on the graph of $f + g$.

S-1. **Graphs and function values**: To get the value of $f(3)$, we enter the function as $Y1 = 2 - X^2$ and use the standard view, that is, the window with a horizontal span from $-10$ to $10$ and the same vertical span. Locating the point on the graph corresponding to $X = 3$, we see from the graph below that $f(3) = -7$. 
S-3. **Graphs and function values**: To get the value of $f(3)$, we enter the function as $Y1 = (X^2 + 2^x)/(X + 10)$ and use the standard view, that is, the window with a horizontal span from $-10$ to $10$ and the same vertical span. Locating the point on the graph corresponding to $X = 3$, we see from the graph below that $f(3) = 1.31$.

![Graph showing $f(3)$](image)

S-5. **Finding a window**: To find an appropriate window setup which will show a good graph of $\frac{x^3}{500}$ with a horizontal span of $-3$ to $3$, first we enter the function as $Y1 = (X^3)/500$, and then we make a table with a table starting value of $-3$ and a table increment value of $1$. Such a table is shown in the figure on the left below. The table of values shows that a vertical span from $-0.06$ to $0.06$ will display the graph, as shown on the right below.

![Table and graph](image)

S-7. **Finding a window**: To find an appropriate window setup which will show a good graph of $\frac{x^4 + 1}{x^2 + 1}$ with a horizontal span of $0$ to $300$, first we enter the function as $Y1 = (X^4 + 1)/(X^2 + 1)$, and then we make a table with a table starting value of $0$ and a table increment value of $50$. Such a table is shown in the figure on the left below. The table of values shows that a vertical span from $0$ to $90,000$ will display the graph, as shown on the right below.

![Table and graph](image)
S-9. **Finding a window**: To find an appropriate window setup which will show a good graph of \( \frac{1}{x^2 + 1} \) with a horizontal span of \(-2\) to \(2\), first we enter the function as \( Y_1 = \frac{1}{x^2 + 1} \), and then we make a table with a table starting value of \(-2\) and a table increment value of \(1\). Such a table is shown in the figure on the left below. The table of values shows that a vertical span from 0 to 1 will display the graph, as shown on the right below.

S-11. **Finding a window**: To find an appropriate window setup which will show a good graph of \( \frac{3x}{50 + x} \) with a horizontal span of 0 to 1000, first we enter the function as \( Y_1 = \frac{3x}{50 + x} \), and then we make a table with a table starting value of 0 and a table increment value of 200. Such a table is shown in the figure on the left below. The table of values shows that a vertical span from 0 to 3 will display the graph, as shown on the right below.
1. **Weekly cost:**

(a) The weekly cost if there are 3 employees is \(2500 + 350 \times 3 = 3550\) dollars.

(b) Let \(n\) be the number of employees and \(C\) the weekly cost in dollars. Because

\[
\text{Weekly cost} = \text{Fixed cost} + \text{Cost per employee} \times \text{Number of employees},
\]

we have \(C = 2500 + 350n\).

(c) We show the graph with a horizontal span of 0 to 10 and a vertical span of 2500 to 7000. In choosing this vertical span we were guided by the table below.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y_1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2500</td>
</tr>
<tr>
<td>1</td>
<td>2850</td>
</tr>
<tr>
<td>2</td>
<td>3200</td>
</tr>
<tr>
<td>3</td>
<td>3550</td>
</tr>
<tr>
<td>4</td>
<td>3900</td>
</tr>
<tr>
<td>5</td>
<td>4250</td>
</tr>
<tr>
<td>6</td>
<td>4600</td>
</tr>
<tr>
<td>7</td>
<td>5050</td>
</tr>
<tr>
<td>8</td>
<td>5500</td>
</tr>
<tr>
<td>9</td>
<td>5950</td>
</tr>
<tr>
<td>10</td>
<td>6400</td>
</tr>
<tr>
<td>11</td>
<td>6750</td>
</tr>
<tr>
<td>12</td>
<td>7000</td>
</tr>
</tbody>
</table>

(d) We want to find what value of the variable \(n\) gives the function value \(C = 4250\). Looking at a table of values or the graph shows that this occurs when \(n = 5\). Thus if there are 5 employees the weekly cost will be \$4250.

2. **Resale value:**

(a) The resale value in the year 2001 is

\[
\text{Resale value} = \text{Value in the year 2000} - \text{Decrease over 1 year} = 18,000 - 1700 = 16,300\text{ dollars.}
\]

Thus the resale value in the year 2001 is \$16,300.

(b) Because

\[
\text{Resale value} = \text{Value in the year 2000} - \text{Decrease each year} \times \text{Number of years since 2000},
\]

we have \(V = 18,000 - 1700t\).

(c) We show the graph with a horizontal span of 0 to 4 and a vertical span of 10,000 to 20,000. In choosing this vertical span we were guided by the table below.
(d) Since 2003 is 3 years since 2000, the resale value in that year is \( V(3) \) in functional notation. The value is \( 18,000 - 1700 \times 3 = 12,900 \) dollars. (This can also be found from a table of values or the graph.)

5. Baking a potato:

(a) The first step is to make a table of values to select a vertical span. As suggested, we use a horizontal span of 0 to 120 minutes. This table with 20 minute increments is shown below. It suggests a vertical span of 0 to 420. We used this window setting to make the graph below, which shows time in minutes on the horizontal axis and temperature on the vertical axis.

(b) The initial temperature of the potato is its temperature when placed in the oven. That is the value of \( P \) when \( t \) is zero:

\[
P(0) = 400 - 325e^{-0/50} = 75 \text{ degrees}.
\]

(c) The change in the potato’s temperature in the first 30 minutes is

\[
\text{Temperature at 30 minutes} - \text{Initial temperature} = P(30) - P(0)
\]
\[
= 221.636 - 75
\]
\[
= 146.636 \text{ degrees}.
\]

The change in the potato’s temperature in the second 30 minutes is
Temperature at 60 minutes − Temperature at 30 minutes = \( P(60) − P(30) \)

\[ = 302.111 − 221.636 \]

\[ = 80.475 \text{ degrees.} \]

The potato’s temperature rose the most in the first 30 minutes.

The average rate of change during the first 30 minutes of baking is

\[
\frac{\text{Change in temp}}{\text{Minutes elapsed}} = \frac{P(30) − P(0)}{30} = \frac{146.636}{30} = 4.89 \text{ degrees per minute.}
\]

The average rate of change during the second 30 minutes of baking is

\[
\frac{\text{Change in temp}}{\text{Minutes elapsed}} = \frac{P(60) − P(30)}{30} = \frac{80.475}{30} = 2.68 \text{ degrees per minute.}
\]

We used three decimal accuracy for the various values of \( P \) to ensure that our final answer would still be accurate to two decimal places after division.

(d) The graph is concave down, and this tells us that, although the temperature is rising, the rate of increase in temperature is decreasing. In other words, the temperature is not rising as fast at later times as it was at first. Note that in Part (c) the average increase from 0 to 30 minutes was 4.89 degrees per minute, whereas the average increase from 30 to 60 minutes was only 2.68 degrees per minute.

(e) The potato will reach a temperature of 270 degrees after about 46 minutes. This can be seen by tracing the graph, as we have done in the graph above.

(f) If the potato is left in the oven a long time, its temperature will match that of the oven. To see what happens to the potato after a long time, we made a new table with a table starting value of 0 and a table increment value of 120. It appears that the limiting value is 400. We see that the temperature of the oven is about 400 degrees.
7. Population growth:

(a) We show the graph with a horizontal span of 0 to 25 and a vertical span of \(-35\) to \(35\). In choosing this vertical span we were guided by the table below.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>18.75</td>
</tr>
<tr>
<td>10</td>
<td>18.75</td>
</tr>
<tr>
<td>15</td>
<td>25</td>
</tr>
<tr>
<td>20</td>
<td>-31.25</td>
</tr>
<tr>
<td>25</td>
<td>-75</td>
</tr>
<tr>
<td>(x = 0)</td>
<td></td>
</tr>
</tbody>
</table>

(b) The growth over a week if the population at the beginning is 4 thousand animals is expressed as \(G(4)\) in functional notation. From a table of values or the graph we find that the value is 16 thousand animals.

(c) From a table of values or the graph we find that the value of \(G(22)\) is \(-11\) thousand animals. This means that if the population at the beginning of a week is 22 thousand animals then the population will decrease by 11 thousand over that week.

(d) Tracing the graph shows that the function increases from \(n = 0\) to about \(n = 10\). Over this interval the graph is concave down. This means that, if the population at the beginning of the week is at most 10 thousand, for larger initial populations the growth over a week is larger, but the rate of increase in this growth actually decreases.

9. The economic order quantity model:

(a) i. Since the demand is \(N = 400\) units per year and the carrying cost is \(h = 24\) dollars per year, the function we want is

\[
Q(c) = \sqrt{\frac{800c}{24}} \text{ items per order.}
\]

Since we do not expect the fixed ordering cost to exceed $25, we use 0 to 25 as the horizontal span. The table below suggests a vertical span of 0 to 30. The graph is shown below. The horizontal axis corresponds to fixed order cost, and the vertical span is number of items per order.
ii. If we evaluate the function above at 6 (using a table of values or the graph), we get that the number of items we need to order at a time is 14.14, or around 14.

iii. From the graph we can see that as the fixed order cost increases so does the number of items we need to order at a time.

(b) i. The exercise tells us that $N = 400$ and $c = 14$. So the function we need is

$$Q = \sqrt{\frac{2 \times 400 \times 14}{h}} = \sqrt{\frac{11200}{h}} \text{ items per order.}$$

Since $h$ ranges from 0 to 25 dollars, we use that for the horizontal span. We used the table of values below to get the vertical span. We used 0 to 150 to make the graph below. The horizontal axis corresponds to carrying cost, and the vertical axis shows number of items per order.

ii. If we evaluate the above function at 15 we get 27.33. So the optimum order size for a carrying cost of $15$ is about 27 items.

iii. From the graph we can see that an increase in the carrying cost decreases the number of items ordered.

iv. Thinking of $Q$ as a function of $h$ alone and evaluating the function at 15 and 18, we see that $Q(15) = 27.325$ and $Q(18) = 24.944$. So the average rate of change from $15$ to $18$ is

$$\frac{\text{Change in number}}{\text{Change in carrying cost}} = \frac{Q(18) - Q(15)}{3} = -0.79 \text{ item per dollar.}$$
Here again we used three decimal places to ensure that our answer is still accurate to two decimal places after division.

v. The graph is concave up, and this tells us that the rate of decrease in the number of items ordered is decreasing as the carrying cost increases.

11. An annuity:

(a) The monthly interest rate is \( r = 0.01 \) and we want a monthly withdrawal of \( M = \$200 \), so the function is

\[
P(t) = 200 \times \frac{1}{0.01} \times \left(1 - \frac{1}{(1 + 0.01)^t}\right) \text{ dollars.}
\]

Note that \( t \) is measured in months.

We use a horizontal span of 0 to 500 months and we chose a vertical span of 0 to 25,000 dollars from the table below. The horizontal axis is number of months of withdrawal, and the vertical axis is dollars invested.

(b) We use the graph to evaluate the function at \( t = 48 \) (4 years is 48 months) and find that we need to invest \$7594.79 so that our child can withdraw \$200 per month for 4 years.

