

# 1

## ARITHMETIC

### CLAST MATHEMATICS COMPETENCIES

- IA1a: Add and subtract rational numbers
- IA1b: Multiply and divide rational numbers
- IA2a: Add and subtract rational numbers in decimal form
- IA2b: Multiply and divide rational numbers in decimal form
- IA3: Calculate percent increase and percent decrease
- IA4: Solve the sentence  $a\%$  of  $b$  is  $c$ , where values for two of the variables are given
- IIA1: Recognize the meaning of exponents
- IIA2: Recognize the role of the base number in determining place value in the base-ten numeration system
- IIA3: Identify equivalent forms of positive rational numbers involving decimals, percents and fractions
- IIA4: Determine the order relation between real numbers
- IIA5: Identify a reasonable estimate of a sum, average, or product of numbers
- IIIA1: Infer relations between numbers in general by examining particular number pairs
- IVA1: Solve real world problems which do not require the use of variables and which do not involve percent
- IVA2: Solve real-world problems which do not require the use of variables and do require the use of percent
- IVA3: Solve problems that involve the structure and logic of arithmetic



## 1.1 OPERATIONS WITH RATIONAL NUMBERS

In this chapter we shall discuss the rational numbers (including fractions, decimals and percents), operations with these numbers and word problems involving them. We start by giving you the basic terminology. There is also a Glossary at the end of the book.

T	TERMINOLOGY--TYPES OF NUMBERS
<b>TYPES OF NUMBERS</b>	<b>EXAMPLES</b>
The <b>natural or counting</b> numbers, <b>1, 2, 3</b> and so on (or <b>1, 2, 3, ...</b> )	8, 49 and 487 are <b>natural</b> numbers.
The <b>whole</b> numbers, <b>0, 1, 2, 3 ...</b>	7, 94 and 1349 are <b>whole</b> numbers.
The <b>integers</b> (the whole numbers and their <b>additive inverses</b> or <b>opposites</b> ) ... <b>-3, -2, -1, 0, 1, 2, 3 ...</b>	-90, 48, 0 and 1876 are <b>integers</b> Note that 90 and -90 are <b>additive inverses</b> or <b>opposites</b> of each other.
<b>Rational numbers</b> are numbers that can be written in the form $\frac{a}{b}$ , where a and b are integers and b is not 0.	34, -45, $\frac{3}{4}$ , $-\frac{1}{5}$ , 0, 3.47, -8.5 and $1\frac{2}{7}$ are <b>rational</b> numbers. Note: all integers are rational numbers.
<b>Prime numbers</b> are numbers which have exactly two <i>different</i> divisors: themselves and 1.	2, 3, 5, 7, 11, 13 and 17 are <b>prime</b> .
<b>Composite numbers</b> are natural numbers greater than 1 that are <b>not</b> prime.	20, 4, 48 and 60 are <b>composite</b> .
A <b>fraction</b> is a number $\frac{a}{b}$ indicating the quotient of the <b>numerator a</b> divided by the <b>denominator b</b> . When <b>a</b> is less than <b>b</b> , the fraction is a <b>proper</b> fraction.	$\frac{3}{4}$ is a <b>proper</b> fraction with a numerator of 3 and a denominator of 4. Note that $\frac{6}{5}$ and $\frac{10}{10}$ are <u>not</u> <b>proper fractions</b> .
A <b>mixed number</b> is an indicated sum of a whole number and a fraction.	$2\frac{3}{5} = 2 + \frac{3}{5}$ is a <b>mixed number</b> .

## A. Adding and Subtracting Rational Numbers

Objective IA1a	CLAST SAMPLE PROBLEMS			
1. $3\frac{4}{7} + 1\frac{2}{5}$	2. $-\frac{2}{3} + \frac{3}{5}$	3. $-9 + 2\frac{3}{4}$	4. $-\frac{1}{4} - \left(3\frac{7}{8}\right)$	
<i>ANSWERS ARE GIVEN AT THE END OF THE PAGE</i>				

We are now ready to perform the four fundamental operations using rational numbers. To do this we need the rules that follow.

1	EQUIVALENT FRACTIONS	
<b>RULE</b>	<b>EXAMPLES</b>	
$\frac{a}{b} = \frac{a \times c}{b \times c}$	$\frac{5}{7} = \frac{5 \times 2}{7 \times 2} = \frac{10}{14}$ and	
You can multiply or divide the numerator <b>a</b> and denominator <b>b</b> of a fraction by a non-zero number <b>c</b> and obtain an equivalent fraction.	$\frac{12}{20} = \frac{12 \div 4}{20 \div 4} = \frac{3}{5}$	
$\frac{a \div c}{b \div c} = \frac{a}{b}$	To reduce $\frac{10}{15}$ , write $\frac{10}{15} = \frac{10 \div 5}{15 \div 5} = \frac{2}{3}$ .	
To <b>reduce</b> fractions, divide numerator and denominator by the same number non-zero number, <b>c</b> in this case.	You can also write $\frac{\cancel{10}^2}{\cancel{15}_3} = \frac{2}{3}$	

2	ADDITION OF SIGNED NUMBERS	
<b>RULE</b>	<b>EXAMPLES</b>	
1. If the numbers have the <b>same</b> sign, <b>add</b> them and give the sum the <b>common</b> sign.	$3 + 8 = +(3 + 8) = 11$ $-3 + (-8) = -(3 + 8) = -11$	
2. If they have <b>different</b> signs, <b>subtract</b> the smaller from the larger and give the sum the sign of the <b>larger</b> number.	$-3 + 8 = +(8 - 3) = 5$ Use the sign of 8, the larger.	
	$3 + (-9) = -(9 - 3) = -6$ Use the sign of 9, the larger.	

3	SUBTRACTION OF SIGNED NUMBERS	
<b>RULE</b>	<b>EXAMPLES</b>	
$a - b = a + (-b)$ , that is, to subtract the number <b>b</b> from the number <b>a</b> , add the additive inverse of <b>b</b> , that is, add <b>(-b)</b>	$3 - 5 = 3 + (-5) = -(5 - 3) = -2$ $-5 - 4 = -5 + (-4) = -(5 + 4) = -9$ $-3 - (-7) = -3 + 7 = +(7 - 3) = 4$	
<b>Note:</b> $a - (-b) = a + b$		

ANSWERS	1. $4\frac{34}{35}$	2. $-\frac{1}{15}$	3. $-6\frac{1}{4}$	4. $-4\frac{1}{8}$
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4	<b>CHANGING MIXED NUMBERS TO FRACTIONS AND VICE VERSA</b>	
<b>RULE</b>	<b>EXAMPLES</b>	
<p>To change a <b>mixed number</b> to an improper fraction, multiply the denominator by the whole number part and add the numerator to obtain the <u>new</u> numerator. Use the same denominator.</p> <p>To change an improper fraction to a <b>mixed number</b>, divide the numerator by the denominator. The quotient is the whole number part, the remainder is the numerator in the fraction part. The denominator remains the same.</p>	$3\frac{4}{5} = \frac{5 \times 3 + 4}{5} = \frac{19}{5}$ $2\frac{7}{10} = \frac{10 \times 2 + 7}{10} = \frac{27}{10}$ $\frac{14}{3} = 4\frac{2}{3}, \text{ because } 14 \text{ divided by } 3 \text{ is } 4, \text{ with a remainder of } 2.$ $\frac{26}{7} = 3\frac{5}{7} \text{ because } 26 \text{ divided by } 7 \text{ is } 3 \text{ with a remainder of } 5.$	

5	<b>FINDING THE LEAST COMMON DENOMINATOR (LCD)</b>	
<b>METHOD 1</b>	<b>EXAMPLES</b>	
<p>Check the multiples of the <b>greater</b> denominator until you get a multiple of the smaller denominator. This number is the LCD. Remember that the multiples of 30, for example, are:</p> $1 \times 30 = \mathbf{30}, \quad 2 \times 30 = \mathbf{60},$ $3 \times 30 = \mathbf{90}, \quad 4 \times 30 = \mathbf{120} \text{ and so on.}$ <p>The multiples of 24 are:</p> $1 \times 24 = \mathbf{24}, \quad 2 \times 24 = \mathbf{48},$ $3 \times 24 = \mathbf{72}, \quad 4 \times 24 = \mathbf{96} \text{ and so on.}$	<p>To find the LCD of <math>\frac{1}{30}</math> and <math>\frac{7}{24}</math>, write the multiples of 30 until you get a multiple of 24. The multiples of 30 are:</p> $1 \times 30 = 30 \text{ (not a multiple of } 24)$ $2 \times 30 = 60 \text{ (not a multiple of } 24)$ $3 \times 30 = 90 \text{ (not a multiple of } 24)$ $4 \times 30 = 120 \text{ (a multiple of } 24, \text{ since } 5 \times 24 = 120).$ <p>Thus, the LCD of <math>\frac{1}{30}</math> and <math>\frac{7}{24}</math> is 120.</p>	

6	<b>FINDING THE LEAST COMMON DENOMINATOR (LCD)</b>	
<b>METHOD 2</b>	<b>EXAMPLES</b>	
<p>Write each denominator as a product of primes using exponents. The LCD is the product of the <b>highest</b> powers of the primes in the factorization. (It helps if you write the powers of the prime factors in a column so they are easier to compare.)</p>	<p>Write 30 and 24 as products of primes.</p> $30 = 2 \times \underline{15} = 2 \times \underline{3} \times \underline{5}$ $24 = 2 \times \underline{12} = 2 \times \underline{2} \times \underline{6} = 2 \times \underline{2} \times \underline{2} \times \underline{3}$ <p>Writing <math>2 \times 2 \times 2</math> as <math>2^3</math> we have</p> $30 = 2 \times 3 \times 5$ $24 = 2^3 \times 3$ <p>The LCD is <math>2^3 \times 3 \times 5 = 120</math> as before.</p>	

7	<b>RULES FOR ADDING AND SUBTRACTING FRACTIONS</b>	
<b>RULE</b>	<b>EXAMPLES</b>	
To add or subtract fractions with the <b>same</b> denominator, write	$\frac{3}{5} + \frac{1}{5} = \frac{3+1}{5} = \frac{4}{5}$	
$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b} \quad \text{and} \quad \frac{a}{b} - \frac{c}{b} = \frac{a-c}{b}$	and $\frac{3}{5} - \frac{1}{5} = \frac{3-1}{5} = \frac{2}{5}$	
To add or subtract fractions with <b>different</b> denominators, write the given fractions as equivalent ones with the LCD as the common denominator and then add or subtract.	To add $\frac{3}{7} + \frac{5}{12}$ , find the LCD of 7 and 12 which is 84, write the fractions with 84 as denominators and add. Thus,	
	$\frac{3}{7} = \frac{3 \times 12}{7 \times 12} = \frac{36}{84}$	
	$\frac{5}{12} = \frac{5 \times 7}{12 \times 7} = \frac{35}{84}$	
	$\begin{array}{r} \frac{36}{84} \\ + \frac{35}{84} \\ \hline \frac{71}{84} \end{array}$	

