Chapter 1

Functions

In this chapter you will learn how to do the following:
1. Plot a function
2. Plot multiple functions simultaneously
3. Change the dimensions of the viewing rectangle
4. Determine intercepts graphically and numerically
5. Determine points of intersection of two graphs
6. Model data using linear regression
7. Model data using power regression

Plot a function

Keystrokes:

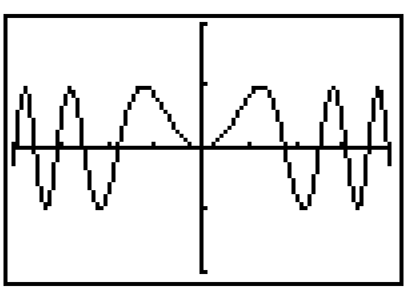
1. Press [Y =] .
2. Type the function to be plotted using the [X.T.ø.n] key for the variable.
3. Press [GRAPH] .

Example:

Plot $y = \sin(x^2)$.

Solution:

[Y =] [SIN] [(] [X.T.ø.n] [^] [2] [)] [GRAPH]



Plot multiple functions simultaneously

Keystrokes:

1. Press [Y =] .
2. Type the function to be plotted using the [X.T.Ø.n] key for the variable.
3. Press [ENTER] .
4. Repeat steps (2) and (3) for subsequent functions.
5. Press [GRAPH] .

Example:

Plot $y = 2x + 3$, $y = 3x + 3$, $y = -2x + 3$ and $y = -x + 3$ simultaneously. What is the effect of the coefficient of x on the graph?

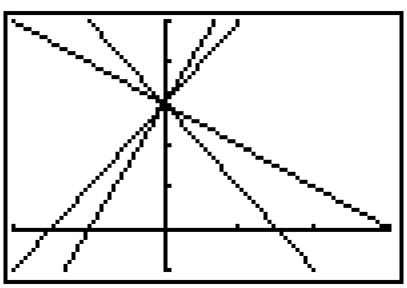
Solution:

[Y =] [2] [X.T.Ø.n] [+] [3] [ENTER]

[3] [X.T.Ø.n] [+] [3] [ENTER]

[(—)] [2] [X.T.Ø.n] [+] [3] [ENTER]

[(—)] [X.T.Ø.n] [+] [3] [GRAPH]



The calculator will graph the lines in the order they were entered. Once the graphs are drawn you can use the [TRACE] key and horizontal arrow keys to move your cursor along a graph. To move between graphs, use the vertical arrow keys. The equation of the graph the cursor is on is shown in the upper left-hand corner of these calculator display.

Lines with positive coefficients slope upward and lines with negative coefficients slope downward. The greater the absolute value of the coefficient of x the steeper the line.

Change the dimensions of the viewing rectangle

The default dimensions of the viewing rectangle are *Xmin* = -10, *Xmax* = 10, *Ymin* = -10, and *Ymax* =10. Frequently a graph may fall outside of the default viewing rectangle and, unless the viewing dimensions are changed, you will be unable to see it. The ZOOM key brings up a menu that allows you to zoom in, zoom out, visually specify the viewing rectangle (zoom box), and perform many other zoom actions. Additionally, you can manually specify the viewing rectangle by following the steps below.

Keystrokes:

1. Press the WINDOW key.

```
WINDOW
 Xmin=-10
 Xmax=10
 Xscl=1
 Ymin=-10
 Ymax=10
 Yscl=1
 Xres=1
```

2. Enter the desired dimensions, using the vertical arrow keys to move between the various data inputs. The meaning of each of the data inputs is detailed below:

Xmin: x-value of left side of viewing rectangle
Xmax: x-value right side of viewing rectangle
Xscl: distance between tick marks on the x-axis
Ymin: y-value of bottom of viewing rectangle
Ymax: y-value of top of viewing rectangle
Yscl: distance between tick marks on the y-axis
Xres: sets the pixel resolution of the graph and is between 1 (high detail) and 8 (low detail)

3. Press GRAPH to re-graph the function using the new viewing rectangle dimensions.

Example:

Water falling from a waterfall x feet high will hit the ground with a speed of $\frac{60}{11}x^{0.5}$ miles per hour (neglecting air resistance). The highest waterfall in the world (Angel Falls) is 3281 feet high. What dimensions of the viewing rectangle will allow the speeds of all the waterfalls in the world to be visually determined?