(c) We evaluate the function at \( t = 120 \) (10 years is 120 months) and find that we need to invest \$13,940.10 so that our child can withdraw \$200 per month for 10 years.

(d) The graph levels off and appears to have a limiting value. If we trace out toward the tail end of the graph, we see that an investment of \$20,000 will be sufficient to establish a \$200 per month perpetuity.

13. Artificial gravity:

(a) i. We want an acceleration of \( a = 9.8 \) meters per second per second, so we use the function

\[
N = \frac{30}{\pi} \times \sqrt{\frac{9.8}{r}} \text{ rotations per minute.}
\]
ii. We use a horizontal span of 10 to 200 meters, and from the table below choose a vertical span of 0 to 10 revolutions per minute. The horizontal axis corresponds to the radius of the station, and the vertical axis to revolutions per minute.

<table>
<thead>
<tr>
<th>X</th>
<th>Y_1</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>9.4539</td>
</tr>
<tr>
<td>45</td>
<td>4.4563</td>
</tr>
<tr>
<td>60</td>
<td>3.2542</td>
</tr>
<tr>
<td>115</td>
<td>2.7876</td>
</tr>
<tr>
<td>150</td>
<td>2.4908</td>
</tr>
<tr>
<td>185</td>
<td>2.1979</td>
</tr>
<tr>
<td>220</td>
<td>2.0155</td>
</tr>
</tbody>
</table>

iii. As the distance \( r \) increases, the number of rotations decreases, although more and more slowly. In practical terms, the larger the space station, the fewer rotations per minute are needed to produce artificial gravity.

iv. If we evaluate the function using the graph, or a table of values, we see that in order to produce Earth gravity for a space station of radius \( r = 150 \) meters, we need \( N = 2.44 \) rotations per minute.

(b) i. Now we are looking at a space station of radius \( r = 150 \) meters. Then gravity as a function of revolutions per minute is given by

\[
N = \frac{30}{\pi} \times \sqrt{\frac{a}{150}} \text{ revolutions per minute.}
\]

ii. We want the horizontal span to be from 2.45 meters per second per second to 9.8 meters per second per second. The table of values below suggests a vertical span of 1 to 3 revolutions per minute. The horizontal axis shows acceleration due to gravity, and the vertical axis shows revolutions per second.

<table>
<thead>
<tr>
<th>X</th>
<th>Y_1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1.1027</td>
</tr>
<tr>
<td>4</td>
<td>1.5584</td>
</tr>
<tr>
<td>6</td>
<td>1.9099</td>
</tr>
<tr>
<td>8</td>
<td>2.2053</td>
</tr>
<tr>
<td>10</td>
<td>2.4568</td>
</tr>
<tr>
<td>12</td>
<td>2.7009</td>
</tr>
</tbody>
</table>

iii. As the desired acceleration increases, the number of rotations needed also increases, although more and more slowly.
15. More on plant growth:

(a) i. We are assuming a rainfall of $R = 100$ millimeters, so our function, measured in kilograms per hectare, is

$$Y = -55.12 - 0.01535N - 0.00056N^2 + 3.946 \times 100$$

$$= 339.48 - 0.01535N - 0.00056N^2.$$  

ii. We use a horizontal span of 0 to 800 kilograms per hectare and from the table of values below, a vertical span of $-50$ to 400 kilograms per hectare. The horizontal axis corresponds to initial biomass and the vertical axis corresponds to growth in biomass.

```
<table>
<thead>
<tr>
<th>N</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>339.48</td>
</tr>
<tr>
<td>150</td>
<td>324.58</td>
</tr>
<tr>
<td>300</td>
<td>294.48</td>
</tr>
<tr>
<td>450</td>
<td>219.17</td>
</tr>
<tr>
<td>600</td>
<td>128.67</td>
</tr>
<tr>
<td>750</td>
<td>12.866</td>
</tr>
<tr>
<td>900</td>
<td>-127.9</td>
</tr>
</tbody>
</table>
```

iii. As the amount of initial plant biomass, $N$, increases, the amount of growth, $Y$, decreases. Practically, there is a limit to how much biomass the land can support, and each kilogram present leaves less room for more plants to grow.

(b) i. We are now looking at a rainfall of $R = 80$ millimeters, so our function now, measured in kilograms per hectare, is

$$Y = -55.12 - 0.01535N - 0.00056N^2 + 3.946 \times 80$$

$$= 260.56 - 0.01535N - 0.00056N^2.$$  

ii. We add this new graph to the one above using the same horizontal and vertical spans as before.

iii. We see that lowered rainfall decreases biomass growth. Yes, this is quite consistent with the prediction made in the previous exercise.
17. **Magazine circulation:**

(a) We use a horizontal span from 0 to 6 years, and we used the table of values shown below to choose a vertical span from 0 to 55 thousand circulation. The horizontal axis is years since 1992 and the vertical axis is circulation in thousands.

<table>
<thead>
<tr>
<th>X</th>
<th>Y1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4.7273</td>
</tr>
<tr>
<td>3</td>
<td>27.368</td>
</tr>
<tr>
<td>15</td>
<td>40.945</td>
</tr>
<tr>
<td>48</td>
<td>50.766</td>
</tr>
<tr>
<td>51.624</td>
<td></td>
</tr>
</tbody>
</table>

(b) Since 18 months is 1.5 years and \( t \) is measured in years, \( C(1.5) \) expresses in functional notation the circulation of the magazine 18 months after it was started. The value of \( C(1.5) \) can be found using the graph, as shown in the figure above on the right. We see that 18 months after it started the magazine has a circulation of 19.67 thousand, or 19,670 magazines.

(c) The graph is concave up for about the first 2 years. (Answers will vary here.) In practical terms, the circulation was increasing more and more quickly during the first two years.

(d) The circulation was increasing the fastest where the graph is steepest. The graph appears to be steepest near \( t = 2 \), or after 2 years. (Answers will vary here but should be consistent with the answer to Part (c).)

(e) To see the limiting value for \( C \), which is in practical terms the level of market saturation, we increase the horizontal span to 20 years and trace the tail end of the graph. We see from the graph below that the limiting value is about 52 thousand magazines. (This can also be seen by scanning down a table.)
19. **Buffalo:**

(a) We show the graph with a horizontal span of 0 to 30 and a vertical span of 0 to 330.

![Graph](image)

(b) The year 2002 corresponds to $t = 0$. From a table of values or the graph we find that the value of the function at $t = 0$ is 21. Thus there were 21 buffalo in the herd in 2002.

(c) We want to find what value of the variable $t$ gives the function value $N = 300$. This can be done by tracing the graph, and the value is between $t = 24$ and $t = 25$. Since $t$ is the number of years since 2002, the number of buffalo will first exceed 300 in the year 2026.

(d) Scrolling down a table on the calculator (or tracing the graph), we see that the limiting value of the function is 315. Thus the population will eventually increase to 315 buffalo.

(e) Tracing the graph shows that it is concave up from about $t = 0$ to about $t = 11$. It is concave down afterwards. This means that the herd is growing at an increasing rate from 2002 to 2013 and that the rate of growth begins to decrease after that time.

21. **Research project:** Answers will vary.
2.3 SOLVING LINEAR EQUATIONS

E-1. **Solving equations linear in one variable**: First we gather the terms involving $x$ to the left side of the equation and those not involving $x$ to the right side, then we factor out $x$, and then we divide by the coefficient of $x$.

(a) We have

\[
xy^3 = xy + y \\
xy^3 - xy = y \\
(y^3 - y)x = y \\
\]

Thus the solution is \( x = \frac{y}{y^3 - y} \). This can be simplified to \( x = \frac{1}{y^2 - 1} \).

(b) We have

\[
3x\sqrt{y} = 2x + \sqrt{y} \\
3x\sqrt{y} - 2x = \sqrt{y} \\
(3\sqrt{y} - 2)x = \sqrt{y} \\
\]

Thus the solution is \( x = \frac{\sqrt{y}}{3\sqrt{y} - 2} \).

(c) We have

\[
yx + z^2x = z - y^2x \\
yx + z^2x + y^2x = z \\
(y + z^2 + y^2)x = z \\
\]

Thus the solution is \( x = \frac{z}{y + z^2 + y^2} \).

(d) We have

\[
\sqrt{9 + y} + x\sqrt{7 + y} = yz + xz \\
x\sqrt{7 + y} - xz = yz - \sqrt{9 + y} \\
(\sqrt{7 + y} - z)x = yz - \sqrt{9 + y} \\
\]

Thus the solution is \( x = \frac{yz - \sqrt{9 + y}}{\sqrt{7 + y} - z} \).
E-3. **Finding inverse functions:** In each case we solve the equation \( f(y) = x \) to find an expression \( y \) giving the inverse.

(a) We have

\[
7y + 5 = x \\
7y = x - 5 \\
y = \frac{x - 5}{7}.
\]

Thus the inverse function of \( f \) is \( y = \frac{x - 5}{7} \).

(b) We have

\[
\frac{3y + 1}{2y - 5} = x \\
3y + 1 = x(2y - 5) \\
3y + 1 = 2xy - 5x \\
3y - 2xy = -5x - 1 \\
(3 - 2x)y = -5x - 1 \\
y = \frac{-5x - 1}{3 - 2x}.
\]

Thus the inverse function of \( f \) is \( y = \frac{-5x - 1}{3 - 2x} \). This can also be written as \( y = \frac{5x + 1}{2x - 3} \).

(c) We have

\[
\frac{2y - 4}{y} = x \\
2y - 4 = xy \\
2y - xy = 4 \\
(2 - x)y = 4 \\
y = \frac{4}{2 - x}.
\]

Thus the inverse function of \( f \) is \( y = \frac{4}{2 - x} \). This can also be written as \( y = \frac{-4}{x - 2} \).

(d) We have

\[
\frac{y}{4y - 3} = x \\
y = x(4y - 3) \\
y = 4xy - 3x \\
y - 4xy = -3x \\
(1 - 4x)y = -3x \\
y = \frac{-3x}{1 - 4x}.
\]
Thus the inverse function of \( f \) is \( y = \frac{-3x}{1 - 4x} \). This can also be written as \( y = \frac{3x}{4x - 1} \).

E-5. Making equations linear:

(a) Multiplying both sides of the equation \( \frac{7}{x + 1} = 3 \) by \( x + 1 \) gives

\[
\frac{7}{x + 1} \times (x + 1) = 3(x + 1)
\]

\[
7 = 3(x + 1)
\]

\[
7 = 3x + 3.
\]

Now we solve for \( x \):

\[
7 = 3x + 3
\]

\[
7 - 3 = 3x
\]

\[
4 = 3x
\]

\[
\frac{4}{3} = x.
\]

Thus the solution is \( x = \frac{4}{3} \).

(b) Multiplying both sides of the equation \( \frac{9}{2x} = \frac{7}{3} \) by \( 6x \) gives

\[
\frac{9}{2x} \times 6x = \frac{7}{3} \times 6x
\]

\[
9 \times 6 = \frac{7 \times 6}{3} \times x
\]

\[
27 = 14x.
\]

Now we solve for \( x \):

\[
27 = 14x
\]

\[
\frac{27}{14} = x.
\]

Thus the solution is \( x = \frac{27}{14} \).