8	<b>TO ADD OR SUBTRACT MIXED NUMBERS</b>	
<b>RULE</b>	<b>EXAMPLES</b>	
To add or subtract mixed numbers, add or subtract the fractional part first, then add or subtract the whole number part. <b>Note:</b> If one of the numbers is a fraction, we can still use this procedure. Thus,	To add $3\frac{3}{4}$ and $2\frac{5}{8}$ , add $\frac{3}{4}$ and $\frac{5}{8}$ first.	
$1\frac{1}{3} + \frac{2}{5} = 1\frac{5}{15} + \frac{6}{15} = 1\frac{11}{15}$	The LCD is 8, so we write	
	$3\frac{3}{4} = 3\frac{6}{8}$	
	$+ 2\frac{5}{8} = + 2\frac{5}{8}$	
	$\begin{array}{r} \phantom{3} \\ + 2 \\ \hline 5\frac{11}{8} = 5 + \frac{11}{8} = 5 + 1\frac{3}{8} = 6\frac{3}{8} \end{array}$	

9	<b>SUBTRACTING MIXED NUMBERS</b>	
<b>RULE</b>	<b>EXAMPLES</b>	
Sometimes you may have to "borrow" from the whole number part. To find $6\frac{1}{4} - 3\frac{4}{5}$ find the LCD 20, and write	To subtract $3\frac{4}{5}$ from $6\frac{1}{4}$ , we find the LCD, 20, as usual, then write:	
$6\frac{1}{4} - 3\frac{4}{5} = 6\frac{5}{20} - 3\frac{16}{20}$ ; but we can not	$6\frac{1}{4} = 6\frac{5}{20} = 5\frac{25}{20}$	
subtract $\frac{16}{20}$ from $\frac{5}{20}$ , so we "borrow" and	$- 3\frac{4}{5} = - 3\frac{16}{20} = - 3\frac{16}{20}$	
write $6\frac{5}{20} = 5 + \frac{20}{20} + \frac{5}{20} = 5\frac{25}{20}$ .	$\begin{array}{r} \phantom{5} \\ - 3 \\ \hline 2\frac{9}{20} \end{array}$	

**CLAST EXAMPLES****Example**

1.  $2\frac{1}{2} + \frac{1}{9} =$

A.  $\frac{2}{11}$     B.  $2\frac{11}{18}$     C.  $2\frac{5}{9}$     D.  $\frac{11}{18}$

Note that  $2\frac{1}{2} + \frac{1}{9}$  must be more than 2, so the answer has to be **B** or **C**.

**Solution**

The LCD of 2 and 9 is 18.

$$\begin{array}{r} 2\frac{1}{2} = 2\frac{9}{18} \\ + \frac{1}{9} = \frac{2}{18} \\ \hline 2\frac{11}{18} \end{array}$$

The answer is **B**.

**Example**

2.  $6 - 2\frac{1}{3} =$

A.  $4\frac{2}{3}$     B.  $4\frac{1}{3}$     C.  $3\frac{2}{3}$     D.  $3\frac{1}{3}$

Note that  $6 - 2$  is 4, so  $6 - 2\frac{1}{3}$  must be less than 4. It has to be **C** or **D**.

**Solution**

First write 6 as  $5 + 1 = 5 + \frac{3}{3} = 5\frac{3}{3}$

$$\begin{array}{r} 6 = 5\frac{3}{3} \\ - 2\frac{1}{3} \\ \hline 3\frac{2}{3} \end{array}$$

$$3\frac{2}{3} \quad (5 - 2 = 3 \text{ and } \frac{3}{3} - \frac{1}{3} = \frac{2}{3})$$

The answer is **C**.

**Example**

3.  $-2 + 1\frac{1}{4} =$

A.  $3\frac{1}{4}$     B.  $\frac{3}{4}$     C.  $-\frac{3}{4}$     D.  $-1\frac{1}{4}$

Note that  $-2 + 1\frac{1}{4}$  has to be negative: eliminate answers A and B. Now,  $-2 + 1\frac{1}{4} = -(2 - 1\frac{1}{4}) = -\frac{3}{4}$ . If you want to do the operations vertically, see the solution.

**Solution**

First write  $-2$  as  $-1 + \frac{-4}{4} = -1\frac{-4}{4}$

$$\begin{array}{r} -2 = -1\frac{-4}{4} \\ + 1\frac{1}{4} \\ \hline -\frac{3}{4} \end{array}$$

$$-\frac{3}{4} \quad \left( \frac{-4}{4} + \frac{1}{4} = \frac{-3}{4} = -\frac{3}{4} \right)$$

The answer is **C**.

**Example**

4.  $-\frac{2}{3} - (-1) =$

- A.
- $-\frac{5}{3}$
- B.
- $-1$
- C.
- $\frac{1}{3}$
- D.
- $-\frac{1}{3}$

**Solution**

By the definition of subtraction,

$$-\frac{2}{3} - (-1) = -\frac{2}{3} + 1 = -\frac{2}{3} + \frac{3}{3} = \frac{1}{3}.$$

The answer is C.

**B. Multiplying and Dividing Rational Numbers****Objective IA1b****CLAST SAMPLE PROBLEMS**

1.  $8\frac{2}{3} \times \frac{3}{4}$

2.  $\frac{3}{5} \times \frac{5}{6}$

3.  $\frac{3}{8} \div \frac{9}{11}$

4.  $-\frac{8}{5} \div \left(-2\frac{1}{2}\right)$

**10****MULTIPLICATION AND DIVISION OF SIGNED NUMBERS****RULE**

When multiplying or dividing two numbers with the **same (like)** signs, the result is **positive**.

When multiplying or dividing two numbers with **different (unlike)** signs, the result is **negative**.

**EXAMPLES**

$$3 \times 4 = 12, \quad 6 \times 9 = 54$$

$$-5 \times (-8) = 40 \quad -6 \times (-4) = 24$$

$$-3 \times 4 = -12 \quad 8 \times (-9) = -72$$

$$(-7) \times 4 = -28 \quad (-3) \times 2 = -6$$

**Note:** We write  $-5 \times (-8)$  instead of  $-5 \times -8$  to avoid having the two symbols  $\times$  and  $-$  together.

**11****MULTIPLICATION OF FRACTIONS****RULE**

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$

To multiply  $\frac{a}{b}$  by  $\frac{c}{d}$  multiply the numerators **a** and **c** and the denominators **b** and **d**

(Write mixed numbers as improper fractions and reduce the answer.)

**EXAMPLES**

$$\frac{3}{5} \times \frac{7}{8} = \frac{3 \times 7}{5 \times 8} = \frac{21}{40}$$

$$1\frac{3}{4} \times \frac{8}{3} = \frac{7}{4} \times \frac{8}{3} = \frac{56}{12} = \frac{14}{3} = 4\frac{2}{3}$$

Note that the answer is written as a mixed number.

**ANSWERS**

1.  $6\frac{1}{2}$

2.  $-\frac{1}{2}$

3.  $\frac{11}{24}$

4.  $\frac{16}{25}$



12	DIVISION OF FRACTIONS	
<b>RULE</b>	<b>EXAMPLES</b>	
$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$	$3\frac{1}{8} \div 5 = \frac{25}{8} \times \frac{1}{5} = \frac{25}{40} = \frac{5}{8}$	
<p>To divide <math>\frac{a}{b}</math> by <math>\frac{c}{d}</math> multiply <math>\frac{a}{b}</math> by <math>\frac{d}{c}</math> (the <b>reciprocal</b> of <math>\frac{c}{d}</math>.)</p> <p>Note: The reciprocal of a number <math>n</math> is <math>\frac{1}{n}</math> since <math>n = \frac{n}{1}</math>. Thus, the reciprocal of <b>5</b> is <math>\frac{1}{5}</math>.</p>	$3\frac{1}{4} \div 1\frac{7}{8} = \frac{13}{4} \div \frac{15}{8} = \frac{13}{4} \times \frac{8}{15} = \frac{26}{15} = 1\frac{11}{15}$ <p>Since the original problem had mixed numbers, the answer should be <math>1\frac{11}{15}</math>.</p> <p>Note that this time we <b>simplified</b> before doing the multiplication.</p>	

**CLAST EXAMPLES**

- Example**
5.  $\frac{1}{3} \times 1\frac{1}{5} =$
- A.  $\frac{2}{15}$     B.  $\frac{1}{4}$     C.  $\frac{2}{5}$     D.  $1\frac{1}{15}$

**Solution**

$$\frac{1}{3} \times 1\frac{1}{5} = \frac{1}{3} \times \frac{6}{5} = \frac{2}{5}$$

The answer is C.

- Example**
6.  $7 \div 2\frac{1}{3} =$
- A.  $\frac{3}{49}$     B.  $\frac{1}{3}$     C. 3    D.  $16\frac{2}{3}$

**Solution:**

$$7 \div 2\frac{1}{3} = 7 \div \frac{7}{3} = \frac{7}{1} \times \frac{3}{7} = 3$$

The answer is C.

**Example**

$$7. \quad \left(-\frac{1}{5}\right) \div \left(-\frac{2}{3}\right) =$$

- A.  $\frac{10}{3}$     B.  $\frac{3}{10}$     C.  $-\frac{3}{10}$     D.  $-\frac{10}{3}$

**Solution**

$$\left(-\frac{1}{5}\right) \div \left(-\frac{2}{3}\right) = \frac{1}{5} \times \frac{3}{2} = \frac{3}{10}$$

Note that the fractions have the **same (like)** signs, so the answer is **positive**.  
The answer is **B**.

**Example**

$$8. \quad (-6) \times 2\frac{1}{3} =$$

- A. -21    B. -14    C.  $-12\frac{1}{3}$     D.  $12\frac{1}{3}$

**Solution**

$$(-6) \times 2\frac{1}{3} = -\overset{2}{\cancel{6}} \times \frac{7}{\cancel{3}} = -14$$

The numbers have **different (unlike)** signs, so their product is negative. The answer is **B**.