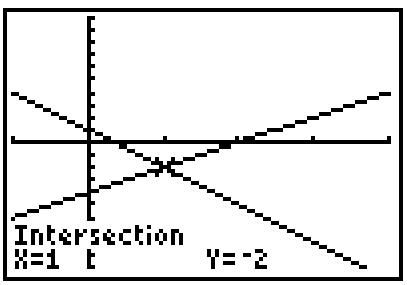
Solution:

Xmin = 0 (Waterfalls are no less than zero feet tall.)
Xmax = 3281 (Waterfalls on earth are no more than 3281 feet tall.)
Ymin = 0 (Falling water travels at a minimum of 0 miles per hour.)
Ymax = 313 (The function is increasing so the largest y-value (312.4 m.p.h.) occurs at the largest x-value (3281 ft).)

Determine points of intersection of two graphs

Keystrokes:

1. Plot the graphs.
2. Press [2nd] the key.
3. Press the [TRACE] key. (Steps (2) and (3) activate the CALC menu.)
4. Press [5] to activate the *intersect* function.
5. The calculator displays the graphs and asks, "First curve?". Use the [TRACE] and vertical arrow keys to select a curve. (The calculator defaults to the first curve defined.)
6. Press [ENTER] .
7. The calculator asks, "Second curve?". Use the [TRACE] and vertical arrow keys to select the second curve. (The calculator defaults to the second curve defined.)
8. Press [ENTER] .
9. The calculator asks, "Guess?". Type in an estimate of the point of intersection.
10. Press [ENTER] . The calculator returns the coordinates of the point of intersection.

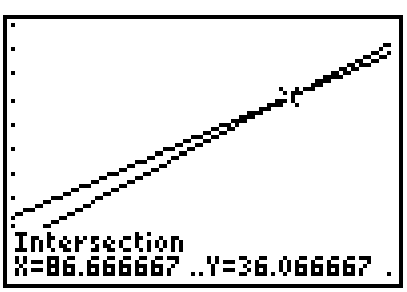


Example:

The median age at first marriage for men is $y_1 = 24.8 + 0.13x$, and for women it is $y_2 = 22.2 + 0.16x$, where x is the number of years since 1980. According to this model, in what year, will the median age at first marriage for men and women be the same?

Solution:

Plotting the functions, using the default curve values and picking 88 as our guess gives the graph below. So in 2066 (86.67 years after 1980), the median marriage age will be 36 years.



Determine intercepts graphically and numerically

X- and *y*-intercepts of a function may be determined graphically or numerically. Which method you use is a matter of personal preference.

Keystrokes:

Graphical:

1. Plot the function using a viewing rectangle that allows you to see all intercepts.
2. Press the [TRACE] key.
3. Use the horizontal arrow keys to move the cursor to the *x*-intercept. Watch the *y*-value as you near the intercept. When the *y*-value becomes zero, the *x*-value is an *x*-intercept of the function.
4. Press [0] [ENTER] to set the *x*-value to zero. The *y*-value displayed is the *y*-intercept.

Alternative Graphical:

1. Plot the function using a viewing rectangle that allows you to see all intercepts.
2. Press [2nd] the key.
3. Press [TRACE] [2] the key. (Steps (2) and (3) activate the CALC menu.)
4. Press to activate the *zero* function.
5. The calculator displays the graphs and asks, "Left bound?". Use the horizontal arrow keys to move the cursor to the left of the *x*-intercept.
6. Press [ENTER] .
7. The calculator asks, "Right bound?". Use the horizontal arrow keys to move the cursor to the right of the *x*-intercept.
8. Press [ENTER] .
9. The calculator asks, "Guess?". Type in an estimate of the *x*-intercept.
10. Press [ENTER] and the calculator returns the value of the *x*-intercept closest to your guess. This process must be repeated for each x-intercept.

Numerical:

1. Plot the function using a viewing rectangle that allows you to see all intercepts.
2. Press [2nd] [GRAPH] to activate the table menu.
3. Use the vertical arrows to scroll through the table. *X*-intercepts occur where $y_1 = 0$ and the *y*-intercept occurs where $x = 0$. (Changing the table setup will be covered in Chapter 2.)