(c) Squaring both sides of the equation

\[3\sqrt{x} = a\]

gives

\[(3\sqrt{x})^2 = a^2\]

\[9x = a^2.\]

Now we solve for \( x \):

\[x = \frac{a^2}{9}.\]
Thus the solution is $x = \frac{a^2}{9}$.

(d) Multiplying both sides of the equation $3\sqrt{x} = 2\sqrt{x} + \frac{4}{\sqrt{x}}$ by $\sqrt{x}$ gives

$$3\sqrt{x} \times \sqrt{x} = \left(2\sqrt{x} + \frac{4}{\sqrt{x}}\right) \times \sqrt{x}$$

$$3x = 2\sqrt{x} \times \sqrt{x} + \frac{4}{\sqrt{x}} \times \sqrt{x}$$

$$3x = 2x + 4.$$

Now we solve for $x$:

$$3x = 2x + 4$$

$$x = 4.$$

Thus the solution is $x = 4$.

S-1. Linear equations: To solve $3x + 7 = x + 21$ for $x$, we follow the procedure described in the solution of Exercise E-1 above: First we gather the terms involving the variable to the left side of the equation and those not involving the variable to the right side, then we factor out the variable, and then finally we divide by the coefficient of the variable.

$$3x + 7 = x + 21$$

$$3x - x = 21 - 7$$

$$2x = 14$$

$$x = 7.$$

S-3. Linear equations: To solve $3 - 5x = 23 + 4x$ for $x$, we follow the procedure described above, just as for Exercise S-1:

$$3 - 5x = 23 + 4x$$

$$-5x - 4x = 23 - 3$$

$$-9x = 20$$

$$x = \frac{20}{-9} = -\frac{20}{9}.$$

S-5. Linear equations: To solve $12x + 4 = 55x + 42$ for $x$, we follow the procedure described above, just as for Exercise S-1:

$$12x + 4 = 55x + 42$$

$$12x - 55x = 42 - 4$$

$$-43x = 38$$

$$x = \frac{38}{-43} = -\frac{38}{43}.$$
S-7. **Linear equations:** To solve for $cx + d = 12$ for $x$, we follow the procedure described above, just as for Exercise S-1:

\[
\begin{align*}
    cx + d &= 12 \\
    cx &= 12 - d \\
    x &= \frac{12 - d}{c}.
\end{align*}
\]

S-9. **Linear equations:** To solve $2k + m = 5k + n$ for $k$, we follow the procedure described above, just as for Exercise S-1, only in this case the variable is named $k$ instead of $x$:

\[
\begin{align*}
    2k + m &= 5k + n \\
    2k - 5k &= n - m \\
    -3k &= n - m \\
    k &= \frac{n - m}{-3} = -\frac{n - m}{3}. \\
\end{align*}
\]

This expression can also be written as $k = \frac{m - n}{3}$.

S-11. **Linear equations:** To solve $2x - 3 = 4 - 2x$ for $x$ we follow the procedure described above, just as for Exercise S-1:

\[
\begin{align*}
    2x - 3 &= 4 - 2x \\
    2x + 2x &= 4 + 3 \\
    4x &= 7 \\
    x &= \frac{7}{4}.
\end{align*}
\]

1. **Gross domestic product:**

   (a) Because $P$ is calculated as the sum of $C, I, G,$ and $E$, we have $P = C + I + G + E$.

   (b) We are given that $C = 7303.7$, $I = 1593.2$, $G = 1972.9$, and $E = -423.6$, all in billions of dollars.

   i. Recall that $E$ is exports minus imports. Because $E$ is negative, imports were larger.

   ii. From the given values we have

   \[P = 7303.7 + 1593.2 + 1972.9 - 423.6 = 10,446.2,\]

   so the gross domestic product was 10,446.2 billion dollars.

   (c) We solve the equation $P = C + I + G + E$ for $E$ by subtracting $C + I + G$ from both sides. The result is $E = P - C - I - G$. 


(d) We are given the same values for $C$, $I$, and $G$ as in Part (b), and we know that $P = 10,886$ (in billions of dollars). The equation from Part (c) gives

$$E = 10,886 - 7303.7 - 1593.2 - 1972.9 = 16.2,$$

so the net sales to foreigners is 16.2 billion dollars.

3. Resale value:

(a) The formula gives $V(3) = 12.5 - 1.1 \times 3 = 9.2$ thousand dollars. This means that the resale value of the boat will be 9.2 thousand dollars, or $9200$, at the start of the year 2004 (because that is 3 years after the start of 2001).

(b) We want to find what value of the variable $t$ gives the value 7 for the function $V$. This can be done either by examining a table of values (or a graph) for the function or by solving the linear equation $12.5 - 1.1t = 7$. We take the second approach:

\[
12.5 - 1.1t = 7 \\
-1.1t = 7 - 12.5 \quad \text{Subtract 12.5 from both sides.} \\
t = \frac{7 - 12.5}{-1.1} \quad \text{Divide both sides by } -1.1. \\
t = 5.
\]

Thus the resale value will be 7 thousand dollars at the start of the year 2006 (because that is 5 years after the start of 2001).

(c) We solve the equation $V = 12.5 - 1.1t$ for $t$ as follows:

\[
V = 12.5 - 1.1t \\
V - 12.5 = -1.1t \quad \text{Subtract 12.5 from both sides.} \\
\frac{V - 12.5}{-1.1} = t \quad \text{Divide both sides by } -1.1.
\]

Thus the formula is $t = \frac{V - 12.5}{-1.1}$. This can also be written as $t = \frac{12.5 - V}{1.1}$ or as $t = 11.36 - 0.91V$.

(d) When we use the value $V = 4.8$ in the answer to Part (c), we obtain $t = \frac{4.8 - 12.5}{-1.1} = 7$. Thus the resale value will be 4.8 thousand dollars at the start of the year 2008 (because that is 7 years after the start of 2001).

5. Gas mileage:

(a) The exercise tells us that we have $g = 12$ gallons of gas in the tank and that our car gets $m = 24$ miles per gallon. So we can travel

\[
d = gm = 12 \times 24 = 288 \text{ miles}.
\]
(b) To solve for \( m \), we divide both sides by \( g \):

\[
d = gm
\]

\[
d \div g = m \text{ miles per gallon.}
\]

i. This equation tells us that the mileage our car gets is the distance traveled divided by the amount of gas used.

ii. If we drive \( d = 335 \) miles using \( g = 13 \) gallons of gas, then our car got

\[
m = \frac{d}{g} = \frac{335}{13} = 25.77 \text{ miles per gallon.}
\]

(c) i. Since \( d = 425 \) miles, we have \( 425 = gm \), so \( g = \frac{425}{m} \).

ii. To choose a window size we need first to pick a reasonable range for gas mileage. We choose 0 to 30 for the horizontal span. The table below suggests a vertical span from 0 to 75. The horizontal axis corresponds to gas mileage, and the vertical axis corresponds to tank capacity.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>ERROR</td>
</tr>
<tr>
<td>8</td>
<td>52.125</td>
</tr>
<tr>
<td>16</td>
<td>28.656</td>
</tr>
<tr>
<td>24</td>
<td>17.708</td>
</tr>
<tr>
<td>32</td>
<td>13.281</td>
</tr>
<tr>
<td>40</td>
<td>10.625</td>
</tr>
<tr>
<td>48</td>
<td>8.8542</td>
</tr>
</tbody>
</table>

The graph is not a straight line.

7. Supply and demand: At the equilibrium price we have \( S = D \), so the expression \( 1.9S - 0.7 \) has the same value as the expression \( 2.8 - 0.6S \); the equilibrium price is then the common value of these expressions. Thus we want to solve the equation \( 1.9S - 0.7 = 2.8 - 0.6S \) for \( S \). Here are the steps:

\[
1.9S - 0.7 = 2.8 - 0.6S
\]

\[
1.9S + 0.6S = 2.8 + 0.7 \quad \text{Add 0.7 and 0.6S to both sides.}
\]

\[
2.5S = 3.5 \quad \text{Combine terms.}
\]

\[
S = \frac{3.5}{2.5} \quad \text{Divide both sides by 2.5.}
\]

\[
S = 1.4.
\]
Thus the equilibrium price occurs at the quantity 1.4 billion bushels; using the formula \( P = 1.9S - 0.7 \) shows that that price is \( 1.9 \times 1.4 - 0.7 = 1.96 \) dollars per bushel.

9. Fire engine pressure:

(a) We are given that \( NP = 80 \) and \( K = 0.51 \). Because the length of the hose is 80 feet, we have \( L = \frac{80}{50} = 1.6 \). Thus \( EP = 80(1.1 + 0.51 \times 1.6) = 153.28 \). The engine pressure is 153.28 psi.

(b) We are given that \( K = 0.73 \) and \( EP = 150 \). Because the length of the hose is 45 feet, we have \( L = \frac{45}{50} = 0.9 \). Thus \( 150 = NP(1.1 + 0.73 \times 0.9) \). Dividing both sides by \( 1.1 + 0.73 \times 0.9 \) gives \( NP = \frac{150}{1.1 + 0.73 \times 0.9} = 85.37 \). The nozzle pressure is 85.37 psi.

(c) To solve the equation \( EP = NP(1.1 + K \times L) \) for \( NP \), we divide both sides by \( 1.1 + K \times L \). The result is \( NP = \frac{EP}{1.1 + K \times L} \).

(d) We are given that \( EP = 160 \) and \( NP = 90 \). Because the length of the hose is 190 feet, we have \( L = \frac{190}{50} = 3.8 \). Thus \( 160 = 90(1.1 + K \times 3.8) \). We solve for \( K \) as follows:

\[
\frac{160}{90} = 1.1 + 3.8K \quad \text{Divide both sides by} \ 90.
\]

\[
\frac{160}{90} - 1 = 3.8K \quad \text{Subtract} \ 1 \ \text{from both sides}.
\]

\[
\frac{160}{90} - 1.1 = 3.8K \quad \text{Divide both sides by} \ 3.8.
\]

\[
0.18 = K.
\]

Thus the “\( K \)” factor is 0.18.

(e) We solve for \( K \) as follows:

\[
EP = NP(1.1 + K \times L)
\]

\[
\frac{EP}{NP} = 1.1 + K \times L \quad \text{Divide both sides by} \ NP.
\]

\[
\frac{EP}{NP} - 1.1 = K \times L \quad \text{Subtract} \ 1.1 \ \text{from both sides}.
\]

\[
\frac{EP}{NP} - 1.1 \frac{EP}{L} = K \quad \text{Divide both sides by} \ L.
\]

Thus \( K = \frac{EP}{NP} - 1.1 \).
11. **Net profit:**

(a) Our gross profit is the number of dolls sold times the selling price. Then we have to subtract the cost of making the dolls, which is the number sold times the cost per doll. We also have to subtract the cost of rent. We find that

\[
\text{Net profit } p = \text{Sales price} \times \# \text{ sold} - \text{Cost per doll} \times \# \text{ sold} - \text{Rent}.
\]

This can also be written as \( P = (d - c)n - R. \)

(b) The exercise tells us that the rent is \( R = 1280 \), it costs \( c = 2 \) each to make the dolls, the selling price is \( d = 6.85 \), and we sell \( n = 826 \) dolls. So our net profit would be

\[
p = dn - cn - R = 6.85 \times 826 - 2 \times 826 - 1280 = 2726.10.
\]

(c) We have

\[
p = dn - cn - R
\]

\[
\begin{align*}
p + cn &= dn - R & \text{Add} \ cn \ \text{to both sides}. \\
p + cn + R &= dn & \text{Add} \ R \ \text{to both sides}.
\end{align*}
\]

\[
\begin{align*}
\frac{p + cn + R}{n} &= d & \text{Divide both sides by} \ n.
\end{align*}
\]

Thus the formula is \( d = \frac{p + cn + R}{n} \). This can also be written as \( d = \frac{p + R}{n} + c. \)

(d) The required net profit is \( p = 4000 \), the rent is \( R = 1200 \), the cost per doll is \( c = 2 \), and we expect to sell \( n = 700 \) dolls. So under these conditions the price we need to get for each doll is

\[
d = \frac{p + cn + R}{n} = \frac{4000 + 700 \times 2 + 1200}{700} = 9.43.
\]

13. **Temperature conversion:**

(a) Here \( K(30) \) is the temperature in kelvins when the temperature on the Celsius scale is 30 degrees. The value is \( 30 + 273.15 = 303.15 \) kelvins.

