5

LOGIC AND SETS

CLAST MATHEMATICS COMPETENCIES

IE1:	Deduce facts of set inclusion or set non-inclusion from a diagram
IIE1:	Identify statements equivalent to the negations of simple and compound statements
IIE2:	Determine equivalence or non-equivalence of statements
IIE3:	Draw logical conclusions from data
IIE4:	Recognize that an argument may not be valid even though its conclusion is true
IIIE1:	Recognize valid reasoning patterns as illustrated by valid arguments in everyday language
IIIE2:	Select applicable rules for transforming statements without affecting their meaning
IVE1:	Draw logical conclusions when facts warrant them

5.1 EQUIVALENT STATEMENTS

The word *logic* is derived from the Greek word *logos* which may be interpreted to mean reason or discourse. Most of the study of logic revolves around the idea of a statement which we shall discuss next.

T	TERMINOLOGY -- STATEMENTS
DEFINITION OF A STATEMENT A **statement** is a declarative sentence which can be classified as true or false.	***EXAMPLES*** I will study today. 2 is an even number.
CONJUNCTIONS (p and q) If two statements are connected by the word "**and**" (or an equivalent word such as "**but**"), the resulting statement is called a **conjunction** and is denoted by $p \wedge q$. The CLAST simply uses the notation ***p* and *q***.	Let ***e*** represent the statement "2 is an even Let ***w*** represent the statement "2 is a whole number". The statement: "2 is an even number **and** 2 is a whole number" is a conjunction which can be written as "$e \wedge w$" or as "***e* and *w***".
DISJUNCTIONS (p or q) If two statements are connected by the word "**or**" (or an equivalent word), the resulting statement is called a **disjunction** and is denoted by $p \vee q$. The CLAST uses the notation ***p* or *q*.**	Let ***h*** be: I will study hard and ***f*** be: I will fail the test The statement "I will study hard or I will fail the test" is a **disjunction** which can be written as "$h \vee f$" or as "***h* or *f***".
CONDITIONALS (If p then q) A statement of the form "If p then q" is called a **conditional** statement and is denoted by $p \rightarrow q$. The statement ***p*** is called the **antecedent** and ***q*** is called the **consequent.**	Let ***s*** be: I study hard and ***p*** be: I pass the test. The statement "If I study hard then I pass the test" is a **conditional** statement which can be written as $s \rightarrow p$.
NEGATIONS (not p) The negation of the statement p is denoted by $\sim p$. The CLAST uses the notation **not p** for the negation of a statement. Note that $\sim\sim p$ means the same as ***p***, that is, the negation of "2 is not even" is "2 is even".	Let ***m*** be: Today is Monday. "Today is **not** Monday" or "It is not true that today is Monday" is the **negation** of ***m*** and it is written as "$\sim m$" or "**not *m***" Let $\sim m$ be : Today is **not** Monday $\sim(\sim m)$ is : Today is Monday

This CLAST skill requires that you negate conjunctions, disjunctions and conditionals. You will be asked to negate statements involving the **universal quantifiers** (*all, no, none* and *every*) and the **existential quantifiers** (*some, some are not, there exists*) and determine which statements are logically equivalent to a given statement. In addition, you will be asked to determine how a given statement involving universal or existential quantifiers is directly transformed into another given statement. As usual, we first give you the terminology you need and several examples. This time, we will even provide a summary showing a statement and its negation.

A. Negating Statements

Objective IIE1	CLAST SAMPLE PROBLEMS
WRITE THE NEGATION OF THE GIVEN STATEMENT:	
1. It is hot and the sun is shining	**2.** If the sun is shining then I will go to the beach
3. It is not raining	**4.** I will study or I will not pass the test
5. Some students do not study	**6.** All students pass the CLAST
7. No students pass the CLAST	**8.** Some dogs have green eyes

1	NEGATING STATEMENTS
NEGATING "p and q" The negation of the *conjunction* "***p* and *q***" is "**not *p* or not *q***", that is, **Not** (p and q) is *equivalent to* **Not** ***p*** *or* **Not *q*** In symbols, $\sim(p \wedge q) \Leftrightarrow \sim p \vee \sim q$	***EXAMPLES*** Negate: "I am not sick and I am tired". Let $\sim s$ be: I am not sick, t be: I am tired. $\sim(\sim s \wedge t) \Leftrightarrow \sim\sim s \vee \sim t$, that is, $s \vee \sim t$ Thus, the negation of "I am not sick and I am tired", is "I am sick or I am not tired".
NEGATING "p or q" The negation of the *disjunction* "***p* or *q***" is "**not *p* and not *q*",** that is, **Not** (p or q) is equivalent to **Not** p and **Not** q In symbols, $\sim(p \vee q) \Leftrightarrow \sim p \wedge \sim q$	Negate: I will study hard or I will not pass this course. Let h be: I will study hard. Let $\sim p$ be: I will not pass this course. $\sim(h \vee \sim p) \Leftrightarrow \sim h \wedge \sim\sim p$, that is, $\sim h \wedge p$. Thus, the negation of "I will study hard or I will not pass this course", is "I will not study hard and I will pass this course".
NEGATING "If p then q" The negation of the statement "**If *p* then *q***" is "***p* and not *q*",** that is, **Not** ($p \rightarrow q$) *is equivalent to* p **and** not q. In symbols, $\sim(p \rightarrow q) \Leftrightarrow p \wedge \sim q$	Negate: If the sun is out then I will go to the beach. Let s be: The sun is out. and b be: I will go to the beach. $\sim(s \rightarrow b) \Leftrightarrow s \wedge \sim b$ Thus, the negation of "If the sun is out then I will go to the beach", is "The sun is out and I will not go to the beach".

Here is a brief summary of the negations we have studied:

	STATEMENT	NEGATION
1.	p	not p
2.	not p	p
3.	p and q	(not p) or (not q)
4.	p or q	(not p) and (not q)
5.	If p, then q	(p) and (not q)

ANSWERS		
	1. It is not hot or the sun is not shining	
	2. The sun is shining and I will not go to the beach	
	3. It is raining	**4.** I will not study and I will pass the test
	5. All students study	**6.** Some students do not pass the CLAST
	7. Some students pass the CLAST	**8.** No dogs have green eyes

CLAST EXAMPLES

Example

1. Select the statement that is the negation of "It is raining and the sun is not out"

A. If it is raining then the sun is not out.

B. It is not raining and the sun is out.

C. It is raining and the sun is not out.

D. It is not raining or the sun is out.

Note: Negating an "and" statement requires an "or" statement. The answer has to be **D.** Look at the solution to see why.

Solution

To solve this type of problem, follow this procedure:

1. Write the statement in symbols.
Let r be: It is raining
Let $\sim$ o be: The sun is not out.
We are to negate: $r \wedge \sim o$

2. Use the proper equivalency to negate the given statement.
We are to negate the conjunction $r \wedge \sim$ o.
$\sim(r \wedge \sim o) \Leftrightarrow \sim r \vee \sim\sim$ o, that is, $\sim r \vee$ o.

3. Write the result in words.
$\sim r \vee o$ means "It is not raining or the sun is out". The answer is **D.**

Example

2. Select the statement that is the negation of: "Wendy will go to the beach or Wendy will not get a tan".

A. If Wendy goes to the beach, Wendy will get a tan.

B. Wendy will not go to the beach and Wendy will get a tan.

C. Wendy will go to the beach or Wendy will get a tan.

D. Wendy will go to the beach and Wendy will get a tan.

Note: Negating an "or" statement requires an "and" statement. The only possible answers are B and D.

Solution

1. Write the statement in symbols.

Let b be: Wendy will go to the beach
and $\sim t$ be: Wendy will not get a tan.
We are to negate: $b \vee \sim t$.

2. $b \vee \sim t$ is a *disjunction* and
$\sim(b \vee \sim t) \Leftrightarrow \sim b \wedge \sim\sim t$, that is, $\sim b \wedge t$.

3. Write the result in words.
$\sim b \wedge t$ means "Wendy will not go to the beach and Wendy will get a tan".

The answer is **B.**

Example

3. Select the statement that is the negation of: "If Joan teaches Algebra, she does not teach Geometry".

A. Joan teaches Algebra and Joan teaches Geometry.
B. If Joan teaches Geometry, then Joan does not teach Algebra.
C. Joan teaches Algebra and she does not teach Geometry.
D. If Joan does not teach Algebra, then Joan does teach Geometry.

Note: The negation of "If p then q" requires an "and" statement. The only possible answers are A and C.

Solution

1. Let a be: Joan teaches Algebra, and $\sim g$ be: Joan does not teach Geometry. We are to negate: $a \to \sim g$

2. $a \to \sim g$ is a *conditional* and $\sim (a \to \sim g) \Leftrightarrow a \wedge \sim\sim g$, that is, $a \wedge g$.