Example:

A company that installs car alarm systems finds that if it installs x systems per week, then its costs will be $C(x) = 210x + 72{,}000$ and its revenue will be $R(x) = -3x^2 + 1230x$ (both in dollars).

a. What are the fixed costs of the company? (Fixed costs typically include cost of building rental, equipment, utilities, etc.)

b. Profit, $P(x)$, is the difference between revenue and cost. Will the profit of the company ever be zero dollars? (The point where profit is zero is called the break-even point.)

Solution:

a. Fixed costs are costs incurred regardless of production level and are found by computing the cost of installing 0 systems. Plotting $C(x) = 210x + 72{,}000$ with viewing rectangle [0,10] [70000, 75000] gives the graph (Figure A). Entering zero returns the y-intercept or fixed cost (Figure B).

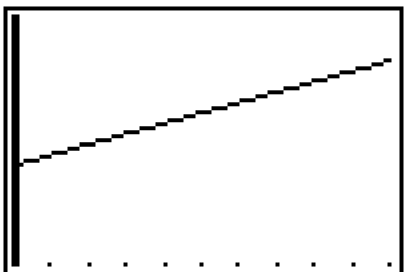


Figure A

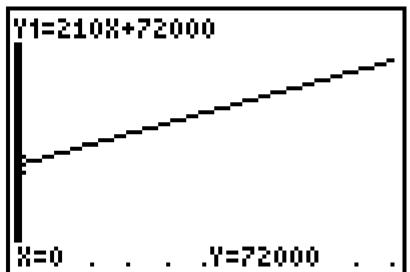


Figure B

b. The profit function $P(x) = (-3x^2 + 1230x) - (210x + 72{,}000)$. Plotting P(x) with viewing rectangle [0, 250] [-1000, 15000] gives the graph below (Figure C).

Using the TRACE key and the horizontal around keys we can determine that the x-intercepts are between 98 and 101 and 239 and 242. Generating a table (Table 1) we can see that the x-intercepts occur at $x = 100, 240$. ($y_1 = 0$ at both places.) So as long as the company installs between 100 and 240 units weekly, it is profitable. Moving through the table values (Table 2) we see that the maximum profit occurs when 170 units are installed.

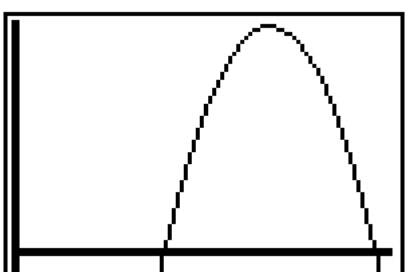


Figure C

X	Y1
65	-18375
100	0
135	11025
170	14700
205	11025
240	0
275	-18375

X=65

Table 1

X	Y1
155	14025
160	14400
165	14625
170	14700
175	14625
180	14400
185	14025

X=155

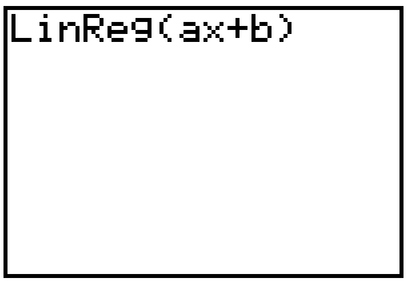
Table 2

Model data using linear regression

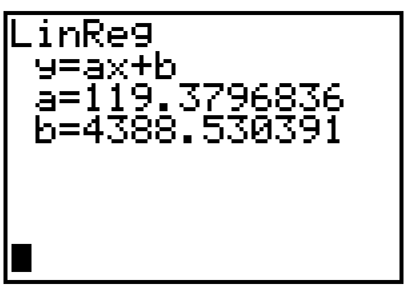
When analyzing real life data is it often useful to develop a mathematical model to represent the data. Regression is a method of finding the equation of a curve that best fits the data. When the data plot appears to have a near constant change between consecutive values, linear regression is used to find the equation of the line that best fits the data.

Keystrokes:

1. Enter the data lists, L_1 and L_2, using the Stat List Editor. (See your manual for details.)
2. Press [STAT] to activate the Statistics Menu.
3. Press the right horizontal arrow to move to the Calculate Menu.
4. Press [4] to activate the LinReg (ax + b) function.