## Section 1.1 Exercises

**WARM-UPS A**

1.  $\frac{1}{3} + \frac{1}{7}$

2.  $\frac{3}{8} + 2\frac{1}{5}$

3.  $5 - 3\frac{1}{4}$

4.  $-2\frac{1}{8} + 6$

5.  $-3 + 1\frac{1}{5}$

6.  $-4 + 1\frac{1}{6}$

7.  $-\frac{3}{4} - (-2)$

8.  $-\frac{2}{5} - (-3)$

9.  $-5 - (-\frac{3}{5})$

10.  $-3 - (-\frac{4}{7})$

**CLAST PRACTICE A****PRACTICE PROBLEMS: Chapter 1, # 1-4**

11.  $1\frac{1}{2} + \frac{1}{7} =$

A.  $\frac{2}{9}$

B.  $\frac{9}{14}$

C.  $1\frac{5}{9}$

D.  $1\frac{9}{14}$

12.  $9 - 4\frac{1}{3} =$

A.  $4\frac{2}{3}$

B.  $5\frac{1}{3}$

C.  $5\frac{2}{3}$

D.  $4\frac{1}{3}$

13.  $-3 + 2\frac{1}{4} =$

A.  $-1\frac{1}{4}$

B.  $-\frac{3}{4}$

C.  $\frac{3}{4}$

D.  $3\frac{1}{4}$

14.  $-\frac{2}{7} - (-1) =$

A.  $-\frac{5}{7}$

B.  $-1$

C.  $-\frac{1}{7}$

D.  $\frac{5}{7}$

**WARM-UPS B**

15.  $\frac{4}{7} \times 2\frac{1}{4}$

16.  $\frac{2}{15} \times 2\frac{1}{2}$

17.  $6 \div 1\frac{1}{3}$

18.  $9 \div 1\frac{1}{2}$

19.  $(-\frac{14}{15}) \div (-\frac{1}{5})$

20.  $(-\frac{1}{27}) \div (-\frac{1}{3})$

21.  $(-\frac{1}{24}) \div (-\frac{1}{2})$

22.  $(-\frac{14}{15}) \div (-\frac{2}{5})$

23.  $-28 \times 2\frac{3}{7}$

24.  $-16 \times 3\frac{3}{4}$

## CLAST PRACTICE B

## PRACTICE PROBLEMS: Chapter 1, # 5-8

25.  $\frac{3}{4} \times 4\frac{1}{2} =$

A.  $4\frac{1}{2}$

B.  $\frac{1}{2}$

C.  $4\frac{1}{8}$

D.  $3\frac{3}{8}$

26.  $8 \div 1\frac{2}{3} =$

A.  $4\frac{4}{5}$

B.  $13\frac{1}{3}$

C.  $\frac{3}{40}$

D.  $\frac{2}{3}$

27.  $\left(-\frac{7}{32}\right) \div \left(-\frac{3}{8}\right) =$

A.  $-\frac{7}{12}$

B.  $\frac{7}{12}$

C.  $-1\frac{5}{7}$

D.  $1\frac{5}{7}$

28.  $(-8) \times 2\frac{1}{2}$

A.  $-16\frac{1}{2}$

B.  $16\frac{1}{2}$

C.  $-25$

D.  $-20$

**EXTRA CLAST PRACTICE**

29.  $-1\frac{1}{3} + \left(-3\frac{4}{7}\right) =$

A.  $2\frac{19}{21}$

B.  $-2\frac{5}{21}$

C.  $-4\frac{11}{21}$

D.  $-4\frac{19}{21}$

30.  $2\frac{1}{5} - \left(-3\frac{4}{9}\right)$

A.  $-\frac{56}{45}$

B.  $\frac{29}{45}$

C.  $5\frac{29}{45}$

D.  $5\frac{5}{14}$

31.  $\left(-4\frac{2}{7}\right) \times \frac{5}{6}$

A.  $-4\frac{5}{21}$

B.  $-4\frac{2}{7}$

C.  $-3\frac{4}{7}$

D.  $3\frac{4}{7}$

## 1.2 EXPONENTS, BASE AND DECIMALS

In Section 1.1 we expressed the number 120 as a product of primes using exponents, as  $120 = 2^3 \times 3 \times 5$ . In the expression  $2^3$ , 2 is called the **base** and 3 is the **exponent**. The exponents 3 tells us how many times the base 2 must be used as a factor. The raised dot  $\cdot$  and parentheses ( ) also indicate multiplication. Thus, we can also write:

$$120 = 2^3 \cdot 3 \cdot 5 \text{ or } 120 = (2^3)(3)(5)$$

$$4 \cdot 4 \cdot 4 = 4^3$$

$4^3$  is read as "four to the third power" or "four cubed"

$$(7)(7)(7)(7) = 7^4$$

$7^4$  is read as "seven to the fourth power" or "seven to the fourth"

T	TERMINOLOGY--EXPONENTS	
	<b>DEFINITION OF EXPONENTS</b>	<b>EXAMPLES</b>
	$\mathbf{a^n = a \times a \times a \dots \times a}$ <p><math>\mathbf{a}</math> is used as a factor <math>\mathbf{n}</math> times (<math>\mathbf{n}</math> is a natural number)</p> <p><b>Note:</b> <math>\mathbf{a^n}</math> is also written as  <math>\mathbf{a^n = (a)(a)(a) \dots (a)}</math>  or as <math>\mathbf{a^n = a \cdot a \cdot a \dots \cdot a}</math></p> <p>We define <math>\mathbf{a^0 = 1}</math> and <math>\mathbf{a^1 = a}</math></p>	$3^2 = 3 \times 3, \quad 5^4 = (5)(5)(5)(5)$  $8^4 = (8)(8)(8)(8)$ $7^5 = 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$  $9^0 = 1, \quad 50^0 = 1 \quad \text{and} \quad (100)^0 = 1$ $10^1 = 10, \quad 50^1 = 50 \quad \text{and} \quad 100^1 = 100$

This section of the CLAST emphasizes the **meaning** and **notation** associated with exponents rather than the **rules** or **laws** of exponents. The examples that follow will illustrate this.

### A. Recognizing the Meaning of Exponents

#### Objective IIA1

#### CLAST SAMPLE PROBLEMS

1.  $5^3 - 2^4 =$

2.  $7^3 \div 3^2 =$

3.  $(6^3)^2 =$

4.  $5(2^3)$

#### ANSWERS

1.  $(5 \times 5 \times 5) - (2 \times 2 \times 2 \times 2)$

2.  $(6 \times 6 \times 6) \div (3 \times 3)$

3.  $6^3 \times 6^3 = (6 \times 6 \times 6) \times (6 \times 6 \times 6)$

4.  $5 \times (2 \times 2 \times 2)$

## CLAST EXAMPLES

- Example**
1.  $(7^3)(6^4) =$
- A.  $(7 \times 6)^{12}$                       B.  $(7 \times 3)(6 \times 4)$
- C.  $(7 \times 7 \times 7)(6 \times 6 \times 6 \times 6)$
- D.  $(7 + 7 + 7)(6 + 6 + 6 + 6)$

**Solution**

By definition  $7^3 = 7 \times 7 \times 7$   
and  $6^4 = 6 \times 6 \times 6 \times 6$ .  
Thus,  
 $(7^3)(6^4) = (7 \times 7 \times 7)(6 \times 6 \times 6 \times 6)$   
The correct answer is **C**.  
Note that you **do not** have to simplify the answer, you just write  $7^3$  and  $6^4$  as indicated products.

- Example**
2.  $6^2 + 5^2 =$
- A.  $(6 + 5)^4$                       B.  $(6)(6) + (5)(5)$
- C.  $(6 + 5)^2$                       D.  $(6)(2) + (5)(2)$

**Solution**

Again, we simply use the definition of exponents to write  $6^2 = (6)(6)$  and  $5^2 = (5)(5)$ .  
Thus,  $6^2 + 5^2 = (6)(6) + (5)(5)$ .  
The correct answer is **B**.  
There is no rule of exponents that covers the *addition* of two numbers.

- Example**
3.  $(9^5)^2 =$
- A.  $9^7$                                       B.  $(9 \times 5)^2$
- C.  $9^{25}$                                       D.  $9^5 \times 9^5$

**Solution**

Note that by definition,  $a^2 = a \times a$   
If you think of  $9^5$  as  $a$ ,  $a^2 = a \times a$   
becomes  $(9^5)^2 = 9^5 \times 9^5$   
and the correct answer is **D**.

## B. Place Value and Base

## Objective IIA2

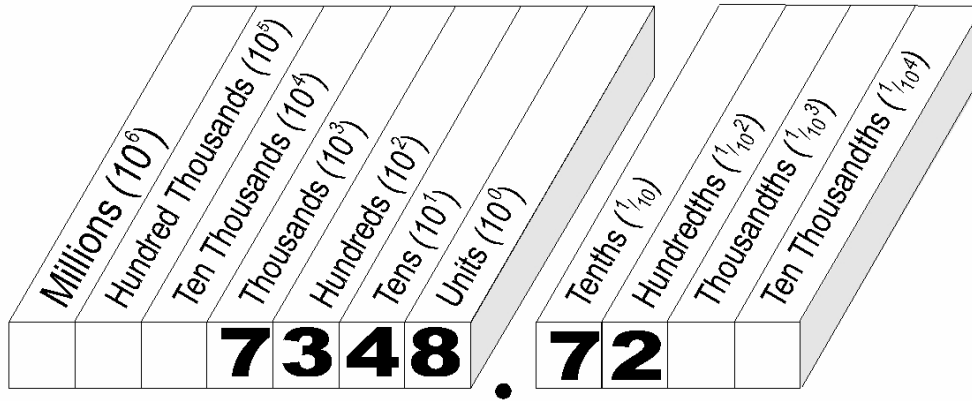
## CLAST SAMPLE PROBLEMS

- Find the place value of the underlined digit  $3\underline{2}86.0038$
- Write 23,285.401 in exponential form
- Find the numeral for  $(3 \times 10^2) + (1 \times 10^0) + \left(5 \times \frac{1}{10^3}\right)$

## ANSWERS

- 2 is in the hundreds place:  $(2 \times 10^2)$
- $(2 \times 10^4) + (3 \times 10^3) + (2 \times 10^2) + (8 \times 10^1) + (5 \times 10^0) + \left(4 \times \frac{1}{10^1}\right) + \left(1 \times \frac{1}{10^3}\right)$
- 301.005

Each of the places (boxes) to the right and left of the decimal point in the diagram has a **place value**. These place values **increase** as you move left from the **units** place, and **decrease** as you move right from the **units** place. For example,



- |                                                               |                                              |
|---------------------------------------------------------------|----------------------------------------------|
| The 7 is in the <b>thousands</b> ( $10^3$ ) place.            | (3 places to the <b>left</b> of the units.)  |
| The 3 is in the <b>hundreds</b> ( $10^2$ ) place.             | (2 places to the <b>left</b> of the units.)  |
| The 4 is in the <b>tens</b> ( $10^1$ ) place.                 | (1 place to the <b>left</b> of the units.)   |
| The 8 is in the <b>units</b> ( $10^0$ ) place.                | (The 8 is <b>0</b> places from the units.)   |
| The 7 is in the <b>tenths</b> ( $\frac{1}{10^1}$ ) place.     | (1 place to the <b>right</b> of the units.)  |
| The 2 is in the <b>hundredths</b> ( $\frac{1}{10^2}$ ) place. | (2 places to the <b>right</b> of the units.) |

Do you see the pattern? You must start at the **units** place!