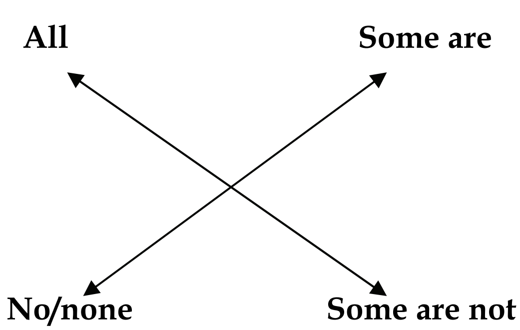
3. In words, $a \wedge g$ is written as "Joan teaches Algebra and Joan teaches Geometry".

The correct answer is **A.**

Statements involving the **universal quantifiers** *all, no, non* and *every*, or the **existential quantifiers** *some* and *there exists at least one* have to be negated in a different way. Here are the rules you need.

2	NEGATING THE UNIVERSAL QUANTIFIER	
STATEMENT	**NEGATION**	***EXAMPLES***
All a's **are** b's	**Some** a's **are not** b's **Not all** a's **are** b's	p: **All** students recycle paper. $\sim p$: **Some** students **do not** recycle paper. or **Not** all students recycle paper.
No a's **are** b's	**Some** a's **are** b's	q: **Some** of us will graduate. $\sim q$: **None** of us will graduate.

3	THE SQUARE OF OPPOSITION
RULE	***EXAMPLES***
These ideas can be remembered by using the Square of Opposition shown below.	To negate "All students are winners" follow the diagonal starting at "All". The answer is: "Some students are not winners". Note that the negation of "All students are not winners" (not shown on the square) is: "Some students are winners". To negate "Some teachers are not fair", follow the diagonal starting at "Some are not". The answer is: "All teachers are fair". To negate "No students are unreliable", follow the diagonal starting at "No". The answer is: "Some students are unreliable".



□CLAST EXAMPLE

Example	Solution
4. The negation of "No students failed this class" is: A. All students failed this class. B. Some students failed this class. C. Some students did not fail this class. D. No students failed this class.	Looking at the square, you can see that the negation of "No" is "Some are". Thus, the negation of "No students failed this class" is "Some students failed this class". The answer is **B.**

B. Selecting Equivalent Statements

Objective IIE2	CLAST SAMPLE PROBLEMS

1. Write two statements that are logically equivalent to:
If you pass the CLAST then you can graduate

2. Write two statements involving the conditional that are NOT logically equivalent to:
If you make a 90, then you make an A

3. Write a statement that is logically equivalent to:
It is not true that Tyrone is not a scholar or Maria is a gentleman

We have already mentioned that the negations of certain statements are **equivalent,** denoted by using the symbol $\Leftrightarrow$. The equivalencies involving the negation of statements and two additional equivalencies regarding conditional statements are covered by this objective. Here is the list of equivalencies you need.

4	LOGICAL EQUIVALENCIES
NEGATING " $\wedge$ ", " $\vee$ " AND " $\rightarrow$ "	***EXAMPLES***
$\sim(p \wedge q) \Leftrightarrow \sim p \vee \sim q$	Not (p **and** q) is equivalent to "not p **or** not q".
$\sim(p \vee q) \Leftrightarrow \sim p \wedge \sim q$	Not (p **or** q) is equivalent to "not p **and** not q".
$\sim(p \rightarrow q) \Leftrightarrow p \wedge \sim q$	Not (if p **then** q) is equivalent to "p **and not** q".
EQUIVALENCIES FOR $p \rightarrow q$	***EXAMPLES***
(1) $p \rightarrow q \Leftrightarrow \sim p \vee q$	"If p then q" is equivalent to "not p or q".
(2) $p \rightarrow q \Leftrightarrow q \vee \sim p$	"If it rains then it pours " is equivalent to: (1) It does not rain or it pours. (2) It pours or it does not rain.
(3) $p \rightarrow q \Leftrightarrow \sim q \rightarrow \sim p$ $\sim q \rightarrow \sim p$ is the **contrapositive** of $p \rightarrow q$.	"If it rains then it pours" is equivalent to: "If it does not pour then it does not rain".

ANSWERS

1. If you do not graduate then you do not pass the CLAST
You do not pass the CLAST or you can graduate

2. If you do not make a 90 then you do not make an A
If you make an A then you make a 90

3. Tyrone is a scholar and Maria is not a gentleman

5	RULES ABOUT EQUIVALENCIES
RULE 1	***EXAMPLES***
If you are asked to find a statement that is logically equivalent to "**if *p* then *q***", look for	Two statements logically equivalent to: If you study hard then you pass the CLAST
(1) "If not q, then not p"	(1) If you do not pass the CLAST then you do not study hard.
(2) "(not p) or q	(2) You do not study hard or you pass the CLAST
RULE 2	***EXAMPLES***
If you are asked to find a statement that is NOT logically equivalent to "**if *p* then *q***", look for one of these forms:	Two statements NOT logically equivalent to: If it rains, then I get wet
(1) If q, then p	(1) If I get wet then it rains
(2) If not p, then not q	(2) If it does not rain, then I do not get wet

We first consider an example that asks for a logically equivalent statement, followed by an example which a statement NOT logically equivalent to the given one.

CLAST EXAMPLES

Example

5. Select the statement that is logically equivalent to "If Tina is in Las Vegas, then she is in Nevada".

A. Tina is in Las Vegas or she is in Nevada.

B. If Tina is not in Nevada, then she is not in Las Vegas.

C. If Tina is in Nevada, then she is in Las Vegas.

D. If Tina is not in Las Vegas, then she is not in Nevada.

Solution

Let the statements be represented by the letters in parentheses. "If Tina is in Las Vegas (v) then she is in Nevada (n)" is of the form $v \to n$. The statements equivalent to $v \to n$ are:

(1) $\sim v \vee n$
(2) $n \vee \sim v$ and
(3) $\sim n \to \sim v$.

The only statement with one of these forms is **B**, which is of the form $\sim n \to \sim v$, that is, "If Tina is not in Nevada ($\sim n$) then she is not in Las Vegas ($\sim v$)". Thus, the correct answer is **B.**

Example	Solution
6. The statement NOT logically equivalent to "If the weather is cool, Larry will go jogging" is: A. Larry will go jogging if the weather is cool. B. If Larry goes jogging, the weather is cool. C. If Larry does not go jogging, the weather is not cool. D. The weather is not cool or Larry will go jogging.	The given statement is of the form $c \to j$. There are three equivalent statements: (1) $\sim c \vee j$ (2) $j \vee \sim c$ (3) $\sim j \to \sim c$ Statement C is written as (3), and statement D is written as (1). Thus, C and D are equivalent to the given statement. Statement A is a different way of saying the given statement. Thus, the only statement NOT logically equivalent to "If the weather is cool, Larry will go jogging" is **B.**

As we mentioned before, the phrase "It is not true that" can be regarded as the negation of the statement that follows it and, as such, symbolized by ~ ( ). Thus, the statement: "It is not true that Bill went to the movies (m) and to the beach (b)" is symbolized by $\sim (m \wedge b) \Leftrightarrow \sim m \vee \sim b$. Here is the rule we need to deal with equivalencies involving the phrase "It is not true that"

6	EQUIVALENCIES INVOLVING "IT IS NOT TRUE THAT"
RULE	***EXAMPLES***
To find a statement equivalent to a given one containing the phrase "It is not true that" replace the phrase with the ~ sign and use one of the following: $\sim (p \wedge q) \Leftrightarrow \sim p \vee \sim q$ $\sim (p \vee q) \Leftrightarrow \sim p \wedge \sim q$ $\sim (p \to q) \Leftrightarrow p \wedge \sim q$ ~ (All p are q) $\Leftrightarrow$ Some p are not q ~ (No p are q) $\Leftrightarrow$ Some p are q ~ (Some p are q) $\Leftrightarrow$ No p are q ~ (Some p are not q) $\Leftrightarrow$ All p are q	The statement "It is not true that Alex is taking Math (m) or English (e)" is symbolized by $\sim (m \vee e)$. An equivalent statement is $\sim m \wedge \sim e$, that is, "Alex is not taking Math and Alex is not taking English". A statement equivalent to "It is not true that if Alex takes Math then he takes English", $\sim (m \to e)$, is $\sim m \wedge \sim e$, that is, "Alex takes Math and he does not take English". A statement equivalent to "It is not true that all students are lazy" is "Some students are not lazy".

□CLAST EXAMPLES

Example

7. Select the statement that is logically equivalent to "It is not true that Bob is not a lawyer and Mary is a doctor".

A. Bob is not a lawyer or Mary is a doctor.
B. Bob is a lawyer or Mary is not a doctor.
C. Bob is a lawyer or Mary is a doctor.
D. Bob is not a lawyer or Mary is not a doctor.

Solution

The statement "It is not true that Bob is not a lawyer and Mary is a doctor" is symbolized by $\sim(\sim l \wedge d)$. An equivalent statement is $\sim\sim l \vee \sim d$, that is, $l \vee \sim d$. This means that "Bob is a lawyer or Mary is not a doctor", which is choice **B.**

Example

8. Select the statement that is logically equivalent to "It is not true that Fran goes to the mall if she gets the money."