(b) To express the Celsius temperature \( C \) in terms of kelvins \( K \) we solve the given equation relating the two:

\[
K = C + 273.15
\]

\[
K - 273.15 = C \ \text{Subtract} \ 273.15 \ \text{from each side}.
\]
Thus we have
\[ C = K - 273.15. \]

(c) We use the expression from Part (b) to replace \( C \) in \( F = 1.8C + 32 \). The result is
\[ F = 1.8(K - 273.15) + 32. \]

This can also be written as \( F = 1.8K - 459.67 \).

(d) If the temperature is \( K = 310 \) kelvins, then by Part (c) the Fahrenheit temperature is
\[ F = 1.8(310 - 273.15) + 32 = 98.33 \text{ degrees}. \]

15. Running ants:

(a) The ambient temperature is \( T = 30 \) degrees Celsius, so in functional notation \( S(30) \) is the speed of the ants. This is calculated as
\[ S(30) = 0.2 \times 30 - 2.7 = 3.3 \text{ centimeters per second}. \]

(b) We have
\[
S = 0.2T - 2.7 \\
S + 2.7 = 0.2T \quad \text{Add 2.7 to each side.}
\]
\[
\frac{S + 2.7}{0.2} = T \quad \text{Divide each side by 0.2.}
\]
Thus the formula is \( T = \frac{S + 2.7}{0.2} \).

(c) The speed is \( S = 3 \) centimeters per second, so the ambient temperature is
\[
T = \frac{S + 2.7}{0.2} = \frac{3 + 2.7}{0.2} = 28.5 \text{ degrees Celsius}. \]

17. Profit:

(a) Assume that the total cost \( C \) is in dollars. Because
\[
\text{Total cost} = \text{Variable cost} \times \text{Number of items} + \text{Fixed costs},
\]
we have \( C = 55N + 200 \).

(b) Assume that the total revenue \( R \) is in dollars. Because
\[
\text{Total revenue} = \text{Selling price} \times \text{Number of items},
\]
we have \( R = 58N \).
(c) Assume that the profit $P$ is in dollars. Because

$$\text{Profit} = \text{Total revenue} - \text{Total cost},$$

we have $P = R - C$. Using the formulas from Parts (a) and (b), we obtain

$$P = 58N - (55N + 200).$$

Because

$$58N - (55N + 200) = 58N - 55N - 200 = 3N - 200,$$

this can also be written as $P = 3N - 200$. The latter is the formula we use in Part (d).

(d) We want to find what value of the variable $N$ gives the function value $P = 0$. Thus we need to solve the equation $3N - 200 = 0$. We solve it as follows:

$$3N - 200 = 0$$
$$3N = 200 \quad \text{Add 200 to both sides.}$$
$$N = \frac{200}{3} \quad \text{Divide both sides by 3.}$$

Thus the break-even point occurs at a production level of about 66.67 thousand widgets per month. This is 66,670 widgets per month.

19. **Plant growth:**

(a) We have

$$Y = -55.12 - 0.01535N - 0.00056N^2 + 3.946R$$
$$Y + 55.12 = -0.01535N - 0.00056N^2 + 3.946R \quad \text{Add 55.12 to each side.}$$
$$Y + 55.12 + 0.01535N = -0.00056N^2 + 3.946R \quad \text{Add 0.01535N to each side.}$$
$$Y + 55.12 + 0.01535N + 0.00056N^2 = 3.946R \quad \text{Add 0.00056N^2 to each side.}$$

$$\frac{Y + 55.12 + 0.01535N + 0.00056N^2}{3.946} = R \quad \text{Divide both sides by 3.946.}$$

Thus the formula is

$$R = \frac{Y + 55.12 + 0.01535N + 0.00056N^2}{3.946}.$$
(c) We use a horizontal span of 0 to 800 kilograms per hectare. Using the table below, we choose a vertical span of 0 to 120 millimeters. The horizontal axis corresponds to initial plant biomass, and the vertical axis corresponds to rainfall.

<table>
<thead>
<tr>
<th>X</th>
<th>Y1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>13.966</td>
</tr>
<tr>
<td>150</td>
<td>17.746</td>
</tr>
<tr>
<td>300</td>
<td>27.908</td>
</tr>
<tr>
<td>450</td>
<td>44.468</td>
</tr>
<tr>
<td>600</td>
<td>67.392</td>
</tr>
<tr>
<td>750</td>
<td>96.714</td>
</tr>
<tr>
<td>900</td>
<td>132.42</td>
</tr>
</tbody>
</table>

(d) As \( N \) increases, \( R \) also increases. In practical terms, the greater the initial plant biomass, the more rainfall it needs just to maintain its size.

(e) If the initial plant biomass is \( N = 400 \) kilograms per hectare, then we can evaluate the function using the graph to get \( R = 38.23 \) millimeters.

(f) If the initial plant biomass \( N \) is 500 kilograms per hectare, then, by using the graph, we find that the corresponding zero isocline rainfall \( R \) needed to sustain that biomass is \( R(500) = 51.39 \) millimeters of rain. Unfortunately, there are only 40 millimeters of rain, so the plants will die back.

21. **Competition between populations:**

(a) We need to solve the equation \( 5(1 - m - n) = 0 \) for \( n \). We solve it as follows:

\[
5(1 - m - n) = 0 \\
1 - m - n = 0 \text{ Divide both sides by } 5. \\
1 - m = n \text{ Add } n \text{ to both sides.}
\]

Thus \( n = 1 - m \) describes this isocline.

(b) We need to solve the equation \( 6(1 - 0.7m - 1.2n) = 0 \) for \( n \). We solve it as follows:

\[
6(1 - 0.7m - 1.2n) = 0 \\
1 - 0.7m - 1.2n = 0 \text{ Divide both sides by } 6. \\
1 - 0.7m = 1.2n \text{ Add } 1.2n \text{ to both sides.} \\
\frac{1 - 0.7m}{1.2} = n \text{ Divide both sides by } 1.2.
\]

Thus \( n = \frac{1 - 0.7m}{1.2} \) describes this isocline.
(c) By Parts (a) and (b), the per capita growth rates will both be zero if \( n = 1 - m \) and \( n = \frac{1 - 0.7m}{1.2} \). This gives the equation \( 1 - m = \frac{1 - 0.7m}{1.2} \). We solve this equation as follows:

\[
1 - m = \frac{1 - 0.7m}{1.2}
\]

\[
1.2(1 - m) = 1 - 0.7m \quad \text{Multiply both sides by 1.2.}
\]

\[
1.2 - 1.2m = 1 - 0.7m \quad \text{Expand.}
\]

\[
1.2 - 1 = -0.7m + 1.2m \quad \text{Add 1.2m and subtract 1 on both sides.}
\]

\[
0.2 = 0.5m \quad \text{Combine terms.}
\]

\[
\frac{0.2}{0.5} = m \quad \text{Divide both sides by 0.5.}
\]

\[
0.4 = m.
\]

Thus \( m = 0.4 \), and from the equation in Part (a) we find that \( n = 1 - m = 1 - 0.4 = 0.6 \). Hence the equilibrium point occurs at \( m = 0.4, n = 0.6 \) thousand animals.

2.4 SOLVING NONLINEAR EQUATIONS

E-1. Factoring quadratics:

(a) We know from the signs that we are looking for factors of the form \((x + a)(x - b)\) with \(0 < a < b\). We want \(ab = 9\) and \(b - a = 8\). Thus we should choose \(a = 1\) and \(b = 9\). Hence \(x^2 - 8x - 9 = (x + 1)(x - 9)\). Of course this can also be written as \(x^2 - 8x - 9 = (x - 9)(x + 1)\).

(b) We know from the signs that we are looking for factors of the form \((x + a)(x + b)\) with \(a, b\) positive. We want \(ab = 9\) and \(a + b = 6\). Thus we should choose \(a = 3\) and \(b = 3\). Hence \(x^2 + 6x + 9 = (x + 3)(x + 3)\), or \(x^2 + 6x + 9 = (x + 3)^2\). \textit{Note:} Here we could also use the perfect-square formula.

(c) We use the following basic formula from the text:

\[
x^2 + (a + b)x + ab = (x + a)(x + b).
\]

Since there is no \(x\) term in \(x^2 - 16\), we have \(a + b = 0\); further, from the constant term we have \(ab = -16\). We choose \(a = -4\) and \(b = 4\). (Of course, the choice \(a = 4\) and \(b = -4\) is also valid.) Hence \(x^2 - 16 = (x - 4)(x + 4)\).

(d) We know from the signs that we are looking for factors of the form \((x - a)(x - b)\) with \(a, b\) positive. We want \(ab = 35\) and \(a + b = 12\). We choose \(a = 7\) and \(b = 5\). (Of course, the choice \(a = 5\) and \(b = 7\) is also valid.) Hence \(x^2 - 12x + 35 = (x - 7)(x - 5)\).
E-3. Solving higher-order equations:

(a) We first bring everything to the left-hand side of the equation: \( x^3 - 3x^2 - 4x = 0 \).

Next we note that each term has a common factor of \( x \), and we factor this out:
\[ x(x^2 - 3x - 4) = 0. \]
The quadratic term factors as \((x + a)(x - b)\) with \(0 < a < b\). We want \( ab = 4 \) and \( b - a = 3 \). Thus we should choose \( a = 1 \) and \( b = 4 \).

Hence \( x^2 - 3x - 4 = (x + 1)(x - 4) \), and the original equation is equivalent to \( x(x + 1)(x - 4) = 0 \). Thus we get three solutions: \( x = 0 \), \( x = -1 \), and \( x = 4 \).

(b) The left-hand side is already factored, and at least one factor must be zero. Thus either \( 2x - 3 = 0 \) or \( 3x + 4 = 0 \) or \( 2x - 5 = 0 \). Solving each of these linear equations gives the three solutions \( x = \frac{3}{2}, x = -\frac{4}{3}, \) and \( x = \frac{5}{2} \).

(c) The first step is to factor the quadratic term \( x^2 + 2x - 15 \). This factors as \((x + a)(x - b)\) with \(0 < b < a\). We want \( ab = 15 \) and \( a - b = 2 \). Thus we should choose \( a = 5 \) and \( b = 3 \). Hence \( x^2 + 2x - 15 = (x + 5)(x - 3) \). Thus the original equation can be written as \((x - 4)(x + 5)(x - 3) = 0\). Thus we get three solutions: \( x = 4 \), \( x = -5 \), and \( x = 3 \).