3 places to the **left** of the units is the thousands or  $10^3$  place.

2 places to the **right** of the units is the hundredths or  $\frac{1}{10^2} = \frac{1}{100}$  place.

If you do see the pattern, you will be able to find the place value associated with the boxed digits in the number  $4\boxed{9}31.68\boxed{5}$

The nine is **two** places to the left of the units, it is in the hundreds or  $10^2$  place, while the 5 is **three** places to the right of the units, it is in the thousandths or  $\frac{1}{10^3}$  place. You can verify this by looking at the diagram.

### CLAST EXAMPLES

Example	Solution
<p>4. Select the place value associated with the underlined digit <math>5.03\text{9}2</math></p> <p>A. <math>\frac{1}{10^9}</math>    B. <math>\frac{1}{10^3}</math>    C. <math>10^9</math>    D. <math>10^3</math></p>	<p>The place value occupied by 9 is <b>three</b> places to the <b>right</b> of the units place. It is the <math>\frac{1}{10^3}</math> (thousandths) place. The correct answer is <b>B</b>.</p>

**Example**

5. Select the place value associated with the underlined digit 83,584.02

- A.  $\frac{1}{10^3}$     B.  $\frac{1}{10^2}$     C.  $10^3$     D.  $10^2$

**Solution**

The place value occupied by 5 is **two** places to the left of the units place. It is the  $10^2$  (hundreds) place.

The correct answer is **D**.

How do you read (or write) the number 7348.72?

Seven thousand, three hundred forty-eight **and** seventy-two hundredths.

Note: The decimal point is read as **and**.

We can also write 7348.72 in **expanded notation** like this:

$$(7 \times 10^3) + (3 \times 10^2) + (4 \times 10^1) + (8 \times 10^0) + (7 \times \frac{1}{10}) + (2 \times \frac{1}{100})$$

**CLAST EXAMPLES****Example**

6. Select the expanded notation for 300.05

A.  $(3 \times 10^2) + (5 \times \frac{1}{10^2})$

B.  $(3 \times 10^3) + (5 \times \frac{1}{10})$

C.  $(3 \times 10^3) + (5 \times \frac{1}{10^2})$

D.  $(3 \times 10^2) + (5 \times \frac{1}{10})$

**Solution**

The number is three hundred **and** five hundredths, that is,  $(3 \times 10^2) + (5 \times \frac{1}{10^2})$ . The correct answer is **A**.

You can also write the value of each place to the right and left of the decimal point above the number 300.05 to find the answer.

$$\begin{array}{cccccc}
 10^2 & 10^1 & 10^0 & \frac{1}{10} & \frac{1}{10^2} & \\
 \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \\
 3 & 0 & 0 & . & 0 & 5 \\
 (3 \times 10^2) & & & + & & (5 \times \frac{1}{10^2})
 \end{array}$$

**Example**

7. Select the numeral for

$$(1 \times \frac{1}{10}) + (1 \times \frac{1}{10^5})$$

A. 0.01010                      B. 0.01001

C. 0.10001                      D. 0.10010

**Solution**

Since the  $\frac{1}{10}$  place value is one place to the right of the units and  $\frac{1}{10^5}$  is five places to the right of the units, the answer must be **0.10001**

↑

**Units place**

The answer is **C**.



**Example**

8. Select the expanded notation for 20,345

- A.  $(2 \times 10^3) + (3 \times 10^2) + (4 \times 10^1) + (5 \times 10^0)$   
 B.  $(2 \times 10^5) + (3 \times 10^3) + (4 \times 10^2) + (5 \times 10^1)$   
 C.  $(2 \times 10^4) + (3 \times 10^2) + (4 \times 10^1) + (5 \times 10^0)$   
 D.  $(2 \times 10^4) + (3 \times 10^3) + (4 \times 10^2) + (5 \times 10^1)$

**Solution**

The 2, 3, 4 and 5 are four (4), two (2), one (1) and zero (0) places from the units place, respectively. Thus, the answer must be:  
 $(2 \times 10^4) + (3 \times 10^2) + (4 \times 10^1) + (5 \times 10^0)$   
 The answer is C.

## Section 1.2 Exercises

**WARM-UPS A**

1.  $(3^5)(2^2) =$                                                   2.  $(4^3)(8^4) =$   
 3.  $2^3 + 9^3 =$                                                   4.  $9^3 + 3^3 =$   
 5.  $(6^3)^2 =$                                                   6.  $(7^4)^2 =$

**CLAST PRACTICE A****PRACTICE PROBLEMS: Chapter 1, # 9-12**

7.  $(5^3)(3^2) =$

- A.  $(5 \times 3)^6$                   B.  $(5 \times 5 \times 5)(3 \times 3)$                   C.  $(5 \times 3)(3 \times 2)$                   D.  $(5 + 5 + 5)(3 + 3)$

8.  $3^2 + 5^2 =$

- A.  $(3 + 5)^2$                   B.  $(3)(2) + (5)(2)$                   C.  $(3 + 5)^4$                   D.  $(3)(3) + (5)(5)$

9.  $(2^3)^2 =$

- A.  $2^5$                                   B.  $2^9$                                   C.  $(2 \times 3)^2$                                   D.  $2^3 \times 2^3$

**WARM-UPS B**

10. What place value is associated with the underlined digit: 73.8425?
11. What place value is associated with the underlined digit: 5.3942?
12. Write the expanded notation for 84.674
13. Write the expanded notation for 294.97
14. Write the numeral for  $(1 \times \frac{1}{10}) + (1 \times \frac{1}{10^2}) + (1 \times \frac{1}{10^5})$
15. Write the numeral for  $(2 \times \frac{1}{10}) + (1 \times \frac{1}{10^3}) + (1 \times \frac{1}{10^5})$
16. Write the expanded notation for 30,478
17. Write the expanded notation for 12,057

**CLAST PRACTICE B****PRACTICE PROBLEMS: Chapter 1, #12**

- 18.** Select the place value associated with the underlined digit: 3.4985
- A.  $\frac{1}{10^3}$       B.  $\frac{1}{10^4}$       C.  $\frac{1}{10^0}$       D.  $\frac{1}{10^5}$
19. Select the place value associated with the underlined digit: 63,493.07
- A.  $\frac{1}{10^3}$       B.  $\frac{1}{10^2}$       C.  $10^2$       D.  $10^3$
20. Select the expanded notation for 500.03.
- A.  $(5 \times 10^2) + (3 \times \frac{1}{10^2})$       B.  $(5 \times 10^3) + (3 \times \frac{1}{10})$
- C.  $(5 \times 10^3) + (3 \times \frac{1}{10^2})$       D.  $(5 \times 10^2) + (3 \times \frac{1}{10})$
21. Select the numeral for  $(1 \times \frac{1}{10}) + (1 \times \frac{1}{10^7})$
- A. 0.0100001      B. 0.100001      C. 0.1000001      D. 1.0000001
22. Select the expanded notation for 40,328.
- A.  $(4 \times 10^3) + (3 \times 10^2) + (2 \times 10^1) + (8 \times 10^0)$
- B.  $(4 \times 10^5) + (3 \times 10^3) + (2 \times 10^2) + (8 \times 10^1)$
- C.  $(4 \times 10^4) + (3 \times 10^2) + (2 \times 10^1) + (8 \times 10^0)$
- D.  $(4 \times 10^4) + (3 \times 10^3) + (2 \times 10^2) + (8 \times 10^1)$

**EXTRA CLAST PRACTICE**

23.  $8^4 - 2^4 =$

A.  $(8 - 2)^4$

B.  $(8 \times 8 \times 8 \times 8) - (2 \times 2 \times 2 \times 2)$

C.  $8 \times 4 - 2 \times 4$

D.  $(8 - 2)^0$

24.  $(6 \times 5)^3 =$

A.  $18 \times 15$

B.  $(6 \times 5) \times (6 \times 5) \times (6 \times 5)$

C.  $(6 \times 5) + (6 \times 5)$

D.  $6 \times 6 \times 6 \times 5$

25. Select the expanded notation for 3002.07

A.  $(3 \times 10^3) + (2 \times 10^1) + \left(7 \times \frac{1}{10^2}\right)$

B.  $(3 \times 10^3) + (2 \times 10^0) + \left(7 \times \frac{1}{10}\right)$

C.  $(3 \times 10^3) + (2 \times 10^0) + \left(7 \times \frac{1}{10^2}\right)$

D.  $(3 \times 10^4) + (2 \times 10^0) + \left(7 \times \frac{1}{10^2}\right)$

26. Select the numeral for  $(5 \times 10^3) + \left(8 \times \frac{1}{10}\right)$ 

A. 500.8

B. 508

C. 5000.08

D. 5000.8

27. Select the place value associated with the underlined digit: 32.8329

A.  $\frac{1}{10^3}$

B.  $\frac{1}{10^2}$

C.  $10^2$

D.  $10^3$

## 1.3 ESTIMATION AND OPERATIONS WITH DECIMALS

In this section we shall study the four fundamental operations using decimals. Before doing so, however, we shall discuss an idea that can be used as a helpful guide in selecting the correct answer, the idea of **estimation**. To estimate sums, averages or products, we need to be familiar with **rounding**.