A. Fran gets the money or she does not go to the mall.

B. Fran does not get the money and she does not go to the mall.

C. Fran gets the money and she does not go to the mall.

D. Fran does not get the money or she does not go to the mall.

Solution

The statement "It is not true that Fran goes to the mall if she gets the money" is symbolized by $\sim(m \rightarrow g)$. An equivalent statement is $m \wedge \sim g$, which means that "Fran gets the money and she does not go to the mall". This is choice **C.**

Example

9. Select the statement that is logically equivalent to "It is not true that all students pass math and some students have calculators".

A. Some students pass math or no students have calculators.

B. Some students do not pass math and no students have calculators.

C. Some students pass math or no students have calculators.

D. Some students do not pass math or no students have calculators.

Solution

The statement "It is not true that all students pass math and some students have calculators" is symbolized by $\sim(p \wedge c)$. An equivalent statement is $\sim p \vee \sim c$ which means that "It is not true that all students pass math or it is not true that some students have calculators".
Now, "It is not true that all students pass math" means "Some students do not pass math". Also, "It is not true that some students have calculators" means that "No students have calculators", thus $\sim p \vee \sim c$ means that "Some students do not pass math or no students have calculators", which is choice **D.**

C. Transforming Statements into Equivalent Ones

Objective IIIE2	CLAST SAMPLE PROBLEMS

Select the rule that transforms statement "i" into statement "ii"

1. i. If you make a 90, then you make an A ii. You did not make a 90 or you make an A

2. i Not all students make A's ii. Some students do not make A's

3. i. It is not true that is raining and Tom is playing golf
ii. It is not raining or Tom is not playing golf

4. i. It is not true that Mary is studying and the music is playing
ii. Mary is not studying or the music is not playing

This CLAST skill does not require you to find statements equivalent to given ones but rather to discover what rule has been used to transform a given statement into an equivalent one. To do this, examine the given statement and state the correct rule used to transform it into an equivalent statement. Consult the Rules given in Box 6. Many of the rules there and some new rules are stated in words in box 7.

7	RULES FOR TRANSFORMATIONS
STATEMENT	***EQUIVALENT STATEMENT***
not (not *p*)	*p*
not (*p* and *q*)	(not *p*) or (not *q*)
not (*p* or *q*)	(not *p*) and (not *q*)
If *p*, then *q*	(not *p*) or *q*
Not all are *p*	Some are not *p*
All are not *p*	None are *p*
Not (If *p*, then *q*)	(*p*) and (not *q*)
If *p*, then *q*	If not *q*, then not *p*

ANSWERS

1. Form for i: If *p* then *q*
Form for ii: (not *p*) or (*q*)

2. Form for i: Not all are *p*
Form for ii: Some are not *p*

3. Form for i: not (*p* and *q*)
Form for ii: (not *p*) or (not *q*)

4. Form for i: not (*p* and *q*)
Form for ii: (not *p*) or (not *q*)

CLAST EXAMPLE

Example

10. Select the rule of logical equivalence that directly transforms (in one step) statement "i" into statement "ii".

i. Not all children are left handed.
ii. Some children are not left-handed.

A. "Not (not *p*)" is equivalent to "*p*"

B. "If *p* then *q*" is equivalent to "If not *q*, then not *p*".

C. "Not all are *p*" is equivalent to "Some are not *p*".

D. "All are not *p*" is equivalent to "None are *p*".

Solution

Statement i is of the form "Not all are *p*" which is equivalent to "Some are not *p*".

The correct response is **C.**

Section 5.1 Exercises

WARM-UPS A

In Exercises 1-8 write the negation of the given statement

1. It is not snowing and the sun is out.

2. It is warm or the air conditioner is broken.

3. Bill will study Math or he will not pass the test.

4. If it does not rain, then we will go to the beach.

5. If the sun is out, the temperature will rise.

6. All athletes are in good shape.

7. Some athletes are in good shape.

8. No athletes are in good shape.

9. Some athletes are not in good shape.

CLAST PRACTICE A **PRACTICE PROBLEMS: Chapter 5, # 1-4**

10. Select the statement that is the negation of: "Ken is a chef or Betty is a teacher".

A. Ken is not a chef or Betty is not a teacher.

B. Ken is not a chef and Betty is a teacher.

C. Ken is a chef and Betty is not a teacher.

D. Ken is not a chef and Betty is not a teacher.

11. The negation of the statement "Jim is coming on Monday and Ann is coming on Friday" is:

A. Jim is not coming on Monday and Ann is not coming on Friday.

B. Jim is not coming on Monday or Ann is coming on Friday.

C. Jim is not coming on Monday or Ann is not coming on Friday.

D. Jim is coming on Monday and Ann is not coming on Friday.

$\boxed{12.}$ Select the statement that is the negation of: "If Rita is offered a free trip, she will accept it."

A. If Rita is offered a free trip, she will not accept it.

B. If Rita is not offered a free trip, she will not accept it.

C. Rita is offered a free trip and she accepts it.

D. Rita is offered a free trip and she does not accept it.

$\boxed{13.}$ The negation of the statement "All the girls and boys are going to the fair" is:

A. Some girls or boys are going to the fair.

B. Some girls and boys are not going to the fair.

C. All the girls and none of the boys are going to the fair.

D. All the girls and boys are not going to the fair.

WARM-UPS B

14. Write three statements equivalent to "If today is Monday, then I will study."

15. Write a statement equivalent to "It is not true that Joe took Math and failed it."

16. Write a statement equivalent to "It is not true that Wendy is intelligent or aloof."

17. Write a statement equivalent to: "It is not true that Pat and Jackie are boys."

CLAST PRACTICE B

PRACTICE PROBLEMS: Chapter 5, # 5-8

18. Select the statement logically equivalent to: "It is not true that both George and Daniel are teachers."

A. If Daniel is not a teacher, George is not a teacher.

B. George is not at teacher or Daniel is not a teacher.

C. If George is not a teacher, Daniel is not a teacher.

D. Daniel is not a teacher and George is not a teacher.

19. The statement logically equivalent to "If Lana fights with her brother, then she will be sorry" is:

A. If Lana is not sorry, then she fights with her brother.

B. Lana does not fight with her brother or she is not sorry.

C. Lana is sorry or she does not fight with her brother.

D. Lana fights with her brother or she is not sorry.

20. Select the statement logically equivalent to "If Mary is in San Francisco, then she is in California."

A. If Mary is not in San Francisco, then she is not in California.

B. If Mary is in California, then she is in San Francisco.

C. If Mary is not in California, then she is not in San Francisco.

D. Mary is in San Francisco or she is in California.

$\boxed{21.}$ The statement NOT logically equivalent to "If the weather is cool, Pedro will wear a sweater" is:

A. If Pedro wears a sweater, the weather is cool.

B. If Pedro does not wear a sweater, the weather is not cool.

C. The weather is not cool or Pedro will wear a sweater.

D. Pedro will wear a sweater if the weather is cool.

WARM-UPS C

22. Indicate which rule of logical equivalence transforms the statement "All bottles are recycled" into "Some bottles are not recycled."

23. Indicate which rule of logical equivalence transforms the statement "Some liquids are flammable" into "It is not true that no liquids are flammable."

24. Indicate which rule of logical equivalence transforms the statement "Not all boys and girls are passing the class" into "Some boys and girls are not passing the class."

25. Indicate which rule of logical equivalence transforms the statement "All cats are not insects" to "No cats are insects"

CLAST PRACTICE C **PRACTICE PROBLEMS: Chapter 5, # 9-10**

26. Select the rule of logical equivalence that directly transforms (in one step) statement "i" into statement "ii".
    i. If x^2 is even, then x is even.
    ii. If x is not even, then x^2 is not even.

A. "If *p*, then *q*" is equivalent to "If not *q*, then not *p*."

B. "If *p*, then *q*" is equivalent to "If *q*, then *p*."

C. "Not (*p* and *q*) is equivalent to "not *p* or not *q*."

D. The correct equivalence rule is not given.

27. Select the rule of logical equivalence that directly transforms (in one step) statement "i" into statement "ii".

    i. Not all boys and girls are passing the class.
    ii. Some boys and girls are not passing the class.

A. "Not (not *p*)" is equivalent to "*p*".

B. "If *p* then *q*" is equivalent to "If not *q* then not *p*."

C. "Not all are *p*" is equivalent to "Some are not *p*."

D. "All are not *p*" is equivalent to "None are *p*."

$\boxed{28.}$ Select the rule of logical equivalence that directly transforms (in one step) statement "i" into statement "ii".

    i. Not all integers are positive numbers.
    ii. Some integers are not positive numbers.