5. Press [2nd] [1] to insert the L_1 list label.
6. Press the comma key.
7. Press to insert the L_2 list label. (To get L_3 - L_6, select 3 - 6 respectively.)
8. Press [2nd] [ENTER] [2] . The equation of the line that best fits the data entered is



displayed.

Example:

The tables below show human life expectancy by birth year. Use linear regression to develop a life expectancy model. Predict life expectancy for females and males born in 2000.

Birth Year	Birth Year (Years Since 1960)	Female Life Expectancy
1960	0	73.1
1970	10	74.7
1980	20	77.5
1990	30	78.8

Birth Year	Birth Year (Years Since 1960)	Male Life Expectancy
1960	0	66.6
1970	10	67.1
1980	20	70.0
1990	30	71.8

Solution:

Entering {0, 10, 20, 30} in list L_1 and {73.1, 74.7, 77.5, 78.8} in list L_2 and following the procedures above results in the following:

```
LinReg
 y=ax+b
 a=.199
 b=73.04
```

So the line $y = 0.199x + 73.04$ is the linear model that best fits the data. According to the model, life expectancy for females born in 2000 is 81.0 years.

Leaving the L_1 unchanged, entering {66.6, 67.1, 70.0, 71.8} in list L_3 and substituting in L_3 in for L_2 is the procedures given earlier results in the following:

```
LinReg
 y=ax+b
 a=.185
 b=66.1
```

So the line $y = 0.185x + 66.1$ is the linear model that best fits the data. According to the model, life expectancy for males born in 2000 is 73.5 years.

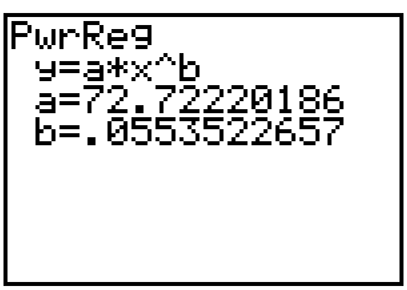
Model data using power regression

When the data plot appears to be nonlinear and to be increasing everywhere or decreasing everywhere, power regression is often used to find the equation of the function of the form $y = ax^n$ that best fits the data. Other forms of regression include logarithmic, exponential, logistic, sinusoidal, quadratic, cubic, and quartic. It is useful to be familiar with the shape of each of these curve types in order to determine which type of regression will give you the most meaningful model.

Keystrokes:

1. Enter the data lists, L_1 and L_2, using the Stat List Editor. (See your manual for details.)
2. Press STAT to activate the Statistics Menu.
3. Press the right horizontal arrow to move to the Calculate Menu.
4. Use the vertical arrows to scroll down to option A and press ENTER to activate the PwrReg function.

5. Press 2nd 1 to insert the L_1 list label.
6. Press the comma key.
7. Press 2nd 2 to insert the L_2 list label. (To get L_3 - L_6, select 3 - 6 respectively.)
8. Press ENTER . The equation of the power function that best fits the data is displayed.



Example:

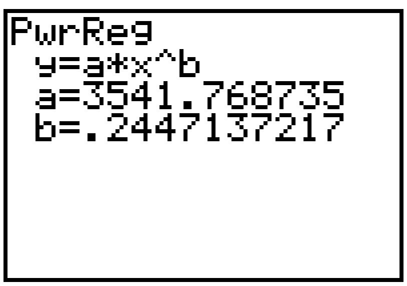
A manufacturer of supercomputers finds that the profit on the sale of the first, the tenth, the twentieth, the thirtieth and the fortieth computer in a given month is as follows:

Supercomputer Number	**Profit ($)**
1	3500
10	6400
20	7400
30	8200
40	8500

Develop a power function model of profit per computer and use the model to predict the profit on the 50^{th} supercomputer sold in a given month.

