

Chapter 1 Problem Solving

Section 1.1 Inductive Reasoning

Exercise 1.1

STUDY TIPS Word problems, sometimes called "story problems" or "statement problems" are at the heart of math anxiety. So says Sheila Tobias, author of *Overcoming Math Anxiety*. Her solution? Figure out some way to help people conquer their fear and disability in solving word problems. Let us do this together and do it now!

We start this book with a discussion of how to solve problems. This section is very important because the procedures and techniques discussed here will be used in the rest of the book. Why should you do this? Read the Getting Started again to remind you. The Exercises at the end of this section are designed to train and help you solve problems. If you have access to a computer, check the Bello Website at

http://college.hmco.com/mathematics/bello/topics/8e/students/web_links/index.html

This site offers links where the topics in the text are discussed, amplified and expanded. This very section contains **six** different links doing just that.

A final word: Before you start the problem set, make sure you are familiar with the **RSTUV** procedure and the meaning of *inductive reasoning*. Ready? Here are the answers to the Problems in Exercise 1.1.

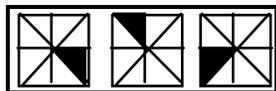
- Step 1.** Understand the problem.
Step 2. Devise a plan.
Step 3. Carry out the plan.
Step 4. Look Back.
- What does the problem ask for? What is the unknown? After all, if you don't know the question, how can you find the answer?
- Go to the "Occasional Plan" row and look at the last column. It says that the transaction fee is 20 cents. Since you

made 15 transactions, the answer is 15×20 cents or \$3.00.

- If you are planning to make 15 transactions per month, the Light Use plan will cost 15×5 cents plus the \$1.95 monthly fee or \$2.70 while the Occasional Plan is \$3.00, thus the Light Use Plan is less expensive.
- To answer this question find the cost of each plan when you make 20, 40 and 60 calls. For 60 calls the cost for the Light Use Plan is
$$\left(\begin{array}{c} \text{Monthly} \\ \text{fee} \end{array} \right) + \left(\begin{array}{c} \text{Number of} \\ \text{Paid Calls} \end{array} \right) \cdot \left(\begin{array}{c} \text{Cost per} \\ \text{call} \end{array} \right)$$
$$\$1.95 + (60 - 10)0.15 = \$1.95 + \$7.50 = \$9.45$$
For 63 calls it will be 45 cents more or \$9.90. After 63 calls the Standard Use Plan (\$9.95 for 100 free calls) is less expensive.
- To get the 2nd term (2), you add 1 to the 1st term.
To get the 3rd term (4), you add 2 to the 2nd term.
To get the 4th term (7), you add 3 to the 3rd term.
To get the 5th term (11), you add 4 to the 4th term.
In general, the pattern is: Add n to the n th term
The 6th and 7th terms are $11 + 5 = 16$ and $16 + 6 = 22$.
- Note that the odd numbered terms are always 1's and the even numbered terms are multiples of 5. Thus, the 7th and 9th terms are 1's and the eighth term is the next multiple of 5 after 15, that is 20. Hence, the next three terms are 1, 20 and 1.

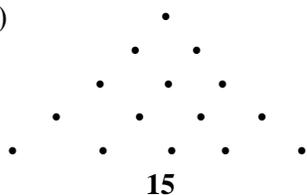
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15. Going clockwise, the shaded region is moved 1 place, 2 places, 3 places and so on. The next three moves will move the shaded region 4, 5 and 6 places. The answer is shown.



17. The numbers in the denominator are obtained by doubling. Thus, the next three terms are $\frac{1}{16}$, $\frac{1}{32}$ and $\frac{1}{64}$. Note that *each term is half the preceding term.*
19. The odd numbered terms are 1, 2, 3, 4, 5, ... and the even numbered terms are 5, 6, 7, 8, 9, ... The next three terms are 7, 4, 8 as shown. 1, 5, 2, 6, 3, $\boxed{7}$, $\boxed{4}$, $\boxed{8}$

21. (a)



- (b) The rows are constructed by adding one more dot than on the preceding row. The next triangular numbers after 10 are $10 + 5 = \boxed{15}$
 $15 + 6 = \boxed{21}$ and $21 + 7 = \boxed{28}$.
- (c) Following the pattern after the 7th triangular number which is 28, the 10th triangular number is:
 $28 + 8 + 9 + 10 = \boxed{55}$.