(d) First we factor each quadratic term. On page 148 of the text, the factorization \( x^2 - 5x + 6 = (x - 3)(x - 2) \) is found. The second quadratic, namely \( x^2 + 7x + 12 \), factors as \((x + a)(x + b)\) with \( a, b \) positive. We want \( ab = 12 \) and \( a + b = 7 \). We choose \( a = 4 \) and \( b = 3 \). Hence \( x^2 + 7x + 12 = (x + 4)(x + 3) \). Thus the original equation can be written as \((x - 3)(x - 2)(x + 4)(x + 3) = 0\). Thus we get four solutions: \( x = 3 \), \( x = 2 \), \( x = -4 \), and \( x = -3 \).

E-5. Rational solutions:

(a) Here the constant term \( a_0 \) and the leading term \( a_n = a_4 \) are both 3. The only divisors of 3 are \( \pm 1 \) and \( \pm 3 \). Hence the only possible rational solutions are \( \pm \frac{1}{3} \), \( \pm 1 \), and \( \pm 3 \). By plugging these into the equation, we find that \( \frac{1}{3} \) and 3 are solutions and that the others are not. Thus the only rational solutions are \( x = \frac{1}{3} \) and \( x = 3 \).

(b) Here the constant term \( a_0 \) is \(-1\) and the leading term \( a_n = a_3 \) is 2. The only divisors of \(-1\) are \( \pm 1 \), and the only divisors of 2 are \( \pm 1 \) and \( \pm 2 \). Hence the only possible rational solutions are \( \pm \frac{1}{2} \) and \( \pm 1 \). By plugging these into the equation, we find that \( \frac{1}{2} \) is a solution and that the others are not. Thus the only rational solution is \( x = \frac{1}{2} \).

(c) Here the constant term \( a_0 \) is \( 1 \) and the leading term \( a_n = a_4 \) is 2. The only divisors of 1 are \( \pm 1 \), and the only divisors of 2 are \( \pm 1 \) and \( \pm 2 \). Hence the only possible rational solutions are \( \pm \frac{1}{2} \) and \( \pm 1 \). By plugging these into the equation, we find that
none of these is a solution. Thus there are no rational solutions of the equation. (In fact, examining the equation shows that there are no real solutions: $2x^4 + 3x^2 + 1$ is at least 1 if $x$ is real because $x^4$ and $x^2$ are nonnegative in that case.)

S-1. **The crossing-graphs method**: To solve \( \frac{20}{1 + 2^x} = x \) using the crossing-graphs method, we enter the \( \frac{20}{1 + 2^x} \) as \( Y_1 \) and \( x \) as \( Y_2 \). Using the standard viewing window, we obtain the graph below, which shows that the solution is \( x = 2.69 \).

S-3. **The crossing-graphs method**: To solve \( 3^x + x = 2^x + 1 \) using the crossing-graphs method, we enter the \( 3^x + X \) as \( Y_1 \) and \( 2^x + 1 \) as \( Y_2 \). Using a horizontal span of \(-2\) to \(2\) and a vertical span of \(0\) to \(5\), we obtain the graph below, which shows that the solution is \( x = 0.59 \).

S-5. **The crossing-graphs method**: To solve \( \frac{5}{x^2 + x + 1} = 1 \) using the crossing-graphs method, we enter the \( \frac{5}{(X^2 + X + 1)} \) as \( Y_1 \) and \( 1 \) as \( Y_2 \). Using a horizontal span of \(-2\) to \(2\) and a vertical span of \(0\) to \(7\), we obtain the graphs below, which show that the solutions are \( x = -2.56 \) and \( x = 1.56 \).
S-7. The single-graph method: To solve \( \frac{5}{x^2 + x + 1} - 1 = 0 \) using the single-graph method, we enter the function as \( Y_1 \). Using a horizontal span of \(-4\) to \(4\) and a vertical span from \(-3\) to \(6\), we obtain the graphs below, which show that the solutions are \( x = -2.56 \) and \( x = 1.56 \).

![Graphs for S-7](image1)

S-9. The single-graph method: To solve \( \frac{-x^4}{x^2 + 1} + 1 = 0 \) using the single-graph method, we enter the function as \( Y_1 \). Using a horizontal span of \(-2\) to \(2\) and a vertical span from \(-2\) to \(2\), we obtain the graphs below, which show that the solutions are \( x = -1.27 \) and \( x = 1.27 \).

![Graphs for S-9](image2)

S-11. The crossing-graphs method: To solve \( x^3 - 5x = 1 - x^2 \) using the crossing-graphs method, we enter the \( X^3 - 5X \) as \( Y_1 \) and \( 1 - X^2 \) as \( Y_2 \). Using a horizontal span of \(-3\) to \(3\) and a vertical span of \(-12\) to \(12\), we obtain the graphs below, which show that the solutions are \( x = -2.71 \), \( x = -0.19 \), and \( x = 1.90 \).

![Graphs for S-11](image3)
PLEASE NOTE: The exercises in this section involve solving nonlinear equations. There are many ways to do this, so it is important that students, graders, and instructors be aware of the various ways to solve equations. For example, some people prefer to use tables and get better and better approximations by refining the table; some will graph and zoom in repeatedly; other will use two graphs and find their intersection; others may graph and trace; and still others will rearrange the expression and find the appropriate root of an equation.

1. A population of foxes:

(a) The number of foxes introduced is the value of $N$ when $t$ is zero:

$$N(0) = \frac{37.5}{0.25 + 0.76^0} = 30 \text{ foxes.}$$

(b) The exercise gave no information about the horizontal span. We have chosen to look at the fox population over a 25-year period. The following table of values led us to choose a vertical span of 0 to 160 foxes. The horizontal axis is time in years, and the vertical axis is number of foxes.

<table>
<thead>
<tr>
<th>$X$</th>
<th>$Y_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>30</td>
</tr>
<tr>
<td>5</td>
<td>74.471</td>
</tr>
<tr>
<td>10</td>
<td>119.32</td>
</tr>
<tr>
<td>15</td>
<td>140.82</td>
</tr>
<tr>
<td>20</td>
<td>147.58</td>
</tr>
<tr>
<td>25</td>
<td>149.84</td>
</tr>
<tr>
<td>30</td>
<td>149.84</td>
</tr>
</tbody>
</table>

The population of foxes grows rapidly at first, and then the growth slows down until there is almost no growth at all.

(c) The fox population reaches 100 individuals when $N(t) = 100$. Thus we need to solve the equation

$$\frac{37.5}{0.25 + 0.76^t} = 100$$
for $t$. We do this using the crossing graphs method. In the figure below we have added the graph of 100 (thick line) and calculated the crossing point. We read from the bottom of the screen that the fox population will reach 100 individuals after 7.58 years.

3. **Revisiting the floating ball**: We need to make a graph with a horizontal span of $-2$ to 7. We used the table of values on the left below to choose a vertical span of $-3300$ to 2100. The graph is on the right below.

In the figures below we have found the other two solutions, $d = -0.98$ foot and $d = 5.80$ feet. The negative solution would have the ball floating above the surface of the water. The positive solution is larger than the diameter of the ball, and so the ball would be completely submerged.
5. **Falling with a parachute**: The man has fallen 140 feet when

\[12.5(0.2^t - 1) + 20t = 140.\]

Thus we need to solve this equation. We use the crossing graphs method. Even with a parachute, it doesn’t take long to fall 140 feet—surely less than 20 seconds. Thus we take a horizontal span of 0 to 20. The distance he falls increases from 0 up to 140, and so we use a slightly larger vertical span of 0 to 160. Below we have graphed \(12.5(0.2^t - 1) + 20t\), and we have added the graph of 140 (thick line). We get the crossing point at \(t = 7.62\). Thus the man falls 140 feet in 7.62 seconds.

7. **Reaction rates**:

(a) The exercise suggests a horizontal span of 0 to 100. The table of values on the left below led us to choose a vertical span of −5 to 25. The graph is on the right below.

(b) The reaction rate when the concentration is 15 moles per cubic meter is represented by \(R(15)\) in functional notation. Using a table of values or the graph gives that the value is 9.25 moles per cubic meter per second.
(c) We want to find what two values of the variable \( x \) give the value 0 for the function \( R \). That is, we want to find the two solutions of the equation \( 0.01x^2 - x + 22 = 0 \). We do this using the single-graph method. As the graphs below indicate, the solutions are \( x = 32.68 \) and \( x = 67.32 \). Thus the reaction is in equilibrium at the concentrations of 32.68 moles per cubic meter and 67.32 moles per cubic meter.

9. Van der Waals equation: The pressure is \( p = 100 \) atm, and the temperature is \( T = 500 \) kelvins. Putting these values into the equation, we have

\[
100 = \frac{0.082 \times 500}{V - 0.043} - \frac{3.592}{V^2}.
\]

We want to know the value of \( V \). That is, we need to solve the equation above.

For the graph of \( \frac{0.082 \times 500}{V - 0.043} - \frac{3.592}{V^2} \) the exercise suggests we use a horizontal span from 0.1 to 1 liter. The table of values below leads us to choose a vertical span of 0 to 400 atm. We have added the graph of 100 (thick line) to the picture and calculated the intersection point at \( V = 0.37 \). Thus one mole of carbon dioxide at this temperature and pressure will occupy a volume of 0.37 liter.

11. Radiocarbon dating: Since the half-life of \( C_{14} \) is \( H = 5.77 \) thousand years, the formula that gives the fraction of the original amount present is

\[
\frac{A}{A_0} = 0.5^{t/5.77}.
\]

(Here \( t \) is measured in thousands of years.) We want to know then this value is \( \frac{1}{3} \). That
is, we want to solve the equation

\[ \frac{1}{3} = 0.5^{t/5.77}. \]

We use the crossing graphs method to do that. Since half of the C\textsubscript{14} will be gone in 5.77 thousand years, and half of that will be gone in another 5.77 thousand years, we know that there will be only a quarter of the original amount of carbon 14 left after 2 \times 5.77 = 11.54 thousand years. Thus the fraction reaches \( \frac{1}{3} \) sometime before 12 thousand years. Hence to graph \( \frac{A}{A_0} \) we use a horizontal span of 0 to 12. The fraction starts at 1 and decreases. Thus we use a vertical span of 0 to 1. In the picture below, we have added the graph of \( \frac{1}{3} \) and calculated the intersection point at \( t = 9.15 \). Thus the tree died about 9.15 thousand years ago.

13. Grazing kangaroos:

(a) i. The exercise suggests a horizontal span of 0 to 2000 pounds per acre. The table of values below for \( G \) led us to choose a vertical span of \(-1\) to \(3\). The graph shows available vegetation biomass on the horizontal axis and daily intake on the vertical axis.