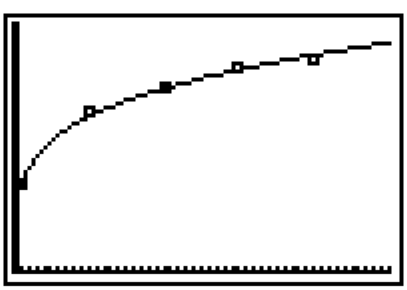
Solution:

Entering {1, 10, 20, 30, 40} in list L_1 and {3500, 6400, 7400, 8200, 8500} in list L_2 and following the procedures given earlier results in the following:



So the curve $y = 3541.77x^{0.245}$ is the power function model that best fits the data. According to the model, the sale of the fiftieth computer will create a $8714.10 profit.

It is often meaningful to see how close your model fits the data. By plotting the model equation and the data simultaneously, you can observe the accuracy of the model. To do this, plot the power function using the procedures previously covered. Then activate the STAT PLOT menu and specify the type of statistics plot you desire. (Procedures are covered in your *TI-83 Graphing Calculator Guidebook*.) In this example, our model very closely fit the data.



Probability

In the following pages you will learn how to do the following:

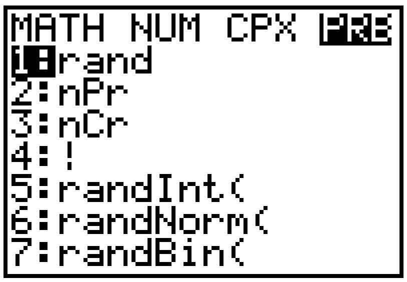
1. Compute factorials
2. Compute permutations
3. Compute combinations
4. Determine a binomial distribution

Compute factorials

Factorials are used to compute the number of ways a set of items may be ordered. n! read "n factorial" means n · (n-1) . . . 3 · 2 · 1 . The largest factorial the TI-83 can compute is 69!. Larger values result in an overflow error.

Keystrokes:

1. Enter the number of items you want to order on the home screen.
2. Press MATH and scroll to PRB to activate the Probability Menu.



3. Scroll to item 4 and press ENTER .

Example:

How many different four-digit PIN numbers can be made using 2, 4, 6, and 8?

Solution:

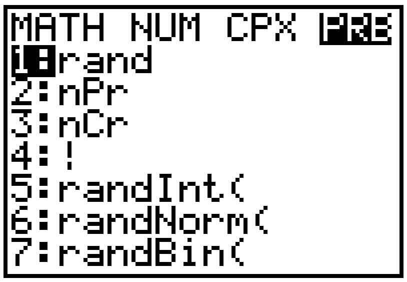
4! = 4 · 3 · 2 · 1 = 24 so there are 24 four-digit PIN numbers that can be made.

Compute permutations

The number of ways that n distinct objects taken r at a time may be ordered is called a permutation and is denoted by ${}_nP_r$.

Keystrokes:

1. Enter the number of items you have to pick from on the home screen.
2. Press MATH and scroll to PRB to activate the Probability Menu.



3. Scroll to item 2 and press ENTER .
4. Enter the number of items you want to choose and press ENTER .

Example:

How many four-digit PIN numbers with distinct digits can be made from the numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9?

Solution:

There are ten numbers to chose from and we want to pick four so we need to compute ${}_{10}P_4$.

$$ {}_{10}P_4 = \frac{10!}{(10-4)!} = \frac{10!}{6!} = 10 \cdot 9 \cdot 8 \cdot 7 = 5040 $$

So there are 5040 different four-digit PIN numbers with distinct digits.

In this example, the PIN numbers using a different ordering of the same digits are counted as distinct. That is, even though the following PIN numbers use the same four digits, they are each counted since their ordering is distinct.

1234, 1243, 1324, 1342, 1423, 1432
2134, 2143, 2314, 2341, 2413, 2431
3124, 3142, 3214, 3241, 3412, 3421,
4123, 4132, 4213, 4231, 4312, 4321

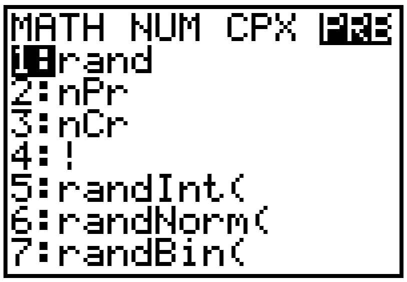
In the next section, we will discover how many four-digit PIN numbers can be made such that no two PIN numbers have the same digits.