23. (a) $1+2+3+4+5+6+7+8 = \boxed{36}$

(b) 36

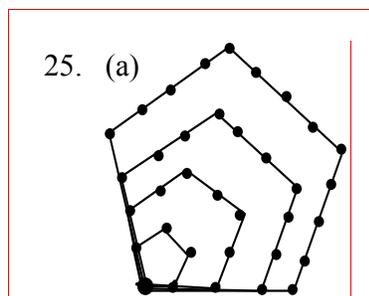
(c) $1 + 2 + 3 + \dots + 12 = \boxed{78}$

(d) 78

(e) $1 + 2 + 3 + \dots + (n - 1) + n$

$$= \frac{n(n+1)}{2}$$

(f) $\frac{100 \cdot 101}{2} = 50 \cdot 101 = 5050$



25. (a)

- (b) At each step, increase the length of the bottom and left lower side of the pentagon by one unit. The number of dots on each side is increased by one unit.
- (c) The 6th pentagonal number is 51.

27. Here is a summary of the information shown in the figure:

Sides 4 5 6 7

Diagonals 1 2 3 4

The number of diagonals is three less than the number of sides. Thus, $10 - 3 = 7$ diagonals can be drawn from one vertex of a decagon.

29. (a)

5	10	20	100
12	17	27	107
36	51	81	321
30	45	75	315
10	15	25	105
5	5	5	5

The final result is always 5.

(b) n $n + 7$
 $3(n + 7) = 3n + 21$
 $3n + 21 - 6 = 3n + 15$
 $\frac{3n + 15}{3} = n + 5$
 $n + 5 - n = 5$

31. (a)

5	10	20	100
10	15	25	105
40	60	100	420
20	30	50	210
10	10	10	10

The final result is always 10.

(b) n $n + 5$
 $4(n + 5) = 4n + 20$
 $\frac{4n + 20}{2} = 2n + 10$
 $2n + 10 - 2n = 10$

33. (a) It is always 4.
 (b) If you pick any number and follow the instructions you eventually get to a number less than or equal to 10. For any of these numbers the pattern leads to the number 4.
35. (a) $(1 + 2 + 3 + 4)^2 = 1^3 + 2^3 + 3^3 + 4^3$
 $(1 + 2 + 3 + 4 + 5)^2 = 1^3 + 2^3 + 3^3 + 4^3 + 5^3$
 $(1 + 2 + 3 + 4 + 5 + 6)^2 = 1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3$
 (b) The square of the sum of the first n counting numbers equals the sum of the cubes of these numbers.
37. The number of units of length of the pendulum is always the square of the number of seconds in the time of the swing.
39. (a) 12, 15, 18
 (b) We can see from the table that each unit increase in size corresponds to a $\frac{1}{3}$ of an inch increase in length. Thus, a 2 unit increase in size (from 6 to 8) corresponds to a $\frac{2}{3}$ in increase in length, from 9 to $9\frac{2}{3}$ in.
41. Answers may vary.
43. Answers may vary.
45. $1 + 1 + 2 + 3 + 5 = 12$. The sixth term is $3 + 5 = 8$, so the seventh term is $5 + 8 = 13$, which is *1 more than the sum of the first five terms*.
47. The fourteenth term is 377 (check this!), so the sum of the first twelve terms is one less or 376.

Section 1.2 Estimation: A Problem-Solving Tool

Exercise 1.2

STUDY TIPS

One of the most powerful tools in solving problems is estimation. Estimation saves time and helps us understand if the answer we are getting is a reasonable one. (Remember the fifth step in the **RSTUV** procedure? **Verify!**) As you can see in the Getting Started, it would be almost impossible to physically count all the people in a crowd, but estimation can do it for us. One of the most basic estimation techniques is the rule for rounding numbers; make sure you understand and know how to use the rule. As for real life examples, reading, know how to read your electric meter is a good way to avoid surprises at the end of the month. Of course, tax estimation is always a good skill to have! We also give many applications of estimation to different areas of endeavor. Here are the solutions to Exercise 1.2.