A. "Not all are *p*" is equivalent to "Some are not *p*."

B. "Not (not *p*)" is equivalent to "*p*".

C. "All are not *p*" is equivalent to "none are *p*."

D. The correct equivalence rule is not given.

29. Select the rule of logical equivalence that directly transforms (in one step) statement "i" into statement "ii".

    i. All tests are not hard.
    ii. No test is hard.

A. "Not (not *p*)" is equivalent to "*p*".

B. "Not some *p*" is equivalent to "All are not *p*."

C. "Not (*p* and *q*)" is equivalent to "Not *p* or not *q*."

D. "All are not *p*" is equivalent to "None are *p*."

30. Select the rule of logical equivalence that directly transforms (in one step) statement "i" into statement "ii".

    i. It is not the case that you study Math or English.
    ii. You do not study Math and you do not study English.

A. "Not all *p*" is equivalent to "Some are not *p*."

B. "None are *p*" is equivalent to "All are not *p*."

C. "Not (*p* and *q*)" is equivalent to "Not *p* or not *q*."

D. "Not (*p* or *q*)" is equivalent to "not *p* and not *q*."

31. Select the rule of logical equivalence that directly transforms (in one step) statement "i" into statement "ii".

    i. If the birds are singing, then it must be spring.
    ii. Birds are not singing or it must be spring.

A. "If *p* then *q*" is equivalent to "Not *p* or *q*."

B. "Not (*p* or *q*)" is equivalent to "Not *p* and not *q*."

C. "Not (*p* and *q*)" is equivalent to "Not *p* or not *q*."

D. "Not (If *p* then *q*)" is equivalent to "*p* and not *q*."

EXTRA CLAST PRACTICE

32. Select the statement that is the negation of "All summer days are hot."

A. Some summer days are hot.
B. Some summer days are not hot.
C. No summer days are hot.
D. If it is not a summer day, then it is not hot.

33. Select the statement that is the negation of "No dogs are canines."

A. Some dogs are canines.
B. Some dogs are not canines.
C. All dogs are canine.
D. If an animal is a canine, then it is not a dog.

34. Select the statement that is logically equivalent to "It is not true that some cats meow or some birds do not fly."

A. No cats meow and some birds fly.
B. Some cats do not meow and some birds fly.
C. No cats meow and all birds fly.
D. Some cats do not meow or some birds fly.

35. Select the rule of logical equivalence than directly (in one step) transforms statement "i" into statement "ii".

i. Not all students have manners.

ii. Some students do not have manners.

A. "If p, then q" is equivalent to "If not q, then not p."

B. "All are not p" is equivalent to "None are p."

C. "Not (not p)" is equivalent to "p".

D. "Not all are p" is equivalent to "some are not p."

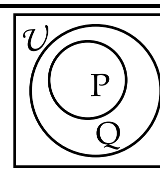
5.2 DRAWING CONCLUSIONS FROM DATA

This CLAST skill involves two types of questions: finding conclusions that can be logically deduced from a set of premises and determining if individual cases meet general requirements or criteria specified in advance. To answer the first type of question, we must learn about sets and their representations as Venn or Euler diagrams.

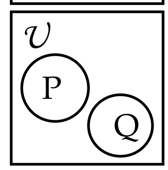
A. Set Inclusion: Venn or Euler diagrams

Objective IE1 CLAST SAMPLE PROBLEMS

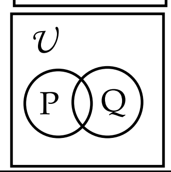
In the diagrams to the right, assume that P, Q and U are sets and no regions are empty



1. What is the relationship between sets P and Q on the diagram?



2 What is the relationship between sets P and Q on the diagram?



3. What is the relationship between sets P and Q on the diagram?

Here is the terminology we need to solve these type of problems.

T	TERMINOLOGY--SETS
THE SET CONCEPT A **set** is a well-defined collection of objects called the **elements** or members of the set.	***EXAMPLES*** The set of natural numbers is {1, 2, 3, 4, **...**}. The elements are 1, 2, 3, and so on.
DESCRIBING SETS Sets are described by **1**. Giving a verbal description. **2**. Listing the elements using braces.	The *set of whole numbers* has just been described by a verbal description. The elements are listed as {0, 1, 2, 3**...**}.
THE UNIVERSAL SET The **universal set** is the set of all elements under discussion and is denoted by U.	When working with numbers, the **universal set** may be the set of real numbers.
SUBSETS The set **A** is a subset of the set **B,** denoted by **A** $\subseteq$ **B**, if *every* element of **A** is also an element of **B.**	When describing animals, the universal set may be the set of all mammals. A subset of this set is the set of dogs (because every dog is a mammal).

ANSWERS

1. Every element (member) of set P is also an element of set Q.
2. No element (member) of set P is an element (member) of set Q
3. At least one element (member) of set P is also an element (member) of set Q

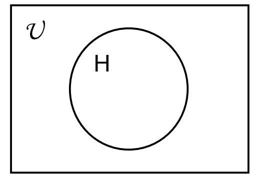
Sets and subsets can be represented graphically using Venn or Euler diagrams. In these diagrams, the universal set U is represented by a rectangle. Circles inside this rectangle represent the sets being considered.

1 VENN DIAGRAMS

DRAWING VENN DIAGRAMS

Let U represent all students enrolled in your school and H represent the students in your history class. Since all students in your history class (represented by the circle labeled H) are enrolled in your school, the circle is drawn inside the rectangle U.

EXAMPLE



2 SUBSETS AND VENN DIAGRAMS

INTERPRETING DIAGRAMS

Let M be the set of students taking Math and E the set of students taking English.
If U represents all students enrolled in your school, the diagrams at the right show four different possibilities for the subsets M and E.

1. No students take Math and English
2. Some students take Math and English.
3. All students taking Math take English.
4. All students taking English take Math

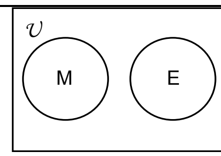


Figure 1

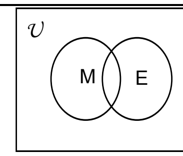


Figure 2

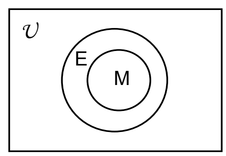


Figure 3

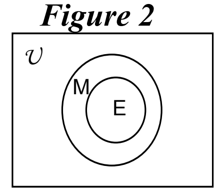


Figure 4

The CLAST asks you to make conclusions about Venn diagrams involving two, three or four subsets. Here are some examples.

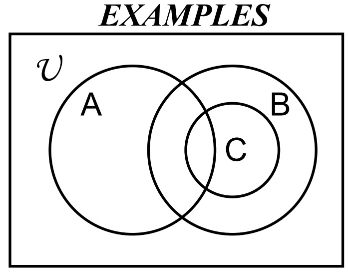
3 THREE OR MORE SUBSETS

INTERPRETING DIAGRAMS

The diagram at the right has a universal set represented by the rectangle and denoted by U and three subsets A, B and C represented by circles. If none of the six regions are empty, here are some conclusions we can reach:

1. Some elements of A are not in B or in C.
2. Some elements of A are in B.
3. Some elements of A are in C.
4. All the elements of C are in B
5. Some elements of B are not in C.

EXAMPLES



Since circle C is inside circle B, every element of C is in B. Also, every element of either A, B or C is in U, but there may be elements of U **not** in A, B or C.

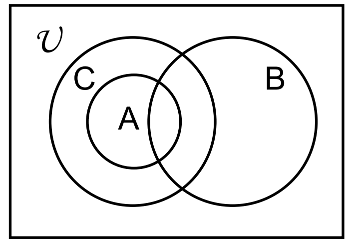
CLAST EXAMPLE

Example

1. Sets A, B and C are related as shown in the diagram. Which of the following statements is true, assuming none of the regions is empty.

A. Any element of A is also a member of C.
B. No element is a member of A, B and C.
C. Any element of U is a member of A.
D. None of these statements is true.

Solution



Since circle A is inside circle C, every element of A is also a member of C. The correct answer is **A.**

B. Drawing Conclusions

Objective IIE3 CLAST SAMPLE PROBLEMS

1. Read the requirements and each applicant's qualifications for obtaining a $120,000 loan needed to buy a house. Then identify which applicant would qualify for the loan.
To qualify for a $120,000 home loan, an applicant must have a gross income of $40,000 if single ($60,000 combined income if married) and debts of no more than $5000

Mr. and Mrs. Jones are married with 3 children. She makes $31,000 and her husband makes $30,000 Their only debts are $3000 on their bank charge card and a $5000 balance on their car loan.

Mr. and Mrs. Tran are married and have no children. Mr. Tran makes $65,000 and his wife does not work outside the home. Their only debt is $5000 on their car loan.

Horacio Martinez is single and makes $30,000. He is free of debt.

