CHAPTER 9  HYPOTHESIS TESTING

The TI-83 Plus and TI-84 Plus fully support hypothesis testing. Use the [STAT] key, then highlight TESTS. The options used in Chapter 9 are given on the two screens.

**TESTING A SINGLE POPULATION MEAN (SECTION 9.1–9.2 OF UNDERSTANDABLE STATISTICS)**

When the value of \( \sigma \) is known, \( z \)-test based on normal distribution is used to test a population mean, provided that the population has a normal distribution or the sample size is large (at least 30). If the value of \( \sigma \) is unknown, then \( t \)-test will be used if either the population has approximately a normal distribution or the sample size is large. Select option 1:Z-Test for \( z \)-test, or 2:T-Test for \( t \)-test. As with confidence intervals, we have a choice of entering raw data into a list using the Data input option or using summary statistics with the Stats option. The null hypothesis is \( H_0: \mu = \mu_0 \). Enter the value of \( \mu_0 \) in the spot indicated. To select the alternate hypothesis, use one of the three \( \mu \) options \( \mu \neq \mu_0, < \mu_0, \) or \( > \mu_0 \). Output consists of the value of the sample mean \( \bar{x} \) and its corresponding \( z \) value. The \( P \) value of the sample statistic is also given.

**Example (Testing a mean, when \( \sigma \) is known)**

Ten years ago, State College did a study regarding the number of hours full-time students worked each week. The mean number of hours was 8.7. A recent study involving a random sample of 45 full time students showed that the average number of hours worked per week was 10.3. Also assume that the population standard deviation was 2.8. Use a 5% level of confidence to test if the mean number of hours worked per week by full-time students has increased.

Because we have a large sample, we will use the normal distribution for our sample test statistic. Select option 1:Z-Test. We have summary statistics rather than raw data, so use the Stats input option. Enter the appropriate values on the screen.
Next highlight **Calculate** and press [ENTER].

We see that the $z$ value corresponding to the sample test statistic $\bar{x} = 10.3$ is $z = 3.83$. The critical value for a 5% level of significance and a right-tailed test is $z_0 = 1.645$. Clearly the sample $z$ value is to the right of $z_0$, and we reject $H_0$. Notice that the $P$ value = $6.325E-5$. This means that we move the decimal 5 places to the left, giving a $P$ value of 0.000063. Since the $P$ value is less than 0.05, we reject $H_0$.

**Graph**

The TI-83 Plus and TI-84 Plus give an option to show the sample test statistic on the normal distribution. Highlight the **Draw** option on the **Z-Test** screen. Because the sample $z$ is so far to the right, it does not appear on this window. However, its value does, and a rounded $P$ value shows as well.

To do hypothesis testing of the mean when $\sigma$ is unknown, we use Student’s $t$ distribution. Select option **2:T-Test**. The entry screens are similar to those for the Z-Test.
LAB ACTIVITIES FOR TESTING A SINGLE POPULATION MEAN

1. A random sample of 65 pro basketball players showed their heights (in feet) to be

<table>
<thead>
<tr>
<th>Height (in feet)</th>
<th>Height (in feet)</th>
<th>Height (in feet)</th>
<th>Height (in feet)</th>
<th>Height (in feet)</th>
<th>Height (in feet)</th>
<th>Height (in feet)</th>
<th>Height (in feet)</th>
<th>Height (in feet)</th>
<th>Height (in feet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.50</td>
<td>6.25</td>
<td>6.33</td>
<td>6.50</td>
<td>6.42</td>
<td>6.67</td>
<td>6.83</td>
<td>6.82</td>
<td>6.17</td>
<td>7.00</td>
</tr>
<tr>
<td>6.17</td>
<td>7.00</td>
<td>5.67</td>
<td>6.50</td>
<td>6.75</td>
<td>6.54</td>
<td>6.42</td>
<td>6.58</td>
<td>6.00</td>
<td>6.75</td>
</tr>
<tr>
<td>6.00</td>
<td>6.75</td>
<td>7.00</td>
<td>6.58</td>
<td>6.29</td>
<td>7.00</td>
<td>6.92</td>
<td>6.42</td>
<td>5.92</td>
<td>6.08</td>
</tr>
<tr>
<td>5.92</td>
<td>6.08</td>
<td>7.00</td>
<td>6.17</td>
<td>6.92</td>
<td>7.00</td>
<td>5.92</td>
<td>6.42</td>
<td>6.00</td>
<td>6.25</td>
</tr>
<tr>
<td>6.00</td>
<td>6.25</td>
<td>6.75</td>
<td>6.17</td>
<td>6.75</td>
<td>6.58</td>
<td>6.58</td>
<td>6.46</td>
<td>6.00</td>
<td>6.75</td>
</tr>
<tr>
<td>6.00</td>
<td>6.42</td>
<td>6.92</td>
<td>6.50</td>
<td>6.33</td>
<td>6.92</td>
<td>6.67</td>
<td>6.33</td>
<td>6.00</td>
<td>6.58</td>
</tr>
<tr>
<td>6.08</td>
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<td></td>
</tr>
</tbody>
</table>