T	TERMINOLOGY--ROUNDING NUMBERS	
<b>ROUNDING NUMBERS</b>	<b>EXAMPLES</b>	
When we wish to <b>round</b> a number we specify the <b>place value</b> to which we are to round by underlining it.	To round 258.34 to the nearest: <b>hundred</b> ,     underline the 2 <u>2</u> 58.34 <b>unit</b> ,         underline the 8         25 <u>8</u> .34 <b>hundredth</b> ,   underline the 4       258. <u>34</u>	

To do the actual **rounding**, we use the following rule:

1	RULE FOR ROUNDING NUMBERS	
<b>RULE</b>	<b>EXAMPLES</b>	
<ol style="list-style-type: none"> <li>1. <b>Underline</b> the place to which you are rounding.</li> <li>2. If the first number to the <i>right</i> of the underlined place is <i>5 or more</i>, <b>add</b> one to the underlined number. Otherwise, <i>do not change</i> the underlined number.</li> <li>3. <i>Change</i> all the numbers to the <i>right</i> of the underlined number to zeros.</li> </ol>	Round 258.34 to the nearest hundred <ol style="list-style-type: none"> <li>1. <b>Underline</b> the 2.     <u>2</u>58.34</li> <li>2. The first number to the <i>right</i> of 2 is 5, so we <b>add</b> one to the underlined digit 2 to get 3.</li> <li>3. <i>Change</i> all the digits to the right of 3 to zeros obtaining 300.00 or 300. Note that if you count by hundreds (100, 200, and so on) 258.34 is closer to 300 than to 200. The procedure is written as:  <div style="text-align: center;"><u>2</u>58 → 300</div> </li> </ol>	

### A. Estimating Sums, Averages or Products

Objective IIA5	CLAST SAMPLE PROBLEMS
<ol style="list-style-type: none"> <li>1. A company employs 20 people. The lowest paid worker earns \$300 per week. The highest paid worker earns \$550 per week. What is a reasonable estimate of the total weekly payroll for the company?</li> <li>2. An investor buys 108 shares of stock. Each stock costs \$48.50. What is a reasonable estimate for the purchase?</li> <li>3. A bank account contains \$290.53. If \$82.90 and \$200.07 are deposited into the account and withdrawals of \$178.50 and \$46.80 are made. What is a reasonable estimate of the amount in the account after the deposits and withdrawals?</li> </ol>	

<b>ANSWERS</b>	1. Between \$6000 and \$11,000	2. \$5000	3. \$350
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The rules for rounding numbers can be used to estimate sums, averages or products. In this section, we **do not** compute averages, but rather estimate what these averages would be. Thus, if the scores on your last three tests are 90, 50 and 73 your average for the three tests would be  $\frac{90 + 50 + 73}{3}$ .

We are not asking for this answer now! Just be aware that the average of 90, 50 and 73 would be **between** the highest (90) and lowest (50) scores involved, that is, the average must be between 50 and 90. It is actually 71!

**CLAST EXAMPLES**

<b>Example</b>	<b>Solution</b>						
<p><b>1.</b> If a unit of water costs \$1.82 and 40.435 units were used, which is a reasonable estimate of the bill? (Water is sold in thousand gallon units).</p> <p>A. \$80,000                      B. \$800</p> <p>C. \$8000                        D. \$80</p>	<p>First, note that the fact that water is sold by thousand gallon units is true, but not important to the problem. Next, we round 40.342 and \$1.82 to the nearest whole number obtaining, <math>40.342 \rightarrow 40</math> and <math>1.82 \rightarrow 2</math>. Multiplying 40 by 2, we get the estimate 80.</p> <p>The answer is <b>D</b>.</p>						
<p style="text-align: center;"><b>Example</b></p> <p><b>2.</b> A student bought cologne for \$7.99, nail enamel for \$2.29, candy for \$3.89, adhesive paper for \$1.89, a curling iron for \$8.69, and sunglasses for \$7.19. Which of the following is a reasonable estimate of the total amount spent?</p> <p>A. \$ 28.00                      B. \$25.00</p> <p>C. \$32.00                        D. \$36.00</p>	<p style="text-align: center;"><b>Solution</b></p> <p>Round all amounts to the nearest dollar.</p> <table style="margin-left: auto; margin-right: auto;"> <tr> <td style="padding-right: 20px;"><math>7.99 \rightarrow 8</math></td> <td><math>\\$1.89 \rightarrow 2</math></td> </tr> <tr> <td><math>2.29 \rightarrow 2</math></td> <td><math>\\$8.69 \rightarrow 9</math></td> </tr> <tr> <td><math>3.89 \rightarrow 4</math></td> <td><math>\\$7.19 \rightarrow 7</math></td> </tr> </table> <p>Since <math>8 + 2 + 4 + 2 + 9 + 7 = 32</math>, the answer is <b>C</b>.</p>	$7.99 \rightarrow 8$	$\$1.89 \rightarrow 2$	$2.29 \rightarrow 2$	$\$8.69 \rightarrow 9$	$3.89 \rightarrow 4$	$\$7.19 \rightarrow 7$
$7.99 \rightarrow 8$	$\$1.89 \rightarrow 2$						
$2.29 \rightarrow 2$	$\$8.69 \rightarrow 9$						
$3.89 \rightarrow 4$	$\$7.19 \rightarrow 7$						
<p style="text-align: center;"><b>Example</b></p> <p><b>3.</b> A bag of rye grass seed covers 2.75 acres. Based on this fact, what is a reasonable estimate of the number of acres that could be covered with <math>153\frac{1}{2}</math> bags of seed?</p> <p>A. 50 acres                      B. 300 acres</p> <p>C. 450 acres                      D. 400 acres</p>	<p style="text-align: center;"><b>Solution</b></p> <p>If <math>153\frac{1}{2}</math> is rounded to the nearest ten, we have <math>153\frac{1}{2} \rightarrow 150</math>. Since a bag of seed covers 3 acres (<math>2.75 \rightarrow 3</math> when rounded to the nearest acre), 150 bags would cover <math>150 \times 3 = 450</math> acres.</p> <p>The answer is <b>C</b>.</p> <p>Note that if you rounded <math>153\frac{1}{2}</math> to the nearest hundred, and 2.75 to the nearest acre, the answer would be <math>200 \times 3 = 600</math> acres. (Not one of the choices!)</p>						

**Example**

4. Five hundred students took an algebra test. All of the students scored less than 92 but more than 63. Which of the following values could be a reasonable estimate of the average score for the students?
- A. 96      B. 63      C. 71      D. 60

**Solution**

Remember, you **do not** have to compute the average. Since the scores ranged from 63 to 92, the answer must be between these two numbers. The only value between 63 and 92 in the answer is 71. Thus, the answer is **C**.

**Example**

5. The mathematics department employs 20 tutors. The lowest paid tutor makes \$150 per month and the best paid earns \$200 per month. Which of the following could be an estimate of the total monthly payroll for the tutors in the math department?
- A. \$3600    B. \$3000    C. \$4000    D. \$6000

**Solution**

If each tutor was paid the lowest rate, the payroll would be  $20 \times \$150$  or \$3000. If each tutor was paid the highest rate, the payroll would be  $20 \times \$200$  or \$4000. But the earnings are **between** \$150 and \$200, so the total payroll must be between \$3000 and \$4000. The only answer between \$3000 and \$4000 is **A**.

**Example**

6. The table shows the actual price of the top 10 stocks by number of shares traded. Which could be a reasonable estimate of the average price per share for these stocks?
- A. \$5      B. \$140    C. \$143    D. \$13

Amdahl	$13\frac{3}{8}$	USBioS	10
ExploLA	$1\frac{1}{4}$	CalifEgy	12
Wang	$2\frac{7}{8}$	Nabors	$7\frac{3}{4}$
EchoBay	$6\frac{1}{2}$	ElenCp	46
Fr.Loom	$37\frac{5}{8}$	AmExpl	$2\frac{3}{4}$

**Solution**

The lowest priced stock is  $1\frac{1}{4}$  and the highest priced is 46. The average price must be between these two numbers. There are two possible answers between  $1\frac{1}{4}$  and 46. They are \$5 and \$13. Which do we choose?

Resist the temptation of getting the actual average, it takes too much valuable time! Just note that 46 and  $37\frac{5}{8}$  will drive the average **up** much more than  $1\frac{1}{4}$  and  $2\frac{3}{4}$  can drive the average **down**. The average is probably not as low as \$5. The correct answer is **D**.

**NOTE:** *Mixed numbers are used for illustrative purposes only. Stocks are now priced in decimals.*

## B. Operations with Decimals

Objectives IA2a, Ia2b		CLAST SAMPLE PROBLEMS		
1. $7.4 + 0.03$	2. $-3.46 - 7.7$	3. $4.58 - 7$	4. $0.006 \times 0.8$	5. $-1.6 \times 0.26$
6. $7 \times 0.025$	7. $1.2 \div 0.6$	8. $0.12 \div 6$	9. $12 \div 0.06$	

We are now ready to perform the four fundamental operations of addition, subtraction, multiplication and division using decimals. Before doing so, memorize and practice with the laws of signs we studied in Section 1.2. Moreover, many of the CLAST answers can be obtained by estimating them, so make sure you understand estimation before you go on.

2	RULE TO ADD OR SUBTRACT DECIMALS	
	<b>RULE</b>	<b>EXAMPLES</b>
	<ol style="list-style-type: none"> <li>Line up the decimal points, that is, write them in the same column.</li> <li>If necessary, attach zeros to the right of the last digit so that they all have the same number of digits after the decimal point.</li> <li>Perform the addition or subtraction working from right to left.</li> <li>Don't forget to estimate your answer.</li> </ol>	To add 1.78 and 0.265 follow the 3 steps given: <ol style="list-style-type: none"> <li>Write and</li> <li>Attach a zero to 1.78. (Now both numbers have three decimal places.)</li> <li>Add the columns from right to left.</li> <li>Since <math>\underline{1.78} \rightarrow 2</math> and <math>\underline{0.265} \rightarrow 0</math>, the answer must be about 2. (It is!)</li> </ol>

### CLAST EXAMPLES

Example	Solution
7. $14.22 - 1.761 =$	<ol style="list-style-type: none"> <li>Align the decimal points.</li> </ol>
A. 12.459                      B. 13.459	$\begin{array}{r} 14.22 \\ - 1.761 \\ \hline \end{array}$
C. 11.459                      D. 12.261	<ol style="list-style-type: none"> <li>Attach a <b>0</b> to 14.22 so both numbers have 3 decimal places.</li> </ol>
Before you do the arithmetic, note that 14.22 can be rounded to 14 and 1.761 can be rounded to 2, so the answer must be about $14 - 2 = 12$ . (A or D). Moreover, $14.22 = 14.220$ which means that the last digit in the answer will be <b>(10 - 1 = 9)</b> . So the answer must be <b>A</b> . Now, look at the solution to see why!	$\begin{array}{r} 14.220 \\ - 1.761 \\ \hline 12.459 \end{array}$

<b>ANSWERS</b>	1. 7.43	2. - 11.16	3. - 2.42	4. 0.0048	5. - 0.416
	6. 0.175	7. 2	8. 0.2	9. 200	

**Example**

8.  $-18.66 - 10.667$   
 A.  $-28.733$                       B.  $-7.993$   
 C.  $-29.327$                       D.  $7.993$

Recall that  $a - b = a + (-b)$ , thus  
 $-18.66 - 10.667 = -18.66 + (-10.667)$   
 Now,  $18.66 \rightarrow 19$  and  $10.667 \rightarrow 11$ ,  
 so the answer is about  $-(19 + 11)$  and must end in  
 7, the last digit you get when we add  $18.660$  and  
 $10.667$ . The answer must be **C**. Now, look at  
 the solution to see why.