This skill requires us to draw conclusions from given data that can be presented in the form of an *argument* or to determine whether individual instances satisfy given general requirements. In order to discuss the first type of problem, we need to define *arguments.*

T **TERMINOLOGY--ARGUMENTS**

ARGUMENTS	***EXAMPLES***
An **argument** is a set of statements, the **premises**, and a claim that another statement, the **conclusion**, can be deduced (follows) from the premises.	If you get engaged, you will get married. You are engaged. Therefore, you will get married. The **premises** are: (1) If you are engaged, you will get married (2) You are engaged. The **conclusion** is: You will get married.

ANSWERS **1.** Mr. Tran is the only person that qualifies

The next step is to determine which arguments are **valid,** that is, when is the conclusion a necessary logical consequence of the premises. Here is the terminology we need.

T TERMINOLOGY--VALID ARGUMENTS	
VALID ARGUMENTS	***EXAMPLE***
An **argument** is valid if the conclusion is *true* whenever all the premises are *true*. If an argument is <u>not</u> valid it is said to be **invalid.**	The following argument is valid: All foreign cars are expensive. My car is a foreign car. Therefore, my car is expensive.

To discover conclusions that can be derived from a set of premises we diagram the premises and show that the conclusion is present. Here are the rules for diagramming statements.

4 DIAGRAMMING ARGUMENTS

RULE

The statements:

(1) All P's are Q's.

(2) No P's are Q's.

(3) Some P's are Q's.

(4) Some P's are not Q's.

are diagrammed as shown at the right.

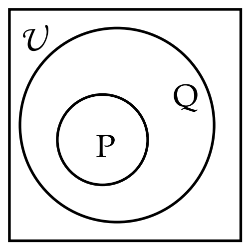
In Figure (1) the set P is inside the set Q, since all P's are Q's.

Figure (2) Shows no common regions, since no P's are Q's.

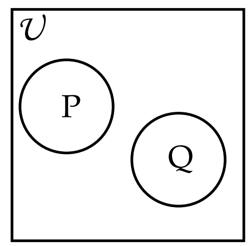
In Figure (3) there is at least one point (the dot) in common, since some P's are Q's

In Figure (4) at least one P represented by the dot, that is not in Q.

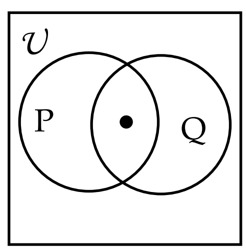
EXAMPLES



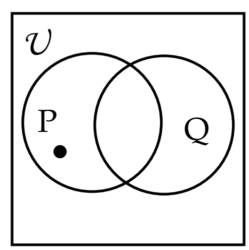
(1) All P's are Q's.



(2) No P's are Q's.



(3) Some P's are Q's.



(4) Some P's are not Q's.

In drawing conclusions about given statements follow this procedure:

1. Choose a letter to represent each statement
2. Determine which of the four forms the statement follows and diagram it
3. Write the conclusion shown in the final diagram in words or, determine which of the given
conclusions is represented by the final the diagram

CLAST EXAMPLES

Example

2. Given that:

i. No people who make assignments are friendly.
ii. All instructors make assignments.

Determine which conclusion can be logically deduced.

A. All instructors are friendly.

B. No instructor is friendly.

C. Some instructors are friendly.

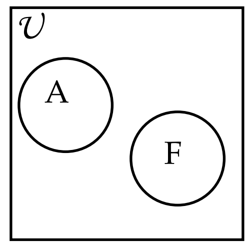
D. None of these answers.

We have to diagram the two premises I and ii and see if the conclusion is shown in the final diagram.

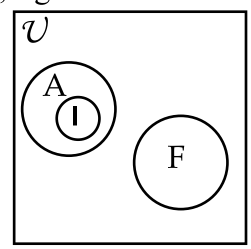
Note that if you have statements involving both universal quantifiers "All" and "No" the conclusion can never be an existential quantifier such as "some are" or "some are not", so response **C** is impossible.

Solution

If we let A be the set of people who make assignments and F the set of people who are friendly, the statement "No people who make assignments are friendly" is of the form (2) and diagrammed as shown below, left. If we let I be the set of all instructors, the statement "All instructors make assignments" is of the form (1) and diagrammed along with the first statement, as shown below, right.



No A's are F's



No A's are F's
All Is are A's

Can we conclude **A**, that all instructors are friendly? If this were the case, circle I would have to be inside circle F. It is not. Can we conclude **B**, that no instructor is friendly? If this were the case, the circle I and circle F would have no common regions. Since this is the case, the correct answer is **B.**

Example

3. Given that:

i. No football player who doesn't go to practice will play in the game.

ii. Some football players don't go to practice.

Which conclusion can be logically deduced?

Solution

Let F be the set of football players that do not go to practice and P the set of football players that will play in the game. The first premise tells us that the sets F and P have no common region. The second premise tell us that there is at least one element in the set F, which we shall represent by a dot. We have diagrammed the two premises.

(continued)

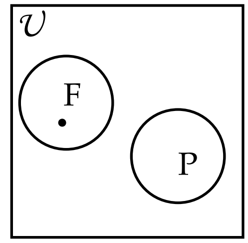
A. No football player will play in the game.

B. Some football players won't play in the game.

C. All football players who go to practice will play in the game.

D. None of these answers.

Note: When you have premises involving the existential quantifier "Some are" or "Some are not" the conclusion can never be a universal quantifier such as "All" or "No", so answers A and C can be eliminated.



The only possible answers are **B** and **D**. The statement in **B**, "Some football players won't play in the game" means that the set F is not empty. Since there is a dot inside F, this is the case and the correct answer is **B.**

The second type of problem can be solved by listing the general requirements as headings for a table in which each of the rows represents the individual cases to be examined. The case that meets all the criteria will be the solution. We illustrate the procedure next.

CLAST EXAMPLE

Example

4. Read the requirements and each applicant's qualifications for obtaining a $25,000 loan, then identify which of the applicants would qualify for the loan.

To qualify for a loan of $25,000 an applicant must have a gross income of $15,000 if single ($18,000 combined if married) and assets of at least $8000.

Mr. J is married with two children and makes $19,000 on his job. His wife does not work.

Ms. V and her husband have assets of $12,000. One makes $17,000, the other makes $6000. Mr. S is a bachelor and works at two jobs. He makes $13,000 on one job and $3000 on the other; he has $7000 in assets.

A. No one B. Mr. J C. Ms. V D. Mr. S

Solution

We list the requirements (Gross Income GI for Singles, combined income C for married and assets A) as headings for the columns in a table which also lists the applicants and their qualifications in each of the rows.

	Single GI = $15,000	**Married** C =$18,000	**Assets** A $\geq$ $8000
Mr. J		$19,000	?
Ms. V		$23,000	$12,000
Mr. S	$16,000		$7000

As you can see, the only person that qualifies is Ms. V with combined income of $17,000 + $6000 = $23,000 and assets of $12,000. Thus, the answer is **C.**

C. Selecting Conclusions to Make a Valid Argument

Objective IIIE1	CLAST SAMPLE PROBLEMS

DETERMINE A LOGICAL CONCLUSION TO MAKE THE ARGUMENT VALID

1. If it is Monday, I must go to school. It is Monday.
2. If I study hard, then I will get an A. I did not get an A.
3. I will take Math or English. I will not take Math.
4. If I study hard, then I will pass the CLAST. If I pass the CLAST then I will graduate.
5. If all students study, then no failing grades are given. Some failing grades are given.
6. If nobody passes the test, then some questions were unfair. If some questions were unfair, then the test should be curved.

In this CLAST skill a set of premises and several conclusions are given and we have to find which of the conclusions will result in a valid argument. The technique used to do this relies on learning several valid *argument forms*, writing the given premises in one of these forms and reaching a valid conclusion. When writing these argument forms, the premises are listed first, then a horizontal bar is drawn and the conclusion is stated following the three little dots, ∴, which mean "therefore". Here are the valid argument forms we need.

5	VALID ARGUMENT FORMS
MODUS PONENS (1) **(A MANNER OF AFFIRMING)** $p \to q$ $\underline{p \quad\quad}$ $\therefore \quad q$	***EXAMPLES*** When the premises are: (1) If you are engaged, you will get married. (2) You are engaged. A valid conclusion will be "You will get married," since the argument is of the form shown at the left.
MODUS TOLLENS (2) **(A MANNER OF DENYING)** $p \to q$ $\underline{\sim q \quad\quad}$ $\therefore \sim p$	When the premises are: (1) If you take Math, you pass the CLAST. (2) You did not pass the CLAST. A valid conclusion will be "You did not take Math."
DISJUNCTIVE SYLLOGISMS (3) $p \vee q \qquad\qquad p \vee q$ $\underline{\sim p \quad} \qquad\qquad \underline{\sim q \quad}$ $\therefore q \qquad\qquad\quad \therefore p$	When the premises are: (1) You take Math or English. (2) You do not take Math. A valid conclusion will be "You take English."
HYPOTHETICAL SYLLOGISMS (4) $p \to q$ $\underline{q \to r}$ $\therefore p \to r$	When the premises are: (1) If you take Math, you pass the CLAST. (2) If you pass the CLAST, you graduate. A valid conclusion will be "If you take Math, you graduate."