(a) Enter the data into list L1. Use 1-Var Stats to determine the sample standard deviation.

(b) Use the T-Test option to test the hypothesis that the average height of the players is greater than 6.2 feet, at the 1% level of significance.

2. In this problem, we will see how the test conclusion is possibly affected by a change in the level of significance.

Teachers for Excellence is interested in the attention span of students in grades 1 and 2, now as compared to 20 years ago. They believe it has decreased. Studies done 20 years ago indicate that the attention span of children in grades 1 and 2 was 15 minutes. A study sponsored by Teachers for Excellence involved a random sample of 20 students in grades 1 and 2. The average attention span of these students was (in minutes) $\bar{x} = 14.2$ with standard deviation $s = 1.5$.

(a) Conduct the hypothesis test using $\alpha = 0.05$ and a left-tailed test. What is the test conclusion? What is the $P$ value?

(b) Conduct the hypothesis test using $\alpha = 0.01$ and a left-tailed test. What is the test conclusion? How could you have predicted this result by looking at the $P$ value from part (a)? Is the $P$ value for this part the same as it was for part (a)?

3. In this problem, let’s explore the effect that sample size has on the process of testing a mean. Run Z-Test with the hypotheses $H_0 : \mu = 200, H_1 : \mu > 200, \alpha = 0.05, \bar{x} = 210$ and $\sigma = 40$.

(a) Use the sample size $n = 30$. Note the $P$ value and $z$ score of the sample test statistic and test conclusion.

(b) Use the sample size $n = 50$. Note the $P$ value and $z$ score of the sample test statistic and test conclusion.

(c) Use the sample size $n = 100$. Note the $P$ value and $z$ score of the sample test statistic and test conclusion.

(d) In general, if your sample statistic is close to the proposed population mean specified in $H_0$, and you want to reject $H_0$, would you use a smaller or a larger sample size?
TESTS INVOLVING A SINGLE PROPORTION (SECTION 9.3 OF UNDERSTANDABLE STATISTICS)

To conduct a hypotheses test of a single proportion, select option 5:1-PropZTest. The null hypothesis is $H_0: p = p_0$. Enter the value of $p_0$. The number of successes is designated by the value $x$. Enter the value. The sample size or number of trials is $n$. The alternate hypothesis will be $\text{prop} \neq p_0$, $< p_0$, or $> p_0$. Highlight the appropriate choice. Finally highlight Calculate and press Enter. Notice that the Draw option is available to show the results on the standard normal distribution.

TEST INVOLVING PAIRED DIFFERENCES (DEPENDENT SAMPLES) (SECTION 9.4 OF UNDERSTANDABLE STATISTICS)

To perform a paired difference test, we put our paired data into two columns, and then put the differences between corresponding pairs of values in a third column. For example, put the “before” data in list $L_1$, the “after” data in $L_2$. Create $L_3 = L_1 - L_2$.

Example

Promoters of a state lottery decided to advertise the lottery heavily on television for one week during the middle of one of the lottery games. To see if the advertising improved ticket sales, the promoters surveyed a random sample of 8 ticket outlets and recorded weekly sales for one week before the television campaign and for one week after the campaign. The results follow (in ticket sales) where B stands for “before” and A for “after” the advertising campaign.