**Solution**

Since  $a - b = a + (-b)$   
 $-18.66 - 10.667 = -18.66 + (-10.667)$   
 As you recall, to add two numbers with the same  
 sign, we add the numbers and give the answer  
 the common sign. Thus,

- |                                                                  |                             |
|------------------------------------------------------------------|-----------------------------|
| 1. Align the decimal points.                                     | $-18.66$                    |
|                                                                  | <u><math>-10.667</math></u> |
|                                                                  |                             |
| 2. Attach a <b>0</b> to 18.66.                                   | $-18.660$                   |
|                                                                  | <u><math>-10.667</math></u> |
|                                                                  |                             |
| 3. Add and give the<br>answer the common<br>sign, which is $-$ . | $-29.327$                   |

**Example**

9.  $16.4 - (-5.78)$   
 A.  $21.18$                       B.  $22.18$   
 C.  $8.32$                       D.  $6.62$

The key here is to remember that:  
 $a - (-b) = a + b$   
 Now,  $16.4 \rightarrow 16$  and  $5.78 \rightarrow 6$ , so the answer  
 must be about  $16 + 6$  or  $22$ . Also, the answer  
 ends in 8, the last digit in the sum  $16.40 + 5.78$ .  
 The answer must be **B**. Now, look at the  
 solution at the right.

**Solution**

- Since  $a - (-b) = a + b$ ,
- $$16.4 - (-5.78) = 16.4 + 5.78$$
- |                               |                            |
|-------------------------------|----------------------------|
| 1. Align the decimal points.  | $16.4$                     |
|                               | <u><math>+ 5.78</math></u> |
|                               |                            |
| 2. Attach a <b>0</b> to 16.4. | $16.40$                    |
|                               | <u><math>+ 5.78</math></u> |
|                               |                            |
| 3. Add                        | $22.18$                    |

<b>3</b>	<b>RULE TO MULTIPLY DECIMALS</b>												
<p style="text-align: center;"><b>RULE</b></p> <p>1. <i>Multiply</i> the two decimal numbers <i>as if they were whole numbers</i>.</p> <p>2. The <i>number of decimal places</i> in the product is the <i>sum</i> of the number of decimal places in the factors.</p>	<p style="text-align: center;"><b>EXAMPLES</b></p> <table style="margin-left: auto; margin-right: auto;"> <tr> <td style="text-align: right; padding-right: 10px;"><math>3.43</math></td> <td><math>3.43</math> has <b>2</b> decimal places.</td> </tr> <tr> <td style="text-align: right; padding-right: 10px;"><math>\times 2.7</math></td> <td><math>2.7</math> has <b>1</b> decimal place.</td> </tr> <tr> <td style="text-align: right; padding-right: 10px;"><math>2401</math></td> <td></td> </tr> <tr> <td colspan="2"> </td> </tr> <tr> <td style="text-align: right; padding-right: 10px;"><u><math>686</math></u></td> <td></td> </tr> <tr> <td style="text-align: right; padding-right: 10px;"><math>9.261</math></td> <td>The answer has <b>2 + 1 = 3</b> decimal places.</td> </tr> </table>	$3.43$	$3.43$ has <b>2</b> decimal places.	$\times 2.7$	$2.7$ has <b>1</b> decimal place.	$2401$				<u><math>686</math></u>		$9.261$	The answer has <b>2 + 1 = 3</b> decimal places.
$3.43$	$3.43$ has <b>2</b> decimal places.												
$\times 2.7$	$2.7$ has <b>1</b> decimal place.												
$2401$													
<u><math>686</math></u>													
$9.261$	The answer has <b>2 + 1 = 3</b> decimal places.												



4	RULE TO DIVIDE DECIMALS	
<b>RULE</b>	<b>EXAMPLES</b>	
<p><b>1.</b> To divide by a whole number place the decimal point directly above the decimal point in the dividend and divide as though dividing whole numbers.</p> <p><b>2.</b> If the divisor is not a whole number, multiply it and the dividend by the appropriate power of 10 (10, 100, and so on) so that the divisor is a whole number. Then proceed as indicated in <b>1</b>.</p> <p>To understand this procedure, you must recall that <math>\frac{a}{b} = \frac{a \times c}{b \times c}</math> and that multiplying by a power of 10 moves the decimal point as many places to the right as there are 0's in the power of 10. Thus,</p> $3.487 \times 10 = 34.87$ $3.487 \times 100 = 348.7$ $3.487 \times 1000 = 3487$ <p>In the problem <math>6\overline{)3.84}</math>, 3.84 is called the <b>dividend</b> and 6 is called the <b>divisor</b>. The answer is called the <b>quotient</b>.</p>	<p>To divide 3.84, the <b>dividend</b>, by 6, the <b>divisor</b>, place the decimal point directly above the decimal point in the dividend and divide as usual.</p> $\begin{array}{r} 0.64 \\ 6\overline{)3.84} \\ \underline{-36} \phantom{0} \\ 24 \\ \underline{-24} \\ 0 \end{array}$ <p>To divide 16.416 by 0.48, we multiply 0.48 by 100 to make it a whole number. Of course, we also have to multiply 16.416 by 100. This can be done because dividing 16.416 by 0.48 means:</p> $\frac{16.416}{0.48}$ <p>and to make the denominator a whole number, we multiply it by 100. (Of course, we multiply the numerator by 100 also). Thus,</p> $\frac{16.416}{0.48} = \frac{16.416 \times 100}{0.48 \times 100} = \frac{1641.6}{48}$ <p>Now, divide as indicated in step <b>1</b>.</p> $\begin{array}{r} 34.2 \\ 48\overline{)1641.6} \\ \underline{-144} \phantom{0} \\ 201 \\ \underline{-192} \phantom{0} \\ 096 \\ \underline{-96} \\ 0 \end{array}$	

**CLAST EXAMPLES****Example**

**10.**  $3.43 \times 2.8 =$

A. 0.9604                      B. 8.504

C. 7.1344                      D. 9.604

First, estimate the answer.  $\underline{3}.43 \rightarrow 3$ ,  $\underline{2}.8 \rightarrow 3$ , thus the answer should be about 9, so it is probably **D**. Now, look at the solution to see why.

**Solution**

$$\begin{array}{r} 3.43 \\ \times 2.8 \\ \hline 2744 \\ 686 \\ \hline 9.604 \end{array}$$

3.43 has **2** decimal places.  
2.8 has **1** decimal place.

The answer has  $2 + 1 = 3$  decimal places, so A and C are eliminated.

The answer is **D**.

**Example**

11.  $(-0.04) \times (-1.2) =$

- A. -0.48    B. -4.8    C. 0.048    D. 4.8

Hint: To save time, **do not** set up the problem in a column. Just multiply 4 by 12 and see the explanation on the right hand side!

**Solution**

First, recall that the multiplication of two numbers with the same sign is **positive**. Now, 0.04 has two decimal places and 1.2 has one decimal place, so the answer must have  $1 + 2 = 3$  decimal places. Since  $4 \times 12 = 48$  and the answer must have 3 decimal places, the answer is 0.048, that is, **C**.

**Example**

12.  $36.75 \div 0.05 =$

- A. 735    B. 73.5    C. 7.35    D. 0.0735

Some students understand the problem better when written as

$$\frac{36.75}{0.05} = \frac{36.75 \times 100}{0.05 \times 100} = \frac{3675}{5} = 735$$

**Solution**

$0.05 \overline{)36.75}$     Multiply 0.05 by 100 and 36.75 by 100

$$\begin{array}{r} 735 \\ 5 \overline{)3675} \\ \underline{-35} \phantom{00} \\ 17 \phantom{00} \\ \underline{-15} \phantom{00} \\ 25 \phantom{00} \\ \underline{-25} \phantom{00} \\ 0 \end{array}$$

Now, divide by the whole number 5. The answer is **A**.

**Example**

13.  $-0.336 \div 0.07 =$

- A. -48    B. -4.8    C. 48    D. 0.48

Again, we might set up the problem as

$$\frac{-0.336}{0.07} = \frac{-0.336 \times 100}{0.07 \times 100} = \frac{-33.6}{7}$$

Note that the answer must be negative, (because -33.6 and 7 have different signs) and it should be close to  $\frac{-35}{7} = -5$ . The answer is probably **B**.

**Solution**

Multiply the dividend and divisor by 100 so we will be dividing -33.6 by the whole number 7, two numbers with different signs. Thus, the answer is **negative**.

Here is the division.

$$\begin{array}{r} 4.8 \\ 7 \overline{)33.6} \\ \underline{-28} \phantom{00} \\ 56 \phantom{00} \\ \underline{-56} \phantom{00} \\ 0 \end{array}$$

Remember to align the decimal point in the quotient with the decimal point in the dividend.

Since the answer must be negative, it must be **B**.