<table>
<thead>
<tr>
<th>( X )</th>
<th>( Y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-2.3</td>
</tr>
<tr>
<td>200</td>
<td>0.34322</td>
</tr>
<tr>
<td>400</td>
<td>1.5309</td>
</tr>
<tr>
<td>600</td>
<td>2.0646</td>
</tr>
<tr>
<td>800</td>
<td>2.3045</td>
</tr>
<tr>
<td>1000</td>
<td>2.421</td>
</tr>
<tr>
<td>1200</td>
<td>2.4605</td>
</tr>
</tbody>
</table>

ii. The graph is concave down. Since the graph is increasing, the kangaroo eats more when the vegetation biomass is greater. Small changes in \( V \) at the lower biomass levels cause the kangaroo to eat much more, while small changes in \( V \) at higher levels do not change the eating habits of the kangaroo very much.
iii. The minimal vegetation biomass is the amount that will cause the kangaroo to consume zero pounds. Thus we want to find the value of $V$ where the graph crosses the horizontal axis. In the left-hand picture below, we have calculated where the graph crosses the horizontal axis as $V = 163.08$ pounds per acre.

iv. The satiation level is the limit of the amount the kangaroo will eat. That is where the curve levels off and becomes horizontal. Tracing toward the end of the graph, we see that $G$ values approach 2.5. So the satiation level for the western grey kangaroo is a daily intake of about 2.5 pounds.

(b) i. In the figure below, we have added the graph of $R$.

ii. The red kangaroo has a minimal vegetation biomass level of $V = 0$ pounds per acre, since that is where the second graph crosses the horizontal axis. Consequently, the red kangaroo is more efficient at grazing than the grey kangaroo.

15. **Growth rate:**

(a) As suggested in the exercise, we use a horizontal span of 0 to 1000 pounds per acre. The table below leads us to choose a vertical span of −1 to 1. The horizontal axis on the graph below corresponds to vegetation biomass and the vertical axis to the per capita growth rate.
(b) The per capita growth rate is zero where the graph crosses the horizontal axis. This point is calculated in the graph below and shows that \( r = 0 \) when \( V = 201.18 \). The population size is stable when the vegetation level is 201.18 pounds per acre.

17. **Breaking even:**

(a) Assume that \( C \) is measured in dollars. Because

\[
\text{Total cost} = \text{Variable cost} \times \text{Number} + \text{Fixed costs},
\]

we have

\[
C = 65N + 700.
\]

(b) Assume that \( R \) is measured in dollars. Because

\[
\text{Total revenue} = \text{Price} \times \text{Number},
\]

we have \( R = pN \), and thus

\[
R = (75 - 0.02N)N.
\]

(c) Assume that \( P \) is measured in dollars. Because

\[
\text{Profit} = \text{Total revenue} - \text{Total cost},
\]

we have \( P = R - C \). Using Parts (a) and (b) gives

\[
P = (75 - 0.02N)N - (65N + 700).
\]

(This can also be written as \( P = -0.02n^2 + 10N - 700 \).)
(d) We want to find what two values of the variable \( N \) give the value 0 for the function \( P \). That is, we want to find the two solutions of the equation \((75 - 0.02N)N - (65N + 700) = 0\). We do this using the single-graph method. As the graphs below indicate, the solutions are \( N = 84.17 \) and \( N = 415.83 \). (We used a horizontal span of 0 to 500, as suggested in the exercise, and a vertical span of \(-800\) to \(650\).)
Thus the two break-even points are 84.17 thousand widgets per month and 415.83 thousand widgets per month.

19. **Water flea**: We are given that \( N_0 = 50 \), so the relation is
\[
e^{0.44t} = \frac{N}{50} \left( \frac{228 - 50}{228 - N} \right)^{4.46}.
\]
We want to find what value of \( t \) corresponds to the value 125 for \( N \). That is, we want to find the solution of the equation
\[
e^{0.44t} = \frac{125}{50} \left( \frac{228 - 50}{228 - 125} \right)^{4.46}.
\]
Note that the right-hand side of this equation is a constant. We solve this equation using the crossing-graphs method, putting the function \( e^{0.44t} \) in \( Y1 \) and the constant \( \frac{125}{50} \left( \frac{228 - 50}{228 - 125} \right)^{4.46} \) in \( Y2 \). As the graph below indicates, the solution is \( t = 7.63 \).
(After examining a table of values, we used a horizontal span of 0 to 8 and a vertical span of 0 to 40.) Thus it takes 7.63 days for the population to grow to 125.
21. Radius of a shock wave:

(a) We are given that \( E = 10^{15} \), and we want to find what value of the variable \( t \) gives the value 4000 for the function \( R \). That is, we want to find the solution of the equation \( 4.16(10^{15})^{0.2}t^{0.4} = 4000 \). We do this using the crossing-graphs method. As the graph on the left below indicates, the solution is \( t = 0.91 \). (From the statement about the period over which the relation is valid, we expect the time to be around 1 second or less. After examining a table of values, we used a horizontal span of 0 to 1 and a vertical span of 0 to 4500.) Thus 0.91 second is required for the shock wave to reach a point 40 meters away.

(b) We are given that \( E = 9 \times 10^{20} \), and we want to find what value of the variable \( t \) gives the value 4000 for the function \( R \). That is, we want to find the solution of the equation \( 4.16(9 \times 10^{20})^{0.2}t^{0.4} = 4000 \). We do this using the crossing-graphs method. As the graph on the right above indicates, the solution is \( t = 9.6 \times 10^{-4} = 0.00096 \). (Because the energy is much greater here than in Part (a), we expect the time required to be much smaller. After examining a table of values, we used a horizontal span of 0 to 0.001 and a vertical span of 0 to 4500.) Thus 0.00096 second is required for the shock wave to reach a point 40 meters away.

(c) We are given that \( t = 1.2 \), and we want to find what value of the variable \( E \) gives the value 5000 for the function \( R \). That is, we want to find the solution of the equation \( 4.16E^{0.2}1.2^{0.4} = 5000 \). We do this using the crossing-graphs method. As the graph below indicates, the solution is \( E = 1.74 \times 10^{15} \). (Because the time required in Part (a) is close to the time required here, we expect the energy to be on the order of \( 10^{15} \) here also. After examining a table of values, we used a horizontal span of 0 to \( 2 \times 10^{15} \) and a vertical span of 0 to 5500.) Thus an energy of \( 1.74 \times 10^{15} \) ergs was released by the explosion.
23. **Research project:** Answers will vary.

### 2.5 Optimization

**E-1. Locating the vertex of a parabola:** In each part we use the fact that the vertex of the graph of $ax^2 + bx + c$ occurs at $x = -\frac{b}{2a}$.

(a) The vertex of $x^2 + 6x - 4$ occurs at $x = -\frac{6}{2 \times 1} = -3$. The vertical coordinate of the vertex is found by getting the function value at $x = -3$:

Vertical coordinate of vertex $= (-3)^2 + 6 \times (-3) - 4 = -13$.

Thus the vertex of the parabola is $(-3, -13)$. Because the leading coefficient 1 is positive, this is a minimum.

(b) The vertex of $3x^2 - 30x + 1$ occurs at $x = -\frac{-30}{2 \times 3} = 5$. The vertical coordinate of the vertex is found by getting the function value at $x = 5$, which is $3 \times 5^2 - 30 \times 5 + 1 = -74$. Thus the vertex of the parabola is $(5, -74)$. Because the leading coefficient 3 is positive, this is a minimum.

(c) The vertex of $-2x^2 + 12x - 3$ occurs at $x = -\frac{12}{2 \times -2} = 3$. The vertical coordinate of the vertex is found by getting the function value at $x = 3$, which is $-2 \times 3^2 + 12 \times 3 - 3 = 15$. Thus the vertex of the parabola is $(3, 15)$. Because the leading coefficient $-2$ is negative, this is a maximum.

**E-3. Determining if the horizontal axis is crossed:**

(a) The vertex of $x^2 - 2x + 5$ occurs at $x = -\frac{-2}{2 \times 1} = 1$. The vertical coordinate of the vertex is found by getting the function value at $x = 1$, which is $1^2 - 2 \times 1 + 5 = 4$.

Thus the vertex is above the horizontal axis. Because the leading coefficient 1 is positive, the vertex is a minimum. This means that the parabola opens upward from the vertex at $(1, 4)$, so it does not cross the horizontal axis. Another way to
say this is that the vertical coordinate 4 of the vertex is the minimum value of the function, so the graph does not cross the horizontal axis.

(b) The vertex of \( x^2 + 4x - 1 \) occurs at \( x = -\frac{4}{2 \times 1} = -2 \). The vertical coordinate of the vertex is found by getting the function value at \( x = -2 \), which is \((-2)^2 + 4 \times (-2) - 1 = -5\). Thus the vertex is below the horizontal axis. Because the leading coefficient 1 is positive, the parabola opens upward from the vertex at \((-2, -5)\), so it crosses the horizontal axis twice.

E-5. **Distance from a line:** To find the \( x \) value giving the point on the line nearest the origin, we find where the minimum of \( D \), the square of the distance function, occurs. To analyze \( D \) we expand the expression and collect terms:

\[
D = x^2 + (x + 1)^2 = x^2 + (x^2 + 2x + 1) = 2x^2 + 2x + 1.
\]

Now \( 2x^2 + 2x + 1 \) is a quadratic function in standard form, and the vertex occurs at \( x = -\frac{2}{2 \times 2} = -\frac{1}{2} \). Because the leading coefficient 2 is positive, this is a minimum. Thus the desired \( x \) value is \( x = -\frac{1}{2} \).

S-1. **Maximum:** To find the maximum value of \( 5x + 4 - x^2 \) on the horizontal span of 0 to 5, we graph the function. A table of values leads us to choose a vertical span of 0 to 15. The graph, below, shows a maximum value of 10.25 at \( x = 2.5 \).

S-3. **Minimum:** To find the minimum value of \( x + \frac{x + 5}{x^2 + 1} \) on the horizontal span of 0 to 5, we graph the function. A table of values leads us to choose a vertical span of 0 to 7. The graph, below, shows a minimum value of 3.40 at \( x = 1.92 \).
S-5. **Maximum**: To find the maximum value of \( x^{1/x} \) on the horizontal span of 0 to 10, we graph the function. A table of values leads us to choose a vertical span of 0 to 3. The graph, below, shows a maximum value of 1.44 at \( x = 2.72 \).

![Graph showing maximum value](image)

S-7. **Maxima and minima**: To find maxima and minima of \( f = x^3 - 6x + 1 \) with a horizontal span from \(-2\) to 2 and a vertical span from \(-10\) to 10, we graph the function. The graph on the left below shows the maximum is at \( x = -1.41, \ y = 6.66 \), while the graph on the right below shows the minimum is at \( x = 1.41, \ y = -4.66 \).

![Graphs showing maxima and minima](image)

S-9. **Endpoint maximum**: To find the maximum value of \( x^3 + x \) on the horizontal span of 0 to 5, we graph the function. A table of values leads us to choose a vertical span of 0 to 130. The graph, below, is increasing, so the maximum value of 130 occurs at the endpoint \( x = 5 \).

![Graph showing endpoint maximum](image)
S-11. **Maxima and minima:** To find maxima and minima of \( f = x(100 - 2x) \) with a horizontal span of 0 to 50, we graph the function. A table of values leads us to choose a vertical span of 0 to 1500. The graph on the left below shows that the maximum is at \( x = 25, \ y = 1250 \), while the other graphs show that the two minima are at \( x = 0, \ y = 0 \) and at \( x = 50, \ y = 0 \), both of which are endpoint minima.

