Chapter 2 Sets

Section 2.1 Sets: A Problem-Solving Tool

Exercise 2.1

STUDY TIPS Most mathematics textbooks start with a discussion about sets, so this section can help you later. The word "set" is an undefined term but at the same time it is listed in the Guinness Book of Records as the "word with the most meanings" (194 in all!). In this section you learn:

(1) How to determine if a set is well defined (Hint: Stay away from subjective words like "good", "bad" and "beautiful")
(2) How to describe a set (There are three ways: writing the set in words, listing the elements separated by commas between braces and using set builder notation.) (See the Table before the C part of Section 1.1) Special caution: the empty set is denoted by $\varnothing$ or $\{\ \}$, but not $\{\varnothing\}$.
(3) Determine if two sets are equal. (When two sets are equal, they have exactly the same elements, but these elements do not have to be written in the same order.)
(4) Find the subsets of a set.

The concept of subset has its own definition, as well as an alternative one used to convince you that the empty set is a subset of every set A (there is no element in the empty set that is not in A.) To find the subsets of a set start by finding the subsets with no elements, then the subsets with one element and so on. How do you know when you are finished? If you have 1 element, you need $2^1 = 2$ subsets, if you have 2 elements you need $2^2 = 4$ subsets and so on. On to the solution of the Exercise 2.1.

1. People do not agree on the meaning of "grouchy", so this description does not define a set.

3. A set

5. A set

7. People do not agree on what is "good", so this description does not define a set.

9. (a) *Incorrect.* The letter D is not an element of A.
   (b) *Correct.* Desi is an element of A.
   (c) *Incorrect.* Jane is an element of A, not the other way around.
   (d) *Correct.* The letter D is not an element of A.
   (e) *Incorrect.* Jane is an element of A

11. x is an element of the set X. Fill the blank with $\in$.

13. A is not an element of the set X. Fill the blank with $\notin$.

15. The set consisting of the first and last letters of the English alphabet.

17. The set consisting of the names of the first biblical man and woman.

19. The set of counting numbers from l to 7. Note that the numbers do not have to be in any specific order.

21. The set of odd counting numbers from 1 to 51.

23. The set of counting numbers starting with 1 and then adding 3 successively until the number 25 is obtained.

25. {Dioxin, Xylene} is the set that was found in everybody's tissue.

27. {1, 2, 3, 4, 5, 6, 7}

29. {0, 1, 2, 3, 4, 5, 6, 7}

31. The word "between" is to be taken literally. The 3 and the 8 are not elements of this set. The answer is {4, 5, 6, 7}.

33. There are no counting numbers between 6 and 7, so this set is empty. The answer may be written as $\varnothing$ or as { }.

35. {4, 5, 6, . . .}

37. {8, 9, 10}

39. {1, 2, 3, 5, 8, 10}

41. {2}

43. {1, 2}

45. The set A = {4, 8, 12, 16, ...} and the set B = {2, 4, 6, 8, ...}, so these sets are not equal.

47. Both A and B are empty, so these sets are equal.

49. (a) Every element of A is an element of B, and every element of B is an element of A. Thus, A = B.
    (b) $\neq$ is correct because 0 is an element of C but not of A.
    (c) $\neq$ is correct because 0 is an element of C but not of B.

51. $\varnothing$, {a}, {b}, {a, b} The first three of these are proper subsets of the given set.

53. $\varnothing$, {l}, {2}, {3}, {4}, {1, 2}, {1, 3}, {1, 4}, {2, 3}, {2, 4}, {3, 4}, {l, 2, 3}, {1, 2, 4}, {1, 3, 4}, {2, 3, 4}, {l, 2, 3, 4} All but {1, 2, 3, 4} are proper subsets of the given set.

55. $\varnothing$, {l}, {2}, {1, 2} The first three of these are proper subsets of the given set.