- 416.38 rounded to the nearest 100 is 400.
 \$30.28 rounded to the nearest dollar is \$30.
 Thus, a reasonable estimate of the value of the investor's stock is $400 \cdot \$30 = \$12,000$.
- $\$7.80 \rightarrow \8.00
 $\$2.29 \rightarrow \2.00
 $\$3.75 \rightarrow \4.00
 $\$1.85 \rightarrow \2.00
 $\$2.90 \rightarrow \underline{\$3.00}$
 Estimate: \$19.00
- $6 \cdot 150 = 900$ gallons is a good estimate.
- $(a) \frac{4256}{14,053} \approx 0.303$ (to 3 dec. places)
 $(b) \frac{4300}{14,100} \approx 0.305$ (to 3 dec. places)
- $\frac{9 \cdot 14}{140} = \frac{9}{10} = 0.900$

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11. It takes 4 to 8 hors d'oeuvres per person, so for 100 persons it takes 100 times as much, that is, 400 to 800 hors d'oeuvres.
13. It takes $\frac{1}{3}$ lb of boneless meat or fish per person, so for 100 persons it takes 100 times as much or $100 \cdot \frac{1}{3} = 33\frac{1}{3}$ lbs. 33 or 34 pounds are needed.
15. It takes $\frac{1}{4}$ lb of raw pasta per person, so for 100 persons it takes 100 times as much or $100 \cdot \frac{1}{4} = 25$ pounds of pasta.

17. (a) The reading is: 5 1 8 2
 (b) $5182 - 5102 = 80$ KWH
 (c) $0.08 \cdot 80 = \$6.40$
 (d) $30 \cdot 6.40 = \$192.00$

19. (a) The reading is: 7 0 0 1
 (b) $7001 - 6951 = 50$ KWH
 (c) $0.08 \cdot 50 = \$4.00$
 (d) $30 \cdot 4 = \$120.00$

21. $\$4090.00 + 0.25(\$40,000 - \$29,700)$
 $= \$4090.00 + 0.25(\$10,300)$
 $= \$4090.00 + \2575.00
 $= \$6665.00$

23. (a) $H = 2.89 \cdot 15 + 27.81$
 $= 43.35 + 27.81$
 $= 71.16$ in
 (b) Rounded to the nearest whole number,
 $H = 3h + 28$
 When $h = 15$, $H = 3 \cdot 15 + 28 = 73$
 The difference is
 $73 - 71.16 = 1.84$ in.

25. $BMI = \frac{705W}{H^2} = \frac{705 \cdot 150}{68^2} \approx 22.87$ (to the nearest hundredth). Since 22.87 is less than 24, the person is normal.

27. $W = \frac{G^2 \cdot L}{330} = \frac{70^2 \cdot 66}{330} = 980$ pounds
 Measured in 100 pound units, this is 9.8 or

about 10 to make it safe. The horse needs 0.6 gallons of water, 1 lb of hay and $\frac{1}{2}$ lb of grain for each 100 lbs of body weight. Thus, the horse needs:
 $0.6 \cdot 10 = 6$ gallons of water
 $1 \cdot 10 = 10$ lbs of hay
 $\frac{1}{2} \cdot 10 = 5$ lbs of grain

29. $C = 596 + 0.0019V + 21.7A$
 $= 596 + 0.0019 \cdot 500,300 + 21.7 \cdot 5$
 $= 596 + 950.57 + 108.50$
 $\approx \$1655$

31. (a) For the first 2 years, we have 24 human years. For the next 3 years, we have $3 \cdot 4 = 12$ human years. Thus, for $2 + 3 = 5$ years, we have $24 + 12 = 36$ human years.
 (b) For the first 2 years, we have 24 human years. For the next 8 years, we have $3 \cdot 8 = 32$ human years. Thus, for $2 + 3 = 5$ years, we have $24 + 32 = 56$ human years.

33. The distance between the intersection of 90 and 128 to the intersection of 90 and 495 is about one inch on the map or about 15 actual miles.

35. The distance between the intersection of 90 and 290 to the intersection of 90 and 86 is about one inch on the map or about 15 actual miles.

37. The distance between the intersection of 90 and 32 to the intersection of 90 and 91 is about 1.5 in on the map or $1.5 \cdot 15 = 22.5$ miles.

39. The distance in Problem 34 is 22.5 miles. If the car makes 20 miles per gallon, it would need $\frac{22.5}{20}$ gallons that would cost $\$2.40 \cdot \frac{22.5}{20} = \2.70 .