1. **The cannon at a different angle:**
   
   (a) We can look at the table of values below to get both horizontal and vertical spans. The cannonball follows the graph of \( y \) until it hits the ground, which corresponds to the horizontal axis. Thus we are only interested in the graph where it is positive. The table below shows that the cannonball strikes the ground just short of 4 miles, so we use 0 to 4 as a horizontal span. The table also shows that the cannonball doesn’t get higher than about 1.5 miles. We use 0 to 2 miles for the vertical span.

   ![Graphs of f(x) = x(100 - 2x) and table of values]

   
<table>
<thead>
<tr>
<th>( x )</th>
<th>( y_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.1</td>
<td>1.1933</td>
</tr>
<tr>
<td>0.5</td>
<td>1.3092</td>
</tr>
<tr>
<td>1</td>
<td>1.691</td>
</tr>
<tr>
<td>1.5</td>
<td>-4.808</td>
</tr>
<tr>
<td>2</td>
<td>-9.002</td>
</tr>
</tbody>
</table>
(b) The cannonball strikes the ground where the graph crosses the horizontal axis. In the left-hand figure below, we have expanded the vertical span a little so that the crossing point is clearly visible, and the calculator shows that this crossing point occurs at $x = 3.22$. Thus the cannonball travels 3.22 miles downrange.

(c) The cannonball reaches its maximum height where the graph reaches a peak. In the right-hand picture below, we see that this occurs when $x = 1.61$ and $y = 1.39$. Thus the cannonball reaches a maximum height of 1.39 miles at 1.61 miles downrange.

3. Marine fishery:

(a) The exercise suggests a horizontal span of 0 to 1.5. The table of values on the left below led us to choose a vertical span of $-0.1$ to $0.1$. The graph is on the right below.

(b) The growth rate if the population size is 0.24 million tons is represented by $G(0.24)$ in functional notation. From a table of values or the graph we see that the value is 0.04 million tons per year.

(c) From a table of values or the graph we see that $G(1.42) = -0.02$. This means that if the population size is 1.42 million tons then the growth rate is $-0.02$ million tons per year, so the population is decreasing at a rate of 0.02 million tons per year.
(d) To find where the largest growth rate occurs, we locate the peak of the graph. In the figure below, we see that the peak occurs when \( n = 0.67 \) and \( G = 0.07 \). Thus the growth rate is the largest at a population size of 0.67 million tons.

![Graph](image)

5. **Forming a pen:**

(a) We use two sides that are \( W \) feet long and one that is \( L \) feet long, so the total amount that we need is \( 2W + L \) feet of fence.

(b) The area of a rectangle is the width times the length, and because that area is to be 100 square feet we have \( WL = 100 \).

(c) To solve \( WL = 100 \) for \( L \) we divide both sides by \( W \). The result is \( L = \frac{100}{W} \).

(d) By Part (a) the total amount of fence needed is \( 2W + L \) feet, and by the equation in Part (c) this gives \( F = 2W + \frac{100}{W} \).

(e) After examining a table of values, we used a horizontal span of 1 to 15 and a vertical span of 0 to 110. The graph is on the left below.

![Graph](image)

(f) In the graph on the right above we have located the minimum point, and we see that it occurs where \( W = 7.07 \) feet. From the equation in Part (c) we find that the corresponding length is \( L = \frac{100}{7.07} = 14.14 \). Thus using 7.07 feet perpendicular to the building and 14.14 feet parallel requires a minimum amount of fence.
7. Maximum sales growth:

(a) First we find a formula for unattained sales. Now

\[ \text{Unattained sales} = \text{Limit} - \text{Sales level}, \]

and we are told that the limit is 4 thousand dollars. Thus the formula for unattained sales is \(4 - s\) thousand dollars. Now \(G\) is proportional to the product of the sales level \(s\) and the unattained sales, and we are told that the constant of proportionality is 0.3. Using the formula for unattained sales, we see that the equation is \(G = 0.3s(4 - s)\).

(b) After examining a table of values, we used a horizontal span of 0 to 4 and a vertical span of 0 to 1.5. The graph is on the left below.

(c) In the graph on the right above we have located the maximum point, and we see that it occurs where \(s = 2.00\). Thus the growth rate is as large as possible at a sales level of 2 thousand dollars.

(d) We choose a limit of 6 thousand dollars, so the new equation is \(G = 0.3s(6 - s)\). In the graph on the left below we have located the maximum point, and we see that it occurs where \(s = 3.00\). (After examining a table of values, we used a horizontal span of 0 to 6 and a vertical span of 0 to 3.) Thus the growth rate is as large as possible at a sales level of 3 thousand dollars. In each of the cases we have examined, the sales level for maximum growth is half of the limit on sales.

If we change the constant of proportionality, say to 0.5, and keep a limit of 6 thousand dollars, then the formula for \(G\) changes to \(G = 0.5s(6 - s)\). In the graph on the right below we have located the maximum point, and we see that it still occurs where \(s = 3.00\). (After examining a table of values, we used a horizontal span of 0 to 6 and a vertical span of 0 to 5.) Thus the growth rate is again as large as possible at a sales level of 3 thousand dollars. This is still half of the limit on sales, which suggests that the relationship doesn’t change if the proportionality constant is changed.
Because the sales level for maximum growth is always half of the limit on sales, we can calculate this limit from the data by taking twice the sales level for maximum growth—that is, by doubling the sales level where the growth rate changes from increasing to decreasing.

9. An aluminum can, continued:

(a) If the radius is 1 inch then the height is

\[ h(1) = \frac{15}{\pi} = 4.77 \text{ inches}. \]

The area is

\[ A(1) = 2\pi + 30 = 36.28 \text{ square inches}. \]

(b) If the radius is 5 inches then the height is

\[ h(5) = \frac{15}{\pi 5^2} = 0.19 \text{ inch}. \]

The area is

\[ A(5) = 2\pi 5^2 + \frac{30}{5} = 163.08 \text{ square inches}. \]

(c) i. The aluminum will have a radius of only a few inches, and so we use a horizontal span of 0 to 4 inches. The table below suggests a vertical span of 0 to 50. In the graph below, the radius is on the horizontal axis, and the surface area is on the vertical axis. The graph shows the surface area needed to make a can holding 15 cubic inches as a function of the radius. From the graph it is evident that, as the radius increases, first the amount of aluminum needed decreases to a minimum, and then it increases.
ii. The can that uses the minimum amount of aluminum is represented by the point at the bottom of the graph. In the figure below, we have located that point at $r = 1.34$ and $A = 33.67$. Thus we get a can of minimum surface area, 33.67 square inches, if we use a radius of 1.34 inches.

![Graph showing minimum point](image)

iii. We know that the radius should be about 1.34 to minimize the amount of aluminum used, so the height in that case would be

$$h(1.34) = \frac{15}{\pi(1.34)^2} = 2.66 \text{ inches.}$$

11. **Profit:**

(a) Assume that $C$ is measured in dollars. Because

$$\text{Total cost} = \text{Variable cost} \times \text{Number} + \text{Fixed costs},$$

we have

$$C = 60N + 600.$$ 

(b) Assume that $R$ is measured in dollars. Because

$$\text{Total revenue} = \text{Price} \times \text{Number},$$

we have $R = pN$, and thus, using $p = 70 - 0.03N$, we find

$$R = (70 - 0.03N)N.$$ 

(c) Assume that $P$ is measured in dollars. Because

$$\text{Profit} = \text{Total revenue} - \text{Total cost},$$

we have $P = R - C$. Using Parts (a) and (b) gives

$$P = (70 - 0.03N)N - (60N + 600).$$

(This can also be written as $P = -0.03n^2 + 10N - 600$.)
(d) We graph the function $P$. In the graph below we used a horizontal span of 0 to 300, as suggested in the exercise, and a vertical span of $-600$ to 300. The graph below shows the maximum, and we see that it occurs at $N = 166.67$. Thus profit is maximized at a production level of 166.67 thousand widgets per month.

![Graph of the function P](image)

13. Laying phone cable, continued:

(a) Now

Total cost = Cost for cable on shore + Cost for cable under water.

Also,

Cost for cable on shore = $300 \times$ Length of cable on shore

= $300L$,

and

Cost for cable under water = $500 \times$ Length of cable under water

= $500W$.

Thus

$C = 300L + 500W$.

(b) We use the formula for $W$ in terms of $L$ from Part (a) of Exercise 12: Because $C = 300L + 500W$, that formula for $W$ gives

$C = 300L + 500\sqrt{1 + (5 - L)^2}$.

(c) Again, we use the formula for $W$ in terms of $L$ from Part (a) of Exercise 12. Because $L = 1$, the amount of cable under water is

$W = \sqrt{1 + (5 - 1)^2} = 4.12$ miles.

By Part (b) of this exercise, the total cost of the project is

$C = 300 \times 1 + 500\sqrt{1 + (5 - 1)^2} = 2361.55$ dollars.
Here is the picture:

(d) Because \( L = 3 \), the amount of cable under water is

\[
W = \sqrt{1 + (5 - 3)^2} = 2.24 \text{ miles.}
\]

The total cost is

\[
C = 300 \times 3 + 500 \sqrt{1 + (5 - 3)^2} = 2018.03 \text{ dollars.}
\]

Here is the picture:

(e) Because City B is only 5 miles downriver from City A, the value of \( L \) will be no more than 5. Thus we use a horizontal span of 0 to 5. The table below led us to choose a vertical span from 1000 to 3000. The horizontal axis is miles of cable laid on shore, and the vertical axis is cost in dollars. The graph shows how much it costs to run cable from City A to City B as a function of miles of cable on shore. From the graph it is evident that, as the amount of cable on shore increases, first the cost decreases to a minimum, and then it increases.
(f) In the picture below, we have located the minimum at \( L = 4.25 \) and \( C = 1900 \).
Thus the minimum cost plan uses 4.25 miles on shore (and costs $1900).

(g) Because \( L = 4.25 \) miles,
\[
W = \sqrt{1 + (5 - 4.25)^2} = 1.25 \text{ miles.}
\]
Here is the picture:

(h) The effect of the increase on the formulas from Parts (a) and (b) of this exercise is to replace 300 by 700. The new formula is
\[
C = 700L + 500\sqrt{1 + (5 - L)^2}.
\]
Because the land cost of $700 per mile is greater than the water cost and the water route is the shortest, the least cost project would lay all the cable under water. This can also be seen by making a graph (shown below) of the new cost function. (We used a vertical span of 1500 to 4500.) As expected (see Parts (c) and (d) of
Exercise 12), this function is always increasing, so the least cost is at the beginning, when \( L = 0 \).

15. Growth of fish biomass:

(a) If a plaice weighs three pounds, then \( w = 3 \), and we want to know the age. That is, we want to solve the equation \( w = 3 \), that is

\[
6.32(1 - 0.93e^{-0.095t})^3 = 3.
\]

The table of values below for \( w \) suggests a horizontal span of 0 to 20 years and a vertical span of 0 to 5 pounds. In the right-hand picture, we have plotted the graph of \( w \), along with that of the constant function 3, and calculated the intersection point. This shows that a 3-pound plaice is about 15.18 years old.

(b) The biomass \( B \) is obtained by multiplying the population size by the weight of a fish:

\[
B = N \times w
\]

\[
B = 1000e^{-0.1t} \times 6.32(1 - 0.93e^{-0.095t})^3.
\]

To make the graph, we use a horizontal span of 0 to 20 years as suggested. From the table below, we chose a vertical span of 0 to 700 pounds.
The horizontal axis is age, and the vertical axis is biomass.