ANSWERS	
	1. I must go to school **2.** I did not study hard. **3.** I will take English. **4.** If I study hard, then I will graduate. **5.** Some students do not study **6.** If nobody passes the test, then the test should be curved.

You need **not** remember the names of these argument forms (note that we numbered them 1, 2, 3 and 4 for easy reference), but you **do** need to memorize the argument form itself.

□CLAST EXAMPLES

Example

5. Select the conclusion that will make the following argument valid.

If prices go up then consumers will not shop. If consumers will not shop, then a recovery will not occur.

A. If prices do not go up, then a recovery will occur.

B. If prices go up, then a recovery will not occur.

C. If consumers will not shop, then prices go up.

D. If prices go up, then a recovery will occur.

Solution

Let p be "prices go up," c be "consumers will not shop" and r be "a recovery will not occur." The argument can be symbolized as:

$$\begin{array}{c} p \to c \\ \underline{c \to r} \\ \therefore\ ? \end{array}$$

According to argument form (4), a valid conclusion for this argument is $p \to r,$ that is, "If prices go up then a recovery will not occur."

This is choice **B.**

Example

6. Select the conclusion that will make the following argument valid.

If all students take Math, then some new instructors are needed. No new instructors are needed.

A. No students take Math.

B. If there are no new instructors, no students take Math.

C. Some students do not take Math.

D. Some students take Math.

Solution

Let m be "All students take Math" and n be "Some new instructors are needed". Note that the negation of "Some new instructors are needed" is "No new instructors are needed". With this in mind, the argument can be symbolized by:

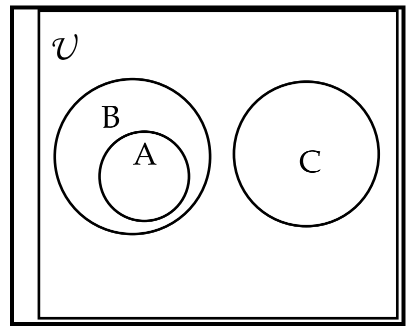
$$\begin{array}{c} m \to n \\ \underline{\sim n} \\ \therefore\ ? \end{array}$$

According to argument form (2), a valid conclusion for this argument is $\sim m$, that is, $\sim$ (All students take Math), which is "Some students do not take Math", choice **C.**

Section 5.2 Exercises

WARM-UPS A

In Exercises 1- 6 refer to the diagram at the right and assume that none of the regions is empty.



1. What is the relationship between the elements of A and those of B?

2. What is the relationship between the elements of C and those of A?

3. What is the relationship between the elements of C and those of B?

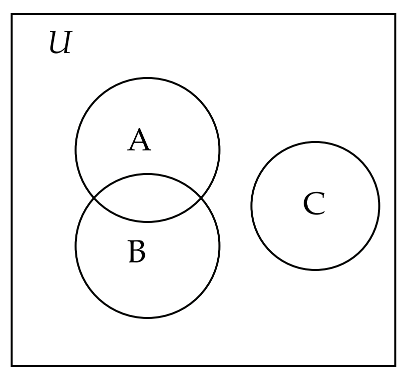
4. What is the relationship between the elements of A and those of U?

5. Is there an element that is in A, B and C?

6. Can there be elements that are not in A, B or C?

CLAST PRACTICE A **PRACTICE PROBLEMS: Chapter 5, # 11**

7. Sets A, B, C and U are related as shown in the diagram at the right. Which statement is true assuming none of the regions is empty?



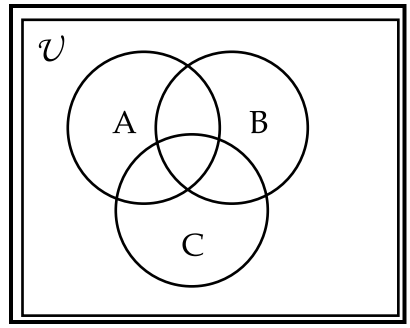
A. If an element is a member of set B, it is also in set A.

B. Any element that is a member of set A is also in set U.

C. If an element is a member of set C, it may also be in set A or set B.

D. If an element is a member of set A, it cannot be a member of set B.

8. Sets A, B, C and U are related as shown in the diagram at the right. What statement can be made regarding the relationship among the sets assuming none of the regions is empty?



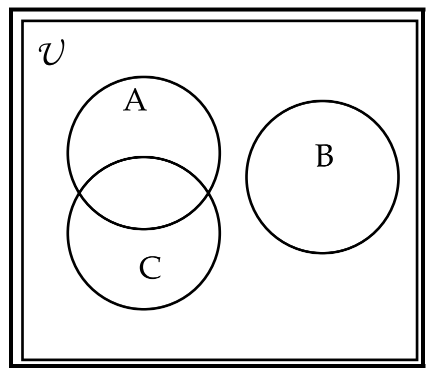
A. It is possible for an element to be a member of sets A, B and C.

B. Any element that is a member of set U, is also in set C.

C. It is not possible for an element to be a member of sets A, B and C.

D. If an element is a member of set A, it is also a member of set B.

9. Sets A, B, C and U are related as shown in the diagram at the right. Which statement is true assuming none of the regions is empty?



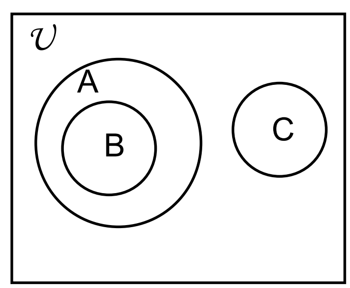
A. No element is a member of all three sets A, B and C.

B. All elements which are members of set B are also in C.

C. No element which is a member of set A is also in C.

D. Any element that is a member of set C is also a member of set A.

$\boxed{10.}$ Sets A, B, C and U are related as shown in the diagram at the right. Which statement is true assuming none of the regions is empty?



i. All elements of B are elements of A.
ii. No element of A is an element of C.
iii. All elements of C are elements of U

A. i only B. i and ii only

C. i and iii only D. i, ii, and iii

WARM-UPS B

11. Given that: i. No people who assign work are lovable.
    ii. All supervisors assign work.

    What conclusion can be logically deduced?

12. Given that: i. All politicians run for office.
    ii. No people who run for office are reliable.

    What conclusion can be logically deduced?

13. Given that: i. No employee who doesn't arrive for work on time will be promoted.
    ii. Some employees don't arrive for work on time.

    What conclusion can be logically deduced?

14. Given that: i. Some students are intelligent
    ii. All intelligent people are snobs.

    What conclusion can be logically deduced?

15. Read the requirements and each applicant's qualifications for obtaining a $40,000 loan, then identify which of the applicants would qualify for the loan.

    To qualify for a loan of $40,000 an applicant must have a gross income of $27,000 if single ($33,000 combined if married) and assets of at least $12,000.

    Ms. A and her husband have assets of $12,000. One makes $28,000, the other makes $10,000.
    Mr. B is married with two children and makes $30,000 on his job. His wife does not work.
    Mr. C is a bachelor and works at two jobs. He makes $20,000 on one job and $2000 on the other; he has $13,000 in assets.

16. Read the requirements and each applicant's qualifications for obtaining a $30,000 loan, then identify which of the applicants would qualify for the loan.

    To qualify for a loan of $30,000 an applicant must have a gross income of $18,000 if single ($20,000 combined if married) and assets of at least $10,000.

    Mr. S is a bachelor and works at two jobs. He makes $10,000 on one job and $8000 on the other; he has $10,000 in assets.
    Ms. A and her husband have assets of $5,000. One makes $20,000, the other makes $18,000.
    Mr. J is married with two children and makes $25,000 on his job. His wife makes $12,000.

□CLAST PRACTICE B **PRACTICE PROBLEMS: Chapter 5, # 12-13**

17. Given that: i. No music student who doesn't practice will learn to play well.
ii. Some music students don't practice.

Determine which conclusion can be logically deduced.

A. Some music students won't learn to play well.
B. No music student will learn to play well.
C. All music students who practice will learn to play well.
D. None of these answers.

18. Given that: i. No college student who doesn't go to class will pass the course.
ii. Some college students don't go to class.

Determine which conclusion can be logically deduced.

A. No college student will pass the course.
B. All college students who go to class will pass the course.
C. Some college students won't pass the course.
D. None of these answers.

19. Given that: i. No student who works late can be a party animal.
ii. All supervisors work late.

Determine which conclusion can be logically deduced.

A. All supervisors are party animals. B. Some supervisors are party animals.

C. No supervisors are party animals. D. None of these answers.

$\boxed{20.}$ Given that: i. No people who teach classes are dumb
ii. All teachers teach classes

Determine which conclusion can be logically deduced.