<table>
<thead>
<tr>
<th>B:</th>
<th>3201</th>
<th>4529</th>
<th>1425</th>
<th>1272</th>
<th>1784</th>
<th>1733</th>
<th>2563</th>
<th>3129</th>
</tr>
</thead>
<tbody>
<tr>
<td>A:</td>
<td>3762</td>
<td>4851</td>
<td>1202</td>
<td>1131</td>
<td>2172</td>
<td>1802</td>
<td>2492</td>
<td>3151</td>
</tr>
</tbody>
</table>

Test the claim that the television campaign increased lottery ticket sales at the 0.05 level of significance.

We want to test to see if $D = B - A$ is less than zero, since we are testing the claim that the lottery ticket sales are greater after the television campaign. We will put the “before” data in $L_1$, the “after” data in $L_2$, and put the differences in list $L_3$ by highlighting the header $L$. entering $L_1 - L_2$, and pressing Enter.

<table>
<thead>
<tr>
<th>L1</th>
<th>L2</th>
<th>L3</th>
</tr>
</thead>
<tbody>
<tr>
<td>3201</td>
<td>3762</td>
<td>-561</td>
</tr>
<tr>
<td>4529</td>
<td>4851</td>
<td>-322</td>
</tr>
<tr>
<td>1425</td>
<td>1202</td>
<td>223</td>
</tr>
<tr>
<td>1272</td>
<td>1131</td>
<td>141</td>
</tr>
<tr>
<td>1784</td>
<td>2172</td>
<td>-388</td>
</tr>
<tr>
<td>1733</td>
<td>1802</td>
<td>-69</td>
</tr>
<tr>
<td>2563</td>
<td>2492</td>
<td>71</td>
</tr>
</tbody>
</table>
Next we will conduct a t-test on the differences in list L₁. Select option 2:T-Test from the TESTS menu. Select Data for Inpt. Since the null hypotheses is that \( d = 0 \), we use the null hypothesis \( H_0: d = \mu_0 \) with \( \mu_0 = 0 \). Since the data is in L₃, enter that as the List. Since the test is a left-tailed test, select \( \mu < \mu_0 \).

Highlight Calculate and press ENTER.

We see that the sample \( t \) value is –1.17 with a corresponding \( P \) value of 0.138. Since the \( P \) value is greater than 0.05, we do not reject \( H_0 \).

**LAB ACTIVITIES USING TESTS INVOLVING PAIRED DIFFERENCES (DEPENDENT SAMPLES)**

1. The data are pairs of values where the first entry represents average salary (in thousands of dollars/year) for male faculty members at an institution and the second entry represents the average salary for female faculty members (in thousands of dollars/year) at the same institution. A random sample of 22 U.S. colleges and universities was used (source: Academe, Bulletin of the American Association of University Professors).

   (34.5, 33.9)  (30.5, 31.2)  (35.1, 35.0)  (35.7, 34.2)  (31.5, 32.4)  
   (34.4, 34.1)  (32.1, 32.7)  (30.7, 29.9)  (33.7, 31.2)  (35.3, 35.5)  
   (30.7, 30.2)  (34.2, 34.8)  (39.6, 38.7)  (30.5, 30.0)  (33.8, 33.8)  
   (31.7, 32.4)  (32.8, 31.7)  (38.5, 38.9)  (40.5, 41.2)  (25.3, 25.5)  
   (28.6, 28.0)  (35.8, 35.1)
(a) Put the first entries in \( L_1 \), the second in \( L_2 \), and create \( L_3 \) to be the difference \( L_1 - L_2 \).

(b) Use the T-Test option to test the hypothesis that there is a difference in salary. What is the \( P \) value of the sample test statistic? Do we reject or fail to reject the null hypothesis at the 5% level of significance? What about at the 1% level of significance?

(c) Use the T-Test option to test the hypothesis that female faculty members have a lower average salary than male faculty members. What is the test conclusion at the 5% level of significance? At the 1% level of significance.