(c) The maximum biomass occurs at the peak of the graph above, and we see from there that it occurs at $t = 13.43$ years.

(d) i. From Part (a), we know that a 3-pound plaice is 15.18 years old, so we can only harvest plaice 15.18 years old or older. In the graph below, we have placed the cursor at $t = 15.18$, and we are looking for the maximum considering the graph only from that point on. But biomass is decreasing beyond 15.18, so 15.18 years is the age giving the largest biomass.

ii. As in Part (a), we can find the age of a 2-pound plaice as the solution to $6.32(1 - 0.93e^{-0.095t})^3 = 2$. We have done this by the crossing graphs method in the left-hand picture below and find that a 2-pound plaice is 11.28 years old. We can harvest plaice 11.28 years old or older. We have marked the graph in the right-hand picture below at this point, and we see that the part of the graph from there on includes the maximum at $t = 13.43$. Thus, if we are able to catch 2-pound plaice, we harvest at the maximum biomass which occurs at 13.43 years.
An alternative solution is to find the weight of a plaice at the age $t = 13.43$, found in Part (c), that gives the maximum biomass. This is $w(13.43)$, about 2.56 pounds. Since the weight increases with age, a 2-pound plaice is younger than 13.43 years, and so (as marked in the right-hand picture above) the relevant portion of the graph of biomass will include the maximum at $t = 13.43$ years.

17. **Rate of growth:**

(a) Since the maximum size of a North Sea cod is about 40 pounds, we use a horizontal span of 0 to 40 pounds. From the table of values below, we choose a vertical span of 0 to 5 pounds per year. The graph shows weight on the horizontal axis and rate of growth on the vertical axis.

(b) The peak of the graph is at $w = 12.70$ pounds and $G = 3.81$ pounds per year. For this part of the exercise, we are interested in cod weighing 5 pounds or more. Thus we should be looking at the graph from $w = 5$ on. This includes the peak of the graph, and so the maximum growth rate is $G = 3.81$ pounds per year.

(c) For cod weighing at least 25 pounds, we should look at the graph of $G$ against $w$ from $w = 25$ on. This is beyond the peak of the graph, and from this point on the graph is decreasing. Thus the maximum value of $G$ occurs when the weight is $w = 25$ pounds. That maximum is 2.95 pounds per year.
19. **Size of high schools:**

(a) The study was for schools with a maximum enrollment of 2400 students, and so we use a horizontal span of 0 to 2400. The table below suggests a horizontal span of 0 to 750 dollars per pupil. The horizontal axis on the graph corresponds to enrollment, and the vertical axis corresponds to cost per pupil.

<table>
<thead>
<tr>
<th>X</th>
<th>Y1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>743</td>
</tr>
<tr>
<td>500</td>
<td>572</td>
</tr>
<tr>
<td>1000</td>
<td>461</td>
</tr>
<tr>
<td>1500</td>
<td>430</td>
</tr>
<tr>
<td>2000</td>
<td>419</td>
</tr>
<tr>
<td>2500</td>
<td>408</td>
</tr>
<tr>
<td>3000</td>
<td>417</td>
</tr>
</tbody>
</table>

(b) The enrollment giving the minimum per-pupil cost occurs at the bottom of the graph, and from the picture above, this is at $n = 1675$ students (with a cost of $C = 406.33$ per pupil).

(c) If a high school had an enrollment of 1200, then its cost would be $C(1200) = 433.40$ per pupil. Increasing the school size to the optimum size of 1675 would lower the cost to $406.33$ per pupil, a savings of $27.07$ per pupil.

21. **Water flea:**

(a) The exercise suggests a horizontal span of 0 to 350, and a table of values led us to choose a vertical span of $-15$ to $15$. The graph is on the left below.

(b) To find where the greatest rate of growth occurs, we locate the peak of the graph. In the figure on the right above, we see that the peak occurs when $N = 73.27$ and $G = 10.36$. Thus the greatest rate of growth occurs at a population level of 73.27.

(c) It is evident from the graph or a table of values that $G = 0$ when $N = 0$. We find the other solution to the equation $G(N) = 0$ using the single-graph method. As the graph below indicates, that solution is $N = 228$. Thus the desired values
are \( N = 0 \) and \( N = 228 \). (Another way to do this part is to notice that in the formula for \( G \) the numerator is factored, and this makes it easy to find the desired solutions.) At the two population levels of 0 and 228, the growth rate is 0, that is, the size of the population is not changing.

(d) From the graph or a table of values we see that if the population size is 300 then the rate of population growth is \(-7.51\) per day. Because the growth rate is negative, at this level the population is decreasing in size (at a rate of 7.51 per day).

23. Research project: Answers will vary.

Chapter 2 Review Exercises

1. Finding a minimum: To find the minimum value of \( f \), we make a table. We enter the function as \( Y1 = X^3 - 9X^2/2 + 6X + 1 \) and use a table starting value of 1 and a table increment value of 1, resulting in the following table.

<table>
<thead>
<tr>
<th>( X )</th>
<th>( Y1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.5</td>
<td>3.5</td>
</tr>
<tr>
<td>4.5</td>
<td>4.5</td>
</tr>
<tr>
<td>5.5</td>
<td>5.5</td>
</tr>
<tr>
<td>6.5</td>
<td>6.5</td>
</tr>
<tr>
<td>7.5</td>
<td>7.5</td>
</tr>
</tbody>
</table>

\( X = 1 \)

The table shows that \( f \) has a minimum value of 3 at \( x = 2 \).
2. A population of foxes:

(a) We show the graph with a horizontal span of 0 to 20 and a vertical span of 0 to 150.

(b) From a table of values or the graph we find that the value of the function at \( t = 9 \) is 112.08, which we round to 112. Thus there are 112 foxes 9 years after introduction.

(c) Tracing the graph shows that it is concave up from \( t = 0 \) to about \( t = 5 \). It is concave down afterwards.

(d) Scrolling down a table on the calculator (or tracing the graph), we see that the limiting value of the function is 150.

3. Linear equations: To solve \( 11 - x = 3 + x \) for \( x \), we follow the usual procedure:

\[
11 - x = 3 + x
-2x = -8
x = \frac{-8}{-2} = 4.
\]

4. Water jug:

(a) On the left below we show the graph with a horizontal span of 0 to 7.5 and a vertical span of 0 to 16.
(b) From a table of values or the graph we find that \( D(3) \), the value of the function at \( t = 3 \), is 5.72. Thus the depth of the water is 5.72 inches above the spigot after the spigot has been open 3 minutes.

(c) The time when the jug will be completely drained corresponds to the point when the graph crosses the horizontal axis. In the graph on the right above we find using the single-graph method that this occurs at \( t = 7.44 \). Thus the jug will be completely drained after 7.44 minutes.

(d) The graph is steeper at the beginning than it is toward the end, so the water drains faster near the beginning.

5. **Maxima and minima:** To find the maximum and minimum values of \( x^2 + 100/x \) with a horizontal span of 1 to 5, we graph the function. A table of values leads us to choose a vertical span of 0 to 115. The graph on the left below shows that the maximum is at the left endpoint \( x = 1 \), where the value is \( y = 101 \). The graph on the right shows that the minimum is at \( x = 3.68 \), where the value is \( y = 40.72 \).

6. **George Deer Reserve population:** Below is a table of values for the function \( N \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>26.147</td>
</tr>
<tr>
<td>4</td>
<td>81.704</td>
</tr>
<tr>
<td>6</td>
<td>148.41</td>
</tr>
<tr>
<td>8</td>
<td>169.3</td>
</tr>
<tr>
<td>10</td>
<td>176.72</td>
</tr>
<tr>
<td>12</td>
<td>177.08</td>
</tr>
</tbody>
</table>

(a) We evaluate the function \( N \) at \( t = 0 \). According to the table that value is 6. Hence there were 6 deer introduced into the deer reserve.

(b) According to the table \( N(4) \) is 81.70, or about 82. This means that there were 82 deer in the reserve 4 years after introduction.
(c) Scrolling down the table on the calculator, we see that the limiting value of the function is 177.43, or about 177, and this is the carrying capacity.

(d) The average rate of increase from \( t = 0 \) to \( t = 2 \) is

\[
\frac{N(2) - N(0)}{2 - 0} = \frac{26.147 - 6}{2} = 10.07 \text{ deer per year.}
\]

Continuing in this way, we get the following table, where the rate of change is measured in deer per year.

<table>
<thead>
<tr>
<th>Interval</th>
<th>Rate of change</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 to 2</td>
<td>10.07</td>
</tr>
<tr>
<td>2 to 4</td>
<td>27.78</td>
</tr>
<tr>
<td>4 to 6</td>
<td>30.85</td>
</tr>
<tr>
<td>6 to 8</td>
<td>12.94</td>
</tr>
</tbody>
</table>

The population increases more and more rapidly, and then the rate of growth decreases.

7. **Making a graph**: To find an appropriate window that will show a good graph of the function, first we enter the function as \( Y_1 = \frac{315}{(1 + 14e^{(-0.23x)})} \), and then we make a table of values. The table shows that a vertical span from 0 to 330 will display the graph, and the graph is shown below.

8. **Forming a pen**:

(a) We use two sides that are \( W \) feet long and two that are \( L \) feet long, so the total amount that we need is \( F = 2W + 2L \) feet of fence.

(b) The area of a rectangle is the width times the length, and because that area is to be 144 square feet we have \( 144 = W \times L \).

(c) To solve \( 144 = W \times L \) for \( W \) we divide both sides by \( L \). The result is \( W = \frac{144}{L} \).

(d) By Part (a) the total amount of fence needed is \( F = 2W + 2L \) feet, and by the equation in Part (c) this gives \( F = 2 \times \frac{144}{L} + 2L \) or \( F = \frac{288}{L} + 2L \).
(e) After examining a table of values, we used a horizontal span of 0 to 24 and a vertical span of 0 to 100. The graph is on the left below.

(f) In the graph on the right above we have located the minimum point, and we see that it occurs where \( L = 12 \) feet. From the equation in Part (c) we find that the corresponding width is \( W = \frac{144}{12} = 12 \) feet. Thus a square (12 by 12) pen requires a minimum amount of fence, which makes sense. The graph illustrates that, as the length increases, first the area decreases to a minimum, and then it increases.

9. The crossing-graphs method: To solve the equation using the crossing-graphs method, we enter \( 6 + 69 \times 0.96^x \) as \( Y_1 \) and 32 as \( Y_2 \). Using a horizontal span of 0 to 30 and a vertical span of 0 to 80, we obtain the graph below, which shows that the solution is \( t = 23.91 \).
10. **Gliding pigeons:**

   (a) We show the graph on the left below with a horizontal span of 0 to 20 and a vertical span of 0 to 10.

   ![Graph](image)

   (b) In the graph on the right above we have located the minimum point, and we see that it occurs where \( u = 12.01 \) meters per second.

   (c) The graph shows that if \( u \) is very small then \( s \) is very large. Hence if the airspeed is very slow the pigeon sinks quickly.

11. **Finding a maximum:** To locate the maximum of \( f \), we make a table. We enter the function as \( Y1 = 12X - X^2 - 25 \) and use a table starting value of 3 and a table increment value of 1, resulting in the following table.

   ![Table](image)

   The table shows that \( f \) has a maximum value of 11 at \( x = 6 \).