A. All teachers are dumb. B. No teacher is dumb.

C. Some teachers are dumb. D. None of these answers

21. Read the requirements and each applicant's qualifications for obtaining a \$25,000 loan, then identify which of the applicants would qualify for the loan.

    To qualify for a loan of \$25,000 an applicant must have a gross income of \$15,000 if single (\$18,000 combined if married) and assets of at least \$8000.

    Ms. A and her husband have assets of \$12,000. One makes \$17,000, the other makes \$6000.
    Mr. J is married with three children and makes \$19,000 on his job. His wife makes \$12,000.
    Mr. S is a bachelor and works at two jobs. He makes \$13,000 on one job and \$3000 on the other; he has \$7000 in assets.

A. Ms. A B. Mr. J C. No one D. Mr. S

WARM-UPS C

22. Find a conclusion that will make the argument valid.

    If you go to college then you get a good job.
    You go to college.

23. Find a conclusion that will make the argument valid.

    I will go to college or to trade school.
    I will not go to trade school.

24. Find a conclusion that will make the argument valid.

    If you are rich, then you travel to Europe.
    If you travel to Europe, then you visit France.

CLAST PRACTICE C **PRACTICE PROBLEMS: Chapter 5, # 14**

$\boxed{25.}$ Select the conclusion that will make the argument valid.

If I drive to work, then I will not be late.
If I am not late, then I do not lose any pay.

A. If I am not late, then I drive to work.
B. If I do not lose any pay, then I drive to work.
C. If I drive to work, then I do not lose any pay.
D. If I do not drive to work, then I lose some pay.

26. Select the conclusion that will make the argument valid.

If the electricity is off, the computer will not work.
The electricity is off.

A. If the electricity is not off, the computer will work.
B. If the computer will not work, the electricity is off.
C. The computer will not work.
D. The computer will work.

27. Select the conclusion that will make the argument valid.

If the Bears win this game, then they will be in the playoffs.
If they are in the playoffs, then their owners will be proud.

A. If their owners are proud, then the Bears will be in the playoffs.
B. If their owners were proud, then the Bears won this game.
C. If the Bears do not win this game, then their owners will not be proud.
D. If the Bears win this game, then their owners will be proud.

28. Select the conclusion that will make the argument valid.

If all people pay their bills on time, then no collection agencies are needed.
Unfortunately, some collection agencies are needed.

A. Some people pay their bills on time.
B. Some people do not pay their bills on time.
C. If there are no collection agencies, then all people will pay their bills on time.
D. All people pay their bills on time.

5.3 VALID AND INVALID ARGUMENTS

In this section we shall study two different CLAST skills. The first one will provide premises for an argument and ask us for the logical conclusion while the second one will provide several arguments with true conclusions and ask us to determine which one of them is valid.

A. Deducing Conclusions

Objective IVE1 — CLAST SAMPLE PROBLEMS

STUDY THE GIVEN INFORMATION AND SELECT A LOGICAL CONCLUSION.
IF NO LOGICAL CONCLUSION IS GIVEN, SELECT OPTION D.

1. If you go to a party, then you have a good time. If you have a good time, then you study better. You go to a party.

A. You study better
B. You don't study better
C. You don't have a good time
D. None of the above is warranted

2. Tom goes to the beach or Maria goes to the movies. If Tom goes to the beach, then he gets a sun tan. Maria does not go to the movies.

A. Tom does not go to the beach
B. Tom gets a sun tan
C. Tom does not get a sun tan
D. None of the above is warranted

3. If Judy goes to the beach, she gets a sun tan. If she gets a sun tan, she wears her white dress. Judy wears her white dress.

A. Judy does not go to the beach
B. Judy does not get a sun tan
C. Judy goes to the beach
D. None of the above is warranted

4. It rains, then it pours. If it pours, people will go to the movies. People go to the mall or it rained. People didn't go to the mall.

A. It did not rain
B. It did not pour
C. People go to the movies
D. None of the above is warranted

ANSWERS **1.** A **2.** B **3.** D **4.** C

The arguments in this skill will be of two kinds: Those using the connectives we have discussed ("if-then," "and," "or"" and so on) and those using the universal or existential quantifiers ("All," "none," "some are," "some are not"). Here is the procedure we need to handle the first type of argument.

1	FINDING CONCLUSIONS FOR ARGUMENTS INVOLVING CONNECTIVES
PROCEDURE	***EXAMPLE***
1. Read the given information and label the statements as p, q, r and so on. 2. Write each of the premises using p, q, r and so on. 3. Use the 5 argument forms $\begin{array}{ccccc} p \to q & p \to q & p \vee q & p \vee q & p \to q \\ \underline{p} & \underline{\sim q} & \underline{\sim p} & \underline{\sim q} & \underline{q \to r} \\ \therefore\ q & \therefore\ \sim p & \therefore\ q & \therefore\ p & \therefore p \to r \end{array}$ to deduce the desired conclusion. Note: If one of the premises involves a conjunction such as $p \wedge q$ it is usually easier to find the conclusion if the conjunction is written as two separate premises p and q.	Consider the argument: If you study Math, you will be successful. If you study Art, you will be successful. You study Art and not Math. What conclusion can you reach? 1. Let m be: You study Math s be: You will be successful a be: You study Art. 2. The premises are written as: (1) $m \to s$ (2) $a \to s$ (3) a (4) $\sim m$ 3. Combining (2) and (3), we can write: $a \to s$ $\underline{a}$ $\therefore$? Which, according to the first argument form, has the conclusion s: You will be successful. Thus, a valid conclusion for this argument is "You will be successful."

□**CLAST EXAMPLE**

Example

1. Study the information given below. If a logical conclusion is given, select that conclusion.

If you ask questions, you will learn a lot.
If you read often, you will ask questions.

A. If you learn a lot, you will ask questions.

B. You will learn a lot.

C. You will not learn a lot.

D. If you read often, you will learn a lot.

Solution

Let a be: You ask questions
L be: You will learn a lot
r be: You read often.

Note that even though the word "then" is not present, a comma is used in its place so that the given statements can be written as:

(1) $a \to L$
(2) $\underline{r \to a}$
$\therefore$?

If we write statement (2) first, we have:

(2) $r \to a$
(1) $\underline{a \to L}$
$\therefore\ r \to L$

If you read often, you will learn a lot. The correct answer is **D.**

The second type of argument uses universal and existential quantifiers. To discover the conclusion, we diagram the premises using a Venn or Euler diagram and see which conclusion is present.

CLAST EXAMPLE

Example

2. Study the information given below. If a logical conclusion is given, select that conclusion. If none of the conclusions given is warranted, select that option.

All sailors are swimmers.
All swimmers wear life jackets.
Sally is wearing a life jacket. Therefore:

A. Sally is not a sailor.

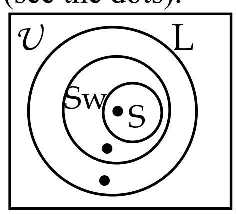
B. Sally is a sailor.

C. Sally is a swimmer.

D. None of these is warranted.

Solution

Let S be the set of all sailors, Sw be the set of all swimmers, and L be the set of people wearing life jackets. All sailors are swimmers mean that the set S is inside the set Sw. All swimmers wear life jackets mean that the set Sw is inside the set L, so S is inside Sw which is inside L. Sally wears a life jacket means that Sally is inside L but there are three possibilities (see the dots).



Clearly, Sally might or might not be a sailor. Also she might or might not be a swimmer. Thus, the answer is **D.**

B. Invalid Arguments

Objective IIE4 — **CLAST SAMPLE PROBLEMS**

DETERMINE THE VALIDITY OF THE GIVEN ARGUMENT USING A DIAGRAM

1. All integers are real numbers. 0 is an integer. Therefore, 0 is a real number

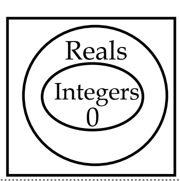
2. All integers are real numbers. 0 is a real number. Therefore, 0 is an integer.

3. All integers are rational numbers. All rational numbers are real numbers. Therefore, all integers are real numbers.

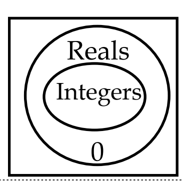
4. All integers are real numbers. All rational numbers are real numbers. Therefore, all integers are rational numbers.

ANSWERS

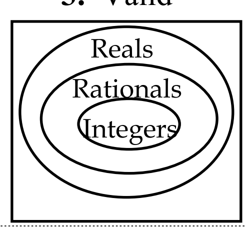
1. Valid



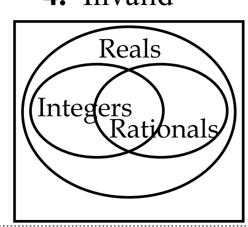
2. Invalid



3. Valid



4. Invalid



The five argument forms we have discussed (see box **1)** may contain statements that are true or false. If these statements can be written in one of the five forms, however, a valid argument will result regardless of the truth or falsity of the individual statements. This CLAST skill will present some arguments in which the conclusion is true but the argument is invalid. To prove this we shall show that after the premises are diagrammed, the conclusion will not be present.