57. Since there are 4 elements in this set, there are 2^4 or 16 subsets.

59. There are 10 elements in A, so A has 2^{10} or 1024 subsets.

61. Note that $32 = 2^5$, so there are 5 elements.

63. Since $64 = 2^6$, there are 6 elements.

65. Yes. Since $\varnothing$ has no elements, there is no element of $\varnothing$ that is not in $\varnothing$. Furthermore, every set is a subset of itself.

67. $B \subseteq A$ because every counting number that is divisible by 4 is also divisible by 2. (A is not a subset of B because a number can be divisible by 2 and not by 4. For instance, 6 is divisible by 2, but not by 4.)

69. (a) There are 5 toppings, so you have 5 choices.
    (b) There are 10 subsets with 2 elements in each. So you have 10 choices. (Try writing these out.)
    (c) You can choose which two toppings you don't want. So the answer is 10 as in (b).

71. Since $256 = 2^8$, you would need 8 different condiments.

73. Answers will vary.

75. Answers will vary.

77. (a) If $g \in S$, then Gepetto shaves himself, which contradicts the statement that Gepetto shaves all those men and only those men of the village who do not shave themselves. Hence, $g \notin S$.
    (b) If $g \in D$, then Gepetto does not shave himself, and so by the same statement, he does shave himself. Thus, there is again a contradiction and $g \notin D$.

79. The word "*non-self-descriptive*" cannot be classified in either way without having a contradiction. If it is an element of S, then it is a self-descriptive word, which contradicts the definition, "non-self-descriptive is a non-self-descriptive word". On the other hand if non-self-descriptive is a non-self-descriptive word, then it is an element of S, which is again a contradiction.

Section 2.2 Set Operations

Exercise 2.2

STUDY TIPS You have to learn how to do *three* operations with sets: (1) form the intersection of sets A and B (make sure all the elements are in A and also in B); (2) form the union of two sets (the elements in the union can be in A or in B or in both); (3) find the difference of sets A and B (the elements have to be in A but not in B.). In addition, you have to learn how to find the complement of a set. Hint: To find the complement A' of a set A, you have to know the universal set U. The complement consists of all the elements that are in U and not in A, that is, the difference of U and A, U – A

1. (a) $A \cap B$, the set of all elements in both A and B, is $\{1, 3, 4\}$.
   (b) $A \cap C$, the set of all elements in both A and C, is $\{1\}$.
   (c) $B \cap C$, the set of all elements in both B and C, is $\{1, 6\}$.

3. (a) $A \cap (B \cup C)$, the set of all elements in both A and $B \cup C$, is $\{1, 3, 4\}$.
   (b) $A \cup (B \cap C)$, the set of all elements in A or in $B \cap C$, is $\{1, 2, 3, 4, 5, 6\}$.

5. $A \cup (B \cup C)$, the set of all elements in A or in $B \cup C$, is $\{1, 2, 3, 4, 5, 6, 7\}$.

7. $A \cap (B \cap C)$, the set of all elements in A and in $B \cap C$, is $\{1\}$.

9. (a) $A \cap B$, the set of all elements in both A and B, is $\{c\}$.
   (b) $A \cap C$, the set of all elements in both A and C, is $\varnothing$. Note: the set $\{a, b\}$ is an element of A, but a and b, separately, are not elements of A.

11. (a) Correct. The set $\{b\}$ is a subset of the set $A \cap B$.
    (b) Incorrect. The set {b} is not an element of the set $A \cap B$.

13. (a) Correct. The set {a, b, c} is a subset of the set $A \cup B$.
    (b) Correct. The set {a, b, c} is an element of the set A, so it is an element of the set $A \cup B$.

15. (a) A', the set of elements in U but not in A, is $\{b, d, f\}$.
    (b) B', the set of elements in U but not in B, is $\{a, c\}$.

17. (a) $(A \cup B)'$, the set of elements in U but not in $A \cup B$, is $\varnothing$. Note that $A \cup B$ includes all the elements in U.
    (b) $A' \cup B'$, the set of elements in A' or in B' is $\{a, b, c, d, f\}$.