41. The distance in Problem 36 is $1\frac{1}{4}$ in. on the map or 18.75 miles. If the car makes 20 miles per gallon, it would need $\frac{18.75}{20}$

gallons that would cost

$$\$2.40 \cdot \frac{18.75}{20} = \$2.25.$$

43. $\frac{268,000}{30} \approx 8933.33$ pounds

45. $\frac{14,000}{30} \approx 466.67$ pounds

47. For a 150 lb male: Multiply the body weight 150 by 10 which is 1500; Add twice the body weight or 300; The BMR is: $1500 + 300 = 1800$.

49. Answers may vary.

51. $23 \cdot 5.5 \cdot 5280 \cdot 2 = 1,335,840$ ft²

Section 1.3 Graph Interpretation: A Problem-Solving Tool

Exercise 1.3

STUDY TIPS Many of the problems you will encounter will involve the interpretation of a graph. This section will tell you how to do it! We shall concentrate on three types of graphs: **circle** (or **pie chart**), **bar graphs** and **line graphs**. Remember, you do not have to know how to construct the graphs, we shall provide them for you. Your job is simply to interpret them. Ready for the problems? Here we go.

- (a) The most preferred mode of transportation is the one that covers the *most area*, that is, the bus. You can reach the same conclusion by noting that the bus is the category with the highest percent (43%).
(b) The least preferred mode of transportation is the one that covers the *least area*, that is, the bike (5%)

(c) First, assume that "drive to work" is the category "car". Since 20% of the people drive a car and 91% do so in Dallas-Fort Worth, the percent difference is $91\% - 20\% = 71\%$.

- (a) The cheese produced the most is Cheddar (36%).
(b) The cheese produced the least is Swiss (2.8%).
(c) The second most popular cheese is Mozzarella (30.6%).
- (a) Bathing (30%)
(b) 30% of 500 = $0.30 \cdot 500 = 150$ gal
(c) The dishwasher uses 3% of the water and the toilet leak uses 5%, thus the toilet leak uses more water
(d) The dishwasher uses 3% of the water, which represents 5 gallons. The faucet uses 12% of the water, which is 4 times as much, that is, the faucet uses $4 \cdot 5 = 20$ gallons of water.
- (a) Paper (40%)
(b) Yard trimmings (18%)
(c) It would contain 40% of 50 = $0.40 \cdot 50$ or 20 lbs of paper; it would also contain 18% of 50 = $0.18 \cdot 50$ or 9 lbs of yard trimmings
- (a) Oil (33%)
(b) Nuclear (5%)
(c) Natural gas (18%)
- (a) "No drinks per day" means "None" and the bar representing None is about 114 units long (actually, it is 114.4). Thus, the systolic blood pressure for young adults consuming no drinks per day is $114.4 \approx 114$.
(b) The bar representing $< 1/\text{day}$ is a little less than the bar of part (a), so the approximate answer is 114.
(c) The bar corresponding to $1 - < 2/\text{day}$ is about 111.2 long. Hint: You may need a ruler here!
(d) The lowest blood pressure corresponds to the category $2 - < 3/\text{day}$; 110