Computing combinations

When order does not matter, combinations are used instead of permutations. The number of sets of n distinct objects taken r at a time is called a combination and is denoted by ${}_nC_r$.

Keystrokes:

1. Enter the number of items you have to pick from on the home screen.
2. Press MATH and scroll to PRB to activate the Probability Menu.



3. Scroll to item 3 and press ENTER .
4. Enter the number of items you want to choose and press ENTER .

Example:

How many four-digit PIN numbers can be made without repetition and without regard to order from the numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9?

Solution:

There are ten numbers to chose from and we want to pick four so we need to compute ${}_{10}C_4$.

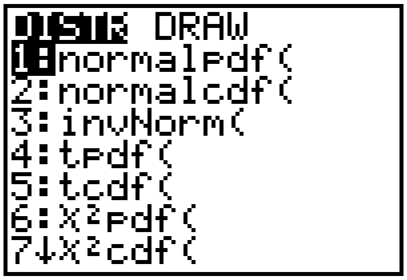
$${}_{10}C_4 = \frac{10!}{(10-4)!\,4!} = \frac{10!}{6!\,4!} = \frac{10\cdot 9\cdot 8\cdot 7}{4\cdot 3\cdot 2\cdot 1} = \frac{5040}{24} = 210$$

So there are 210 different four-digit PIN numbers with distinct digits. The number of combinations is significantly smaller than the number of permutations because PIN numbers containing the same digits in a different order are not counted. That is, all twenty-four PIN numbers shown in the previous section are considered to be the same PIN when using combinations.

Determine a binomial distribution

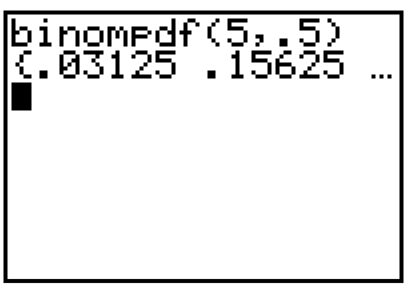
Keystrokes:

1. Press [2nd] [VARS] to activate the Distributions Menu.

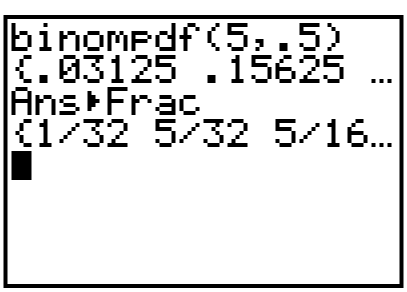


2. Scroll to 0 and press [ENTER].

3. Enter the number of Bernoulli trials.
4. Press [,]
5. Enter the probability of success for each trial and press [ENTER].



6. To change the decimal approximations to fractions, press [MATH], scroll to 1, and press [ENTER].

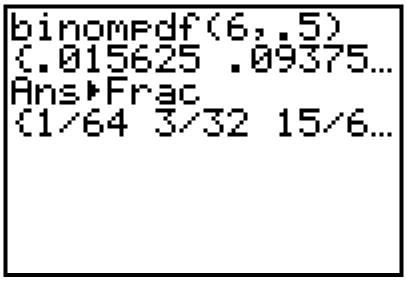


Example:

A family has six children. What the probability that they are all boys? What is the probability that there are three boys and three girls? What is the probability that there are less boys than girls?

Solution:

Let X be the number of boys and assume the probability a child is a boy is 0.50.



The probability distribution is $\left\{\frac{1}{64},\frac{3}{32},\frac{15}{64},\frac{5}{16},\frac{15}{64},\frac{3}{32},\frac{1}{64}\right\}$. The probability that they are all boys is $\frac{1}{64}$ or 0.015625. So there is roughly a 1.5% chance that they are all boys.

The same distribution may be used to answer the second and third question. The probability that three of the children are boys is $\frac{5}{16}$ or 0.3125. So there is roughly a 31% chance that there are three boys and three girls in the family.

If there are 0, 1, or 2 boys there are less boys than girls. By adding their respective probabilities ($\frac{1}{64},\frac{3}{32},\frac{15}{64}$) we determine that the probability that there are less boys than girls is $\frac{11}{32}$ or 0.34375. So there is roughly a 34% chance that the girls outnumber the boys.