19. (a) $(A \cap B) \cup C'$, the set of elements in $A \cap B$ or in C', is $\{c, e\}$. Note that $A \cap B$ is $\{e\}$.
    (b) $C \cup (A \cap B)'$, the set of elements in C or not in $A \cap B$ is $\{a, b, c, d, f\}$.

21. (a) $A' \cap B$, the set of elements not in A but in B, is $\{b, d, f\}$.
    (b) $A \cap B'$, the set of elements in A but not in B, is $\{a, c\}$.

23. (a) The elements in C' are c and e and those in $(A \cap B)'$ are a, b, c, d, and f. Thus, $C' \cup (A \cap B)' = \{a, b, c, d, e, f\}$.
    (b) $C' = \{c, e\}$, $A \cup B = U$, so $(A \cup B)' = \varnothing$. Thus, $C' \cup (A \cup B)' = \{c, e\}$.

25. (a) This is the set U with the elements in A taken out, that is, $\{b, d, f\}$.
    (b) This is the set U with the elements in B taken out, that is, $\{a, c\}$.

27. (a) This is the set of elements not in B. The answer is $\{2, 3\}$.

(b) This is the set U with the elements in B taken out. The answer is the same as in Part (a), {2, 3}.

29. $\varnothing'$, the set of all elements in U that are not in the empty set, is U.

31. The set of all elements that are in both A and the empty set is $\varnothing$.

33. This is the set of all elements that are in both A and U, that is, A.

35. This is the set of elements that are both in A and not in A, that is, $\varnothing$.

37. This is a double negative; the elements that are not in "not A" are, of course, in A. Thus, the answer is A.

39. To include A, you must have the elements 1, 2, 3. Then, to include B, you must have the element 4, and to include C, you must have the element 5. Thus, the smallest set that can be used for U is {1, 2, 3, 4, 5}.

41. {Beauty, Consideration, Kindliness, Friendliness, Helpfulness, Loyalty}

43. This is the set of traits that are in both M_W and M_m, so the answer is {Intelligence, Cheerfulness, Congeniality}.

45. {Intelligence, Cheerfulness}

47. {Is aware of others, Follows up on action}

49. {Follows up on action}

51. (a) M', the set of employees who are not male, is F.
(b) F', the set of employees who are not female, is M.

53. (a) Male employees who work in the data processing department.
(b) Female employees who are under 21.

55. These employees would have to be in both sets D and S. Thus, the answer is $D \cap S$.

57. These employees would have to be in sets M and D, both. Thus, the answer is $M \cap D$.

59. Male employees or employees who are 21 or over.

61. (a) $F \cap S = \{04, 08\}$ is the set of full-time employees who do shop work.
(b) $P \cap (O \cup I) = \{02, 05, 07\}$ is the set of part-time employees who do outdoor field work or indoor office work.

63. {Jimi Hendrix, Led Zeppelin, The Who}

65. 3

67. {4, 7}

69. {3}

71. {8}

73. {1, 2, 3, 6, 7, 8, 9}

75. {1, 2, 3, 4, 5, 6, 7, 8,, 9, 10}

77. A and B have no elements in common.

79. All the elements of A are elements of B, and all the elements of B are elements of A.

81. (a) This is the set of characteristics that are in both columns of the table: {Long tongue, Skin-covered horns, Native to Africa}
(b) The same answer as in part (a).
(c) This is the set of characteristics that occur in either column of the table: {Tall, Short, Long neck, Short Neck, Long tongue, Skin-covered horns, Native to Africa}
(d) G' = {Short, Short neck}
(e) O' = {Tall, Long neck}