## Section 1.3 Exercises

### WARM-UPS A

1. An investor owns 416.38 shares of a mutual fund valued at \$30.28 per share. Find a reasonable estimate of the value of the investor's stock to the nearest hundred dollars.
2. Water is sold in thousand gallon units. If a unit of water costs \$1.88 and 50.439 units were used, find a reasonable estimate of the bill to the nearest hundred dollars.
3. A student bought artichokes for \$7.80, cucumbers for \$2.29, lettuce for \$3.75, tomatoes for \$1.85, and broccoli for \$2.90. Find a reasonable estimate of the total amount the student spent on vegetables by rounding each price to the nearest dollar.
4. A student bought a towel for \$8.99, soap for \$2.39, toothpaste for \$3.79, shampoo for \$1.79, a pair of shorts for \$8.79 and a hat for \$9.99. Find a reasonable estimate of the total purchases by rounding each quantity to the nearest dollar.
5. A herbicide is to be applied at the rate of 5.75 gallons per acre. Based on this fact, find, to the nearest hundred gallons, a reasonable estimate for the amount of herbicide needed for  $154\frac{1}{2}$  acres.
6. A bag of bahia grass covers 1.75 acres. Based on this fact, what is a reasonable estimate of the number of acres that could be covered with  $158\frac{1}{2}$  bags of seed? Answer to the nearest hundred acres.
7. A class of 70 students took a test. The highest score was 92 and the lowest 72. If 60, 70, 80 and 90 represent the lowest possible D, C, B and A grades, respectively, what would be a reasonable estimate of the class average?
8. A geometry class with 60 students took a test. All the students scored less than 80 but more than 60. If 60, 70, 80 and 90 represent the lowest possible D, C, B and A grades, respectively, what would be a reasonable estimate of the class average?
9. A company employs 40 people. The lowest paid person earns \$400 per month, while the highest paid earns \$600 per month, the monthly payroll for this company has to be about \$ \_\_\_\_\_.

10. On a certain day, the Pizza Hot sold 80 pizzas. The lowest priced pizza is \$6 and the highest priced \$10. If they only sell \$6 and \$10 pizzas, a good estimate of their income for the day should be about \$ \_\_\_\_\_.

11. The chart shows the closing prices of 10 leading stocks. The average price of the five stocks in the left hand column must be between \_\_\_\_ and \_\_\_\_.

Amdahl	$13\frac{3}{8}$	USBio	10
ExploLA	$1\frac{1}{4}$	CalifEgy	12
Wang	$2\frac{7}{8}$	Nab	$7\frac{3}{4}$
EchoBay	$6\frac{1}{2}$	ElenCp	46
Fr.Loom	$37\frac{5}{8}$	AmExpl	$2\frac{3}{4}$

12. The average price of the five stocks in the right hand column must be between \_\_\_\_ and \_\_\_\_.

**CLAST PRACTICE A****PRACTICE PROBLEMS: Chapter 1, #13, 14**

13. Twentieth Century Ultra Mutual fund is currently selling for \$15.18 per share. If an investor buys 215.326 shares of this fund, a reasonable estimate for the cost would be:

A. \$3000                      B. \$320                      C. \$4000                      D. \$30,000

14. A student bought mangos for \$7.60, bananas for \$1.99, nectarines for \$3.39, grapes for \$2.88, strawberries for \$7.75, apricots for \$3.76 and oranges for \$5.90. Which of the following would be a reasonable estimate of the total amount the student spent on fruits?

A. \$26                      B. \$30                      C. \$34                      D. \$36

15. A fertilizer has to be applied at the rate of 4.78 gallons per acre. If the fertilizer is applied at the given rate, what would be a reasonable estimate for the amount of fertilizer needed for  $257\frac{1}{2}$  acres?

A. 100 gallons              B. 1250 gallons              C. 1000 gallons              D. 125 gallons

16. 500 students took a placement test. All of the students scored more than 37 but less than 48. Which of the following would be a reasonable estimate of the average score for the students?

A. 53                      B. 37                      C. 42                      D. 36

17. A company employs 10 people. The lowest paid person earns \$175 per week and the highest paid earns \$200 per week. Which of the following would be a reasonable estimate of the total weekly payroll for the 10?

- A. \$1750            B. \$1875            C. \$2000            D. \$180

18. Here are the prices of the 5 most active stocks on the American Exchange and their closing prices per share in dollars.

Daxon Corp:  $13\frac{5}{8}$ , Wang Labs:  $2\frac{1}{4}$ , Exploration LA:  $1\frac{7}{8}$ , Amdhal:  $13\frac{3}{4}$ , Chambers:  $8\frac{5}{8}$ .

What would be a reasonable estimate of the average price per share for these stocks?

- A. \$2            B. \$14            C. \$13            D. \$8

### WARM-UPS B

19.  $0.23 + 5.737 =$

20.  $5.26 + 0.615 =$

21.  $9.64 - 2.671 =$

22.  $18.30 - 2.497 =$

23.  $-19.23 - 7.839 =$

24.  $-10.72 - 4.131 =$

25.  $5.13 - (-3.496) =$

26.  $3.58 - (-5.651) =$

27.  $6.15 \times 9.6 =$

28.  $6.26 \times 15.5 =$

29.  $-4.05 \times 1.8 =$

30.  $-6.25 \times 7.4 =$

31.  $14.585 \div 0.05 =$

32.  $17.35 \div 0.05 =$

33.  $-0.372 \div 0.06 =$

34.  $-0.522 \div 0.09 =$

### CLAST PRACTICE B

### PRACTICE PROBLEMS: Chapter 1, #15-21

35.  $2.78 + 0.238 =$

- A. 5.16            B. 3.018            C. 1.146            D. 1.02018

36.  $7.72 - 1.202 =$

- A. 6.518            B. 5.870            C. 7.518            D. 5.518

$$\boxed{37.} \quad -17.24 - 8.848 =$$

- A. -25.872      B. -8.392      C. -26.088      D. 8.392

$$38. \quad 5.378 - (-1.37) =$$

- A. 6.748      B. 6.415      C. 4.008      D. -4.008

$$39. \quad 4.04 \times 1.1 =$$

- A. 4.4044      B. 4.0804      C. 0.444      D. 4.444

$$40. \quad -6.25 \times 7.5 =$$

- A. 46.875      B. -46.875      C. 4.6875      D. -4.6875

$$41. \quad 7.2 \div 0.5 =$$

- A. 1.44      B. 0.00144      C. 14.4      D. 0.144

$$\boxed{42.} \quad -0.039 \div 0.03 =$$

- A. -1.3      B. -13      C. 13      D. 0.13

### EXTRA CLAST PRACTICE

$$43. \quad 9 - 0.291 =$$

- A. 1.509      B. 8.709      C. 9.291      D. 9.819

$$44. \quad (-0.05) \times (-7.22) =$$

- A. 36.10      B. -0.3610      C. 0.3610      D. -36.10

$$45. \quad -18 \div (-0.06)$$

- A. 3      B. 30      C. 3000      D. 300

46. Reynaldo has 5 items to buy with a \$20 bill. The most expensive item he buys is \$3.69 and the least expensive item is \$1.18. Which of the following is a reasonable estimate of his bill?

- A. \$10      B. \$5      C. \$20      D. \$3

## 1.4 EQUIVALENCE, ORDER AND SEQUENCES

Rational numbers can be written as fractions, mixed numbers, decimals or percents. In this section we shall study how these three types of representations are related, compare rationals as to their magnitude (size) and infer some relations between number pairs.

### A. Equivalent Forms of Rational Numbers

#### Objective IIA3

#### CLAST SAMPLE PROBLEMS

- |                                      |                                       |                                      |
|--------------------------------------|---------------------------------------|--------------------------------------|
| 1. Change 2.45 to a percent          | 2. Write 0.06 as a fraction           | 3. Write 1.9 as a fraction           |
| 4. Change 3% to a decimal            | 5. Change 17.3% to a decimal          | 6. Change 46% to a fraction          |
| 7. Change $\frac{1}{4}$ to a decimal | 8. Change $1\frac{2}{5}$ to a decimal | 9. Change $\frac{2}{5}$ to a percent |

Is 0.17 a rational number? We can prove that it is by writing 0.17 as a fraction. To read 0.17, read the number to the right of the decimal point (seventeen) and follow by the place value of the last digit (hundredths). Thus, 0.17 is "seventeen hundredths," that is,  $0.17 = \frac{17}{100}$ . The number  $\frac{17}{100}$  also means 17 *percent* and written as 17%. Thus,  $0.17 = \frac{17}{100} = 17\%$ .

Here are the rules you need to convert from one form to another.

1	TO CONVERT RATIONALS FROM ONE FORM TO ANOTHER	
	<i>RULE</i>	<i>EXAMPLES</i>
	To convert a <i>fraction</i> to a <i>decimal</i> , divide the numerator by the denominator.	To convert $\frac{3}{4}$ to a decimal, divide 3 by 4. The result is 0.75 <div style="text-align: right; margin-top: 10px;"> <math display="block">\begin{array}{r} 0.75 \\ 4 \overline{) 3.00} \\ \underline{- 28} \phantom{0} \\ 20 \\ \underline{- 20} \\ 0 \end{array}</math> </div>

#### ANSWERS

- |                    |                   |                    |         |          |
|--------------------|-------------------|--------------------|---------|----------|
| 1. 245%            | 2. $\frac{3}{50}$ | 3. $\frac{19}{10}$ | 4. 0.03 | 5. 0.173 |
| 6. $\frac{23}{50}$ | 7. 0.25           | 8. 1.4             | 9. 40%  |          |

1	<b>TO CONVERT RATIONALS FROM ONE FORM TO ANOTHER (CONT.)</b>	
<p>To convert a <i>decimal</i> to a <i>fraction</i>, write the decimal as the numerator of the fraction (omit the decimal point) and the denominator as a 1 followed by as many zeros as there are decimal places in the decimal.</p> <p>Note: If you <i>understand</i> the diagram on page 12 you will know that 0.27 is read as "twenty-seven hundredths". Thus,</p> $0.27 = \frac{27}{100}.$ No rule is needed!	$0.7 = \frac{7}{10}$ (Note that 0.7 is read as "seven tenths") $0.18 = \frac{18}{100}$ (Note that 0.18 is read as "eighteen hundredths".) $0.131 = \frac{131}{1000}$ (Note that 0.131 is read as "one hundred thirty one thousandths")	
<p style="text-align: center;"><b><i>RULE (DECIMAL TO PERCENT)</i></b></p> <p>To convert a <i>decimal</i> to a <i>percent</i> move the decimal point 2 places to the right and add the % symbol</p>	<p style="text-align: center;"><b><i>EXAMPLES</i></b></p> $0.89 = 89\%$ $8.9 = 890\%$ (Note that we added a <b>0.</b> ) $0.08 = 8\%$	
<p style="text-align: center;"><b><i>RULE (PERCENT TO DECIMAL)</i></b></p> <p>To convert a <i>percent</i> to a <i>decimal</i>, move the decimal point 2 places left and drop the % symbol.</p>	<p style="text-align: center;"><b><i>EXAMPLES</i></b></p> $49\% = 0.49$ $3\% = 0.03$ (Note that we added a <b>0.</b> ) $147\% = 1.47$	
<p style="text-align: center;"><b><i>RULE (FRACTION TO PERCENT)</i></b></p> <p>To convert a <i>fraction</i> to a <i>percent</i> change the fraction to a decimal (make sure you have <i>at least</i> two decimal places), then change the decimal to a percent.</p>	<p style="text-align: center;"><b><i>EXAMPLES</i></b></p> $\frac{3}{4} = 0.75 = 75\%$ $\frac{1}{5} = 0.20 = 20\%$	
<p style="text-align: center;"><b><i>RULE (PERCENT TO FRACTION)</i></b></p> <p>To convert a <i>percent</i> to a <i>fraction</i> write the number over 100 and reduce the fraction (if possible)</p>	<p style="text-align: center;"><b><i>EXAMPLES</i></b></p> $42\% = \frac{42}{100} = \frac{21}{50}$ $0.7\% = \frac{0.7}{100} = \frac{0.7 \times 10}{100 \times 10} = \frac{7}{1000}$ $158\% = \frac{158}{100} = \frac{79}{50}$	



The following chart shows the equivalent relationship among some common decimals, fractions and percents. Try to memorize as many as you can!