2. An audiologist is conducting a study on noise and stress. Twelve subjects selected at random were given a stress test in a room that was quiet. Then the same subjects were given another stress test, this time in a room with high-pitched background noise. The results of the stress tests were scores 1 through 20, with 20 indicating the greatest stress. The results, where \( B \) represents the score of the test administered in the quiet room and \( A \) represents the scores of the test administered in the room with the high-pitched background noise, are shown below.

<table>
<thead>
<tr>
<th>Subject</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>13</td>
<td>12</td>
<td>16</td>
<td>19</td>
<td>7</td>
<td>13</td>
<td>9</td>
<td>15</td>
<td>17</td>
<td>6</td>
<td>14</td>
</tr>
<tr>
<td>A</td>
<td>18</td>
<td>15</td>
<td>14</td>
<td>18</td>
<td>10</td>
<td>12</td>
<td>11</td>
<td>14</td>
<td>17</td>
<td>8</td>
<td>16</td>
</tr>
</tbody>
</table>

Test the hypothesis that the stress level was greater during exposure to noise. Look at the \( P \) value. Should you reject the null hypotheses at the 1% level of significance? At the 5% level?

**TESTS OF DIFFERENCE OF MEANS (INDEPENDENT SAMPLES) (SECTION 9.5 OF UNDERSTANDABLE STATISTICS)**

We consider the \( \bar{x}_1 - \bar{x}_2 \) distribution. The null hypothesis is that there is no difference between means so \( H_0: \mu_1 = \mu_2 \), or \( H_0: \mu_1 - \mu_2 = 0 \).

**Example**

Sellers of microwave French fry cookers claim that their process saves cooking time. McDougle Fast Food Chain is considering the purchase of these new cookers, but wants to test the claim. Six batches of French fries cooked in the traditional way. These times (in minutes) are

15 17 14 15 16 13

Six batches of French fries of the same weight were cooked using the new microwave cooker. These cooking times (in minutes) are

11 14 12 10 11 15
Test the claim that the microwave process takes less time. Use \( \alpha = 0.05 \).

Put the data for traditional cooking in list \( L_1 \) and the data for the new method in List \( L_2 \). Since we have small samples, we want to use the Student’s \( t \) distribution. Select option 4:2-SampTTest. Select Data for Inpt, use the alternate hypothesis \( \mu_1 > \mu_2 \), select Yes for Pooled:

The output is on two screens.

Since the \( P \) value is 0.008, which is less than 0.05, we reject \( H_0 \) and conclude that the new method cooks food faster.

**TESTING A DIFFERENCE OF PROPORTIONS (SECTION 9.5 OF UNDERSTANDABLE STATISTICS)**

To conduct a hypothesis test for a difference of proportions, select option 6:2-PropZTest and enter appropriate values.
LAB ACTIVITIES FOR TESTING DIFFERENCE OF MEANS (INDEPENDENT SAMPLES) OR PROPORTIONS

1. Calm Cough Medicine is testing a new ingredient to see if its addition will lengthen the effective cough relief time of a single dose. A random sample of 15 doses of the standard medicine was tested, and the effective relief times were (in minutes):

   42  35  40  32  30  26  51  39  33  28
   37  22  36  33  41

A random sample of 20 doses was tested when the new ingredient was added. The effective relief times were (in minutes):

   43  51  35  49  32  29  42  38  45  74
   31  31  46  36  33  45  30  32  41  25

Assume that the standard deviations of the relief times are equal for the two populations. Test the claim that the effective relief time is longer when the new ingredient is added. Use $\alpha = 0.01$.

2. Publisher’s Survey did a study to see if the proportion of men who read mysteries is different from the proportion of women who read them. A random sample of 402 women showed that 112 read mysteries regularly (at least six books per year). A random sample of 365 men showed that 92 read mysteries regularly. Is the proportion of mystery readers different between men and women? Use a 1% level of significance.

   (a) Find the $P$ value of the test conclusion.
   
   (b) Test the hypothesis that the proportion of women who read mysteries is greater than the proportion of men. Use a 1% level of significance. Is the $P$ value for a right-tailed test half that of a two-tailed test? If you know the $P$ value for a two-tailed test, can you draw conclusions for a one-tailed test?