CLAST EXAMPLE

Example

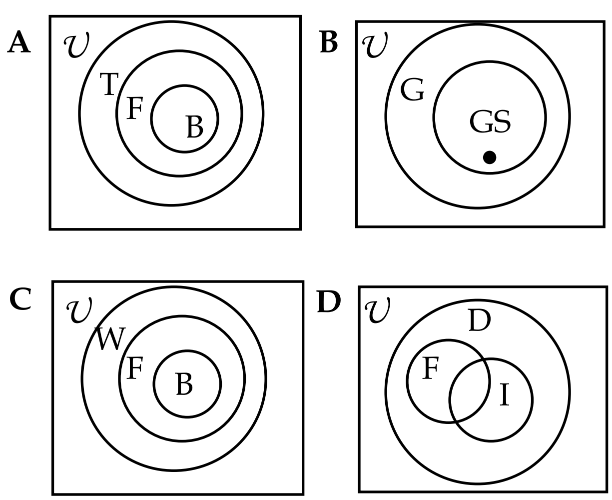
3. All of the following arguments A-D have true conclusions, but one of the arguments is not valid. Select the argument that is not valid.

A. All fish taste good and all bass are fish. Therefore, all bass taste good.

B. Every Girl Scout is a girl. Susan is a Girl Scout. Therefore, Susan is a girl.

C. All fish live in water and all bass are fish. Therefore, all bass live in water.

D. All flies are dirty and insects are dirty. Therefore, all flies are insects.



Solution

We have diagrammed the four arguments.

In **A** the set of fish F must be inside the set of fish that taste good T and the set of bass B must be inside the set of fish F as shown. Since the set B is inside the set T, it does follow that all bass taste good.

In **B** the set of all Girl Scouts GS must be inside the set of girls G and Susan, represented by the dot, is a Girl Scout, so she must be inside the circle GS. Since Susan is inside the set G it does follow that Susan is a girl.

In the diagram for **C** the set of fish F is inside the set of things living in water W, since all fish live in water. The set of bass B is inside the set of fish F, since all bass are fish. Since the set B is inside the set W we can conclude that all bass live in water.

The invalid one must be **D!** Let's see why. Since all flies F are dirty D, set F is inside set D. Since all insects I are dirty the set I is inside the set D. However, there is no indication that F is inside of I or vice versa so we have drawn them as shown. The conclusion "All flies are insects," which is a true statement, does not follow from the diagram, so the argument is invalid.

Section 5.3 Exercises

□WARM-UPS A
In Problems 1-6, find a valid conclusion that follows from the premises.

1. The car will start or it will need a new battery.
The car did not need a new battery.

2. If we want to save money, we will shop at the discount store.
We will not shop at the discount store.

3. If the battery is dead, the car will not start.
The battery is dead.

4. If you use the right bait, you will catch some fish.
If you catch some fish, you will have a nice dinner.

5. All dogs are mammals.
All mammals have legs.

6. No Math student who doesn't go to class will pass the course.
Some Math students don't go to class.

□CLAST PRACTICE A **PRACTICE PROBLEMS: Chapter 5, #15-16**

7. Study the given information. If a logical conclusion is given, select that conclusion.

If you are charming, you will have friends. If you help others, you will have friends.
You help others and you are not charming. Therefore,

A. You will have friends. B. You are charming.

C. You will not have friends. D. You will not help others.

8. Study the given information. If a logical conclusion is given, select that conclusion.

If you ask questions, you will learn a lot. If you read often, you will learn a lot.
You do not read often but you do ask questions. Therefore,

A. You listen to others. B. You will learn a lot.

C. You will not learn a lot. D. None of these answers.

9. Study the given information. If a logical conclusion is given, select that conclusion.

All pilots are navigators.
All navigators read maps.
John is a navigator. Therefore,

A. John is a pilot. B. John reads maps.
C. John cannot navigate. D. None of these answers.

10. Study the information. If a logical conclusion is given, select that conclusion.

All parents are good cooks.
All good cooks read cookbooks.
John is reading a cook book. Therefore,

A. John cannot cook. B. John can cook.
C. John is a parent. D. None of these answers.

WARM-UPS B
In Problems 11-16, decide which arguments are valid and which are not valid.

11. Every junior in an institution of higher learning in Florida has to pass the CLAST.
Jackie is a junior in an institution of higher learning in Florida.
Therefore, Jackie has to pass the CLAST.

12. All penguins have legs and all birds have legs.
Therefore, all penguins are birds.

13. Pam or Lois is an Olympic swimmer. Lois is not an Olympic swimmer.
Therefore, Pam is an Olympic swimmer.

14. All trees are plants. All plants should be watered.
Therefore, all trees should be watered.

15. All pirates were sailors. Some women were pirates.
Therefore, some women were sailors.

16. No person who teaches classes make money. All teachers teach classes.
Therefore, no teachers make money.

CLAST PRACTICE B **PRACTICE PROBLEMS: Chapter 5, #17-18**

17. All of the following arguments A-D have true conclusions, but one of the arguments is not valid. Select the argument that is not valid.

    A. All monsters are scary. Some kids are monsters.
    Therefore, some kids are scary.

    B. All cats have legs and all mammals have legs.
    Therefore, all cats are mammals.

    C. All cats are mammals and all mammals have legs.
    Therefore, all cats have legs.

    D. Tallahassee is in Florida and Florida is in the United States.
    Therefore, Tallahassee is in the United States.

18. All of the following arguments A-D have true conclusions, but one of the arguments is not valid. Select the argument that is not valid.

    A. All robbers are criminals and all criminals break the law.
    Therefore, all robbers break the law.

    B. All hazards are harmful. All pollution is a hazard.
    Therefore, all pollution is harmful.

    C. All horses are mammals and all horses have hair.
    Therefore, all mammals have hair.

    D. All birds have eyes and all penguins are birds.
    Therefore, all penguins have eyes.

19. All of the following arguments A-D have true conclusions, but one of the arguments is not valid. Select the argument that is not valid.

    A. All boys are noisy and children are noisy.
    Therefore, all boys are children.

    B. All cats have tails and all Siamese are cats.
    Therefore, all Siamese have tails.

    C. Every dog is an animal. The beagle is a dog.
    Therefore, the beagle is an animal.

    D. All dogs are beautiful and all Poodles are dogs.
    Therefore, all Poodles are beautiful.

$\boxed{20.}$ All of the following arguments have true conclusions, but one of the arguments is not valid. Select the argument that is not valid.

A. All birds have feathers and all penguins are birds.
Therefore, all penguins have feathers.

B. All fleas have six legs and insects have six legs.
Therefore, all fleas are insects.

C. All birds have legs and all penguins are birds.
Therefore, all penguins have legs.

D. Every borough of New York City is in New York State.
The Bronx is a borough of New York City.
Therefore, the Bronx is in New York State.

EXTRA CLAST PRACTICE

21. Study the information given below. If a logical conclusion is given, select that conclusion. If none of the conclusions is warranted, select option D.

If I pass this test, then I will pass this course. I pass this test or I go the movies. I did not go to the movies.

A. I did not pass this test.
B. I did not pass this course.
C. I did pass this course.
D. None of the conclusions is warranted.

22. Study the information given below. If a logical conclusion is given, select that conclusion. If none of the conclusions is warranted, select option D.

If all tests are proctored, then no student cheats. If no student cheats, then no A's are given. Some A's are given.

A. No student cheats.
B. Some tests are not proctored.
C. All tests are proctored.
D. None of the conclusions is warranted.

23. Study the information given below. If a logical conclusion is given, select that conclusion. If none of the conclusions is warranted, select option D.

Tom eats meat or he eats vegetables.
If Tom eats vegetables, then he is healthy.
If Tom is healthy, then he can run the marathon.
Tom does not eat vegetables.

A. Tom does not eat meat.

B. Tom is healthy.

C. If Tom runs the marathon, then he eats vegetables.

D. None of the conclusions is warranted.

24. All of the following arguments A-D have true conclusions, but one of the arguments is not valid. Select the argument that is not valid.

A. All birds have wings and all sparrows are birds; therefore, all sparrows have wings.

B. All sparrows have wings and all birds have wings; therefore, all sparrows are birds.

C. All snakes are reptiles and all reptiles have a scaly skin; therefore, snakes have a scaly skin.

D. All mammals have hair. Cows are a mammal. Therefore, cows have hair.

25. All of the following arguments A-D have true conclusions, but one of the arguments is not valid. Select the argument that is not valid.

A. All dogs are predatory. Rottweilers are predatory. Therefore, Rottweilers are dogs.

B. All cats are felines and all felines are mammals; therefore, all cats are mammals.

C. All frogs are amphibians and all amphibians breathe by lungs, gill, or skin; therefore, all frogs breath by lungs, gills, or skin.

D. All mammals have hair and all dogs are mammals; therefore, all dogs have hair.