83. This is the second item in the 35 and older row: 685,000.

85. This is the set of 12 - 17 year old females: $F \cap A$.

87. There are no persons that are both male and female. This set is empty.

89. $41,339

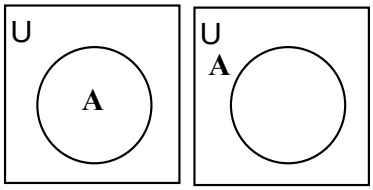
91. $28,403

93. Average earnings of males with a High School degree; $32,521

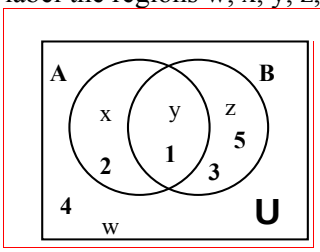
Section 2.3 Venn Diagrams

Exercise 2.3

STUDY TIPS In this section you will learn to make pictures associated with the operations of *intersections* and *unions*. These pictures can be used to verify the equality of certain sets. Think of sets *A* and *B* as sets of points inside two circles. To make a picture (called a **Venn Diagram)** of an intersection, draw vertical lines inside of *A* and horizontal lines inside of *B*. Where the lines **intersect** is the intersection. If there is no intersection, the sets are called **disjoint.** To form the **union**, draw vertical lines inside of *A* and continue drawing vertical lines inside *B*, the result is the **union** of *A* and *B*. What about the complement of *A*? As before, you need the universal set *U*, represented by a rectangle outside the circles *A* and *B*. The complement of *A* consists of all points in *U* **not** in *A*.
A word of warning: Sometimes students get confused by the placing of the name of the set on the diagram. Both diagrams show a set *A* inside a universal set *U*. It does not matter if the set *A* is labeled outside or inside the circle. In both cases, we have a set *A* inside a universal set *U*.

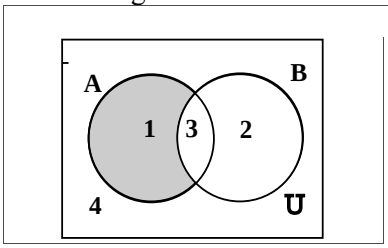


l. **Step I**: Draw a diagram with two circles and label the regions w, x, y, z, as shown.

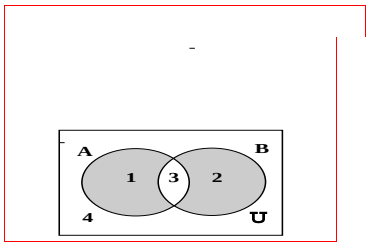


Step 2: Select the element that is in both *A* and *B*, the number l. Write l in region y.
Step 3: Select the element that is in *A* but not in *B*, the number 2. Write 2 in region x.
Step 4: Select the elements that are in *B* but not in *A*, the numbers 3 and 5. Write 3 and 5 in region z.
Step 5: Select the element that is not in *A* or *B*, the number 4, and write 4 in region w. This completes the diagram.

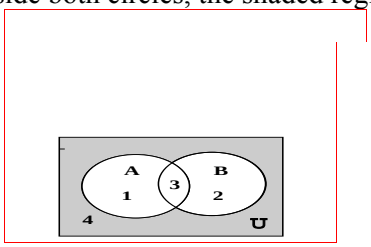
3. Draw a Venn diagram labeled as in the figure. Then look at the regions corresponding to the various sets. *A*: regions 1, 3; *B′* (regions outside of *B*) regions 1, 4; $A \cap B'$ (regions in both *A* and *B′*): region l. Shade region 1 for the desired diagram.



5. Draw a diagram as in Problem 3. Then find the regions corresponding to the various sets. $A \cup B$ (regions in *A* or *B*): regions 1, 2, 3; $A \cap B$ (regions in both *A* and *B*): region 3; $(A \cup B) - (A \cap B)$: Take the region in $A \cap B$ away from the regions in $A \cup B$, leaving regions l, 2. Shade regions l, 2 for the desired diagram.



7. Draw a diagram as in Problem 3. The region corresponding to A' is the entire region outside of circle A. The region corresponding to B' is the entire region outside of circle B. Thus, the region corresponding to $A' \cap B'$ is the region outside both circles, the shaded region.



9. A: regions 1, 3, 5, 7;