- (e) The longest bar (highest blood pressure) corresponds to $> 3/\text{day}$; about 120
13. (a) 39
(b) $13 + 14 + 2 = 29$
(c) 20–.29; 14
15. (a) 20–29 (the longest bar)
(b) 13–15
(c) Less than 50
(d) 90+; Answers vary. (You do not see many 90+ people driving!)
17. (a) \$75K+ (more than \$75,000); 77.7% in 2000, 60.3% in 1998
(b) Under 15K (less than \$15,000); 12.7% in 2000, 7.1% in 1998
(c) The difference is about $\$75,000 - \$15,000 = \$60,000$
19. (a) The Cookie Dough ice cream is about 280 calories ($\frac{1}{2}$ cup) or 560 per cup. The Cherry Garcia yogurt is about 150 calories ($\frac{1}{2}$ cup) or 300 per cup. The difference (per cup) is $560 - 300 = 260$ calories
(b) 2 cups of the Cookie Dough ice cream has $2 \cdot 560 = 1120$ calories. 2 cups of the Cherry Garcia yogurt has $2 \cdot 300 = 600$ calories. The difference is $1120 - 600 = 520$ calories.
21. (a) Cuban toast (the longest bar)
(b) Cheese toast (second longest bar)
(c) $60 + 20 + 10 + 15 = 105$ breakfasts. Since each breakfast uses $\frac{1}{4}$ of a loaf of bread, we need $\frac{1}{4} \cdot 105 = 26\frac{1}{4}$ loaves or 27 loaves of Cuban bread.
23. (a) African Americans
(b) White
(c) About 20%; about 15%. Hint: Use a ruler to measure!
(d) Answers will vary
25. (a) About 24%; about 82%; $82\% - 24\% = 58\%$ (Answer may vary)
- (b) About 4%; about 22%; $22\% - 4\% = 18\%$
(c) Less than 12 years is decreasing; the other two are increasing.
27. (a) 65 years and older
(b) 18 to 64 years
(c) About 30%
(d) 65 years and older; under 18 years
(e) None
(f) 18 to 64 years
29. (a) 60
(b) About 10
(c) About 2
31. (a) 4 kg
(b) 10 kg
(c) About 10 Mo.
33. Go to 6 on the horizontal axis and up until you meet the top curve. The intersection occurs at about \$570.
35. About \$300
37. Answers may vary.
41. (a) Years 1-7
(b) Years 2-7
(c) Years 1-7
(d) Years 4-7
(e) Year 7 of either the breast cancer or the stroke group; 0.005
(f) Breast cancer; years 0-4

Chapter 1 Practice Test

STUDY TIPS Find out if your actual test will be multiple choice or fill in the blank, and how long you will have to take it. Then take the practice test using the time limit set by your instructor. Find your weaknesses and remedy them before your actual test! We will have more test taking tips later. Here are the answers to the practice test.

Select the unknown

Think of a plan

Use the techniques you are
studying to carry out the plan
Verify the answer

2. The process of arriving at a general conclusion on the basis of repeated observations of specific examples

3. Look at the difference between successive terms as shown

	1	2	7	19	41	76
Diff	1	5	12	22	35	
Diff		4	7	10	13	
Diff			3	3	3	

The third differences are constant (3), so the next number can be constructed by addition. Add the last diagonal from bottom to top. We obtain the next number in the pattern, $3 + 13 + 35 + 76 = 127$.

	1	2	7	19	41	76	127
Diff	1	5	12	22	35	51	
Diff		4	7	10	13	16	
Diff			3	3	3	3	

Now, we can use the 127 to continue the last three rows as shown. The next term now is $3 + 16 + 51 + 127 = 197$. If you do this one more time, you will find the next term to be 289. Once you get four terms you can show that all the following terms can be obtained from the formula:

$$a_{n+1} = 3 + 3a_n - 3a_{n-1} + a_{n-2}$$

Thus, the next three terms after 76 are 127, 197, 289.

4. (a) Select a number: n
 Multiply by 4: $4n$
 Add 6 to the product: $4n + 6$
 Divide the sum by 2: $\frac{4n+6}{2} = 2n+3$
 Subtract 3 from the quotient:
 $2n + 3 - 3 = 2n$
- (b) Using 1, the final result is 2
 Using 10, the final result is 20
 Using 100, the final result is 200
- (c) The final result is twice the original number.
5. (a) $319.26 \rightarrow 319.3$
 Since the 6 after the 2 is greater than 5,

add one to the 2.

- (b) $319.26 \rightarrow 300$
 Since the 1 after the 3 is less than 5, leave the 3 alone; add 0's.

6. (a) 6064 KWH
 (b) $6064 - 6002 = 62$ KWH
 (c) $0.10 \cdot 62 = \$6.20$
 (d) $30 \cdot 620 = \$186.00$
7. (a) Female: $H = 28.6 + 2.5(15) \approx 66$ in
 (rounded from 66.1)
 (b) Male: $H = 32.2 + 2.4(15) \approx 68$ in
 (rounded from 68.2)
8. (a) Public Safety
 (b) $1/2$
 (c) Sales Tax
9. (a) About 50 million
 (b) About 51 million
 (c) About 82 million
 (d) About 64 million
 (e) About $64 - 50 = 14$ million
10. (a) About 6.91%
 (b) About 6.76%
 (c) About $6.91 - 6.76 = 0.15\%$
 (d) They seem to be decreasing