FRACTION	DECIMAL	PERCENT
$\frac{1}{4}$	0.25	25%
$\frac{1}{2}$	0.50	50%
$\frac{3}{4}$	0.75	75%
$\frac{1}{10}$	0.10	10%
$\frac{2}{10}$	0.20	20%
$\frac{3}{10}$	0.30	30%
$\frac{1}{3}$	$0.333\dots = 0.\overline{3}$	$33\frac{1}{3}\%$
$\frac{2}{3}$	$0.666\dots = 0.\overline{6}$	$66\frac{2}{3}\%$

### CLAST EXAMPLES

#### Example

1.  $0.19 =$

- A.  $\frac{19}{100}\%$    B.  $1\frac{9}{10}$    C.  $\frac{19}{10}$    D.  $\frac{19}{100}$

Note: Another correct answer is 19%.  
The CLAST does not tell you what type of answers they want, you may need to **look** for the correct answer.

#### Solution

0.19 is read as "19 hundredths".

Thus,  $0.19 = \frac{19}{100}$ . (If you use the rule, you would write 19 as the numerator with a denominator of 1 followed by two zeros, since there are two decimal places in 0.19.) In either case, the answer is **D**.

#### Example

2.  $350\% =$

- A. 0.350   B. 3.50   C. 350.0   D. 3500

Note: All answer choices are decimals, so you must change 350% to a decimal.

#### Solution

To change 350% to a decimal, move the decimal point 2 places left in 350. and drop the % symbol. Thus,  $350\% = 3.50$ . The answer is **B**.

If we wanted answers in fractional form, we would select  $\frac{350}{100} = \frac{7}{2}$ .

**Example**

3.  $\frac{23}{25} =$

- A. 0.92   B. 0.092   C. 9.2%   D. 0.92%

Note that it is immaterial if the answer is a percent or a decimal, you have to start by dividing 23 by 25 **unless** you notice that you can get a denominator of 100 by multiplying numerator and denominator by 4. Thus,

$$\frac{23}{25} = \frac{23 \times 4}{25 \times 4} = \frac{92}{100} = 0.92$$

**Solution**

If you want to convert a fraction whose denominator is a divisor of 100, multiply numerator and denominator by a number that will make the denominator 100 and then convert the fraction to a decimal. Otherwise, divide 23 by 25.

$$\begin{array}{r} 0.92 \\ 25 \overline{)23.00} \\ \underline{-225} \phantom{0} \\ 50 \\ \underline{-50} \\ 0 \end{array}$$

The answer is **A**.

**B. Order Relations (Comparing Numbers)**

Objective IIA4	CLAST SAMPLE PROBLEMS						
<b>Compare:</b>	1. $\frac{2}{5}$ and $\frac{4}{9}$	2. $-\frac{2}{5}$ and $-\frac{4}{9}$	3. 3.42 and 3.423	4. $5.\overline{12}$ and $5.1\bar{2}$	5. $\frac{3}{5}$ and 0.62	6. $\sqrt{27}$ and 5	7. 8.9 and $\sqrt{85}$

T	TERMINOLOGY--TRICHOTOMY LAW	
	<p style="text-align: center;"><b>TRICHOTOMY LAW</b></p> <p>For any two numbers <b>a</b> and <b>b</b>, exactly one of the following must occur:                      (1) <b>a = b</b>   (2) <b>a &lt; b</b>   (3) <b>a &gt; b</b></p> <p><b>a &lt; b</b> is read as "<b>a</b> is less than <b>b</b>"</p> <p><b>a &gt; b</b> is read as "<b>a</b> is greater than <b>b</b>"</p>	<p style="text-align: center;"><b>EXAMPLES</b></p> <p>Given the two numbers -3 and 4, we know that <math>-3 &lt; 4</math> (a negative number is always <b>less</b> than a positive number).                      Given the numbers 8 and 5 we know that <math>5 &lt; 8</math> or equivalently, <math>8 &gt; 5</math>.                      Given the decimals 0.34 and 0.84, we know that <math>0.34 &lt; 0.84</math> or <math>0.84 &gt; 0.34</math></p>
	<p>When a bar is placed over the decimal part of a number, it indicates that the number under the bar repeats indefinitely.</p>	<p><math>1.\bar{7} = 1.777\dots</math> (The 7 repeats.)  <math>0.\overline{32} = 0.323232\dots</math> (The 32 repeats.)  <math>4.1\bar{8} = 4.1888\dots</math> (The 8 repeats.)</p>

<b>ANSWERS</b>	1. $\frac{2}{5} < \frac{4}{9}$	2. $-\frac{2}{5} > -\frac{4}{9}$	3. $3.42 < 3.423$	4. $5.\overline{12} < 5.1\bar{2}$
	5. $\frac{3}{5} < 0.62$	6. $\sqrt{27} > 5$	7. $8.9 < \sqrt{85}$	

T	<b>TERMINOLOGY--SQUARE ROOTS</b>	
The square root of natural number $a$ , denoted by $\sqrt{a}$ is a number $b$ such that $b^2 = a$ . If the result is a whole number, the number has a <i>perfect</i> square root, otherwise the number is an <i>irrational</i> number (not rational).	$\sqrt{16} = 4$ because $4^2 = 16$ $\sqrt{4} = 2$ because $2^2 = 4$ $\sqrt{64} = 8$ because $8^2 = 64$ $\sqrt{8}$ , $\sqrt{2}$ , $\sqrt{15}$ , $\sqrt{17}$ and $\sqrt{131}$ are irrational numbers.	

The CLAST asks to compare *two decimals*, *two fractions*, a *fraction* and a *decimal* or an *irrational* number and a *decimal*. Here are the rules we need.

2	<b>TO COMPARE TWO FRACTIONS</b>	
<b>RULE</b>	<b>EXAMPLES</b>	
<p>To compare <i>two fractions</i> write them with a common denominator. The fraction with the larger numerator is greater. Alternatively, we can use the following "cross multiplication" rules:</p> $\frac{a}{b} < \frac{c}{d} \text{ means } a \times d < b \times c$ $\frac{a}{b} > \frac{c}{d} \text{ means } a \times d > b \times c$	<p>To compare <math>\frac{10}{18}</math> and <math>\frac{5}{9}</math>, write <math>\frac{5}{9}</math> with a denominator of 18. Thus, <math>\frac{5}{9} = \frac{5 \times 2}{9 \times 2}</math> or <math>\frac{10}{18}</math>.</p> <p>In this case, <math>\frac{5}{9} = \frac{10}{18}</math></p> <p>To compare <math>\frac{9}{13}</math> and <math>\frac{17}{22}</math>, we use the "cross multiplication rule".</p> <p><math>9 \times 22 = 198</math> and <math>13 \times 17 = 221</math>.</p> <p>Since <math>9 \times 22 = 198 &lt; 13 \times 17 = 221</math>, then</p> $\frac{9}{13} < \frac{17}{22}$	

3	<b>TO COMPARE TWO DECIMALS</b>	
<b>RULE</b>	<b>EXAMPLES</b>	
<p>To compare <i>two decimals</i>, write them in a column with the decimal points aligned. Compare corresponding digits starting at the left. When two digits differ, the number with the larger digit is the larger of the two numbers.</p>	<p>To compare <math>5.\overline{37}</math> and <math>5.3\overline{7}</math></p> <p><math>5.\overline{37} = 5.373737</math> (the 37 repeats)</p> <p><math>5.3\overline{7} = 5.377777</math> (the 7 repeats)</p> <p>The first three digits, 5, 3 and 7 are the same in both numbers. The fourth digit in the first number is 3, in the second number it is 7.</p> <p>Thus, the second number, <math>5.3\overline{7}</math>, is greater.</p> <p>We then write <math>5.3\overline{7} &gt; 5.\overline{37}</math></p>	

4	<b>TO COMPARE A FRACTION AND A DECIMAL</b>	
<b>RULE</b>	<b>EXAMPLES</b>	
To compare a <i>fraction</i> and a <i>decimal</i> , change the fraction to a decimal by dividing the numerator by the denominator and use the rule to compare decimals given in box 3 above.	To compare 0.20 and $\frac{1}{4}$ , write $\frac{1}{4}$ as a decimal by dividing 1 by 4.	
	$\begin{array}{r} 0.25 \\ 4 \overline{) 1.00} \\ \underline{- 8} \phantom{0} \\ 20 \\ \underline{- 20} \\ 0 \end{array}$	
	Since $0.20 < 0.25$ , we have $0.20 < \frac{1}{4}$	

5	<b>TO COMPARE AN IRRATIONAL WITH A DECIMAL</b>	
<b>RULE</b>	<b>EXAMPLES</b>	
To compare an irrational number of the form $\sqrt{a}$ with a decimal, compare the decimal with the closest perfect square root to $\sqrt{a}$ and use the fact that	To compare $\sqrt{70}$ and $7.\bar{8}$ note that	
$\sqrt{a} > \sqrt{b}$ means that $a > b$	$\sqrt{64} = 8 > 7.\bar{8}$	
and $\sqrt{a} < \sqrt{b}$ means that $a < b$ .	Since $\sqrt{70} > \sqrt{64} > 7.\bar{8}$ , $\sqrt{70} > 7.\bar{8}$	
	To compare 4.25 and $\sqrt{15}$ note that	
	$\sqrt{16} = 4 < 4.25$	
	Since $\sqrt{15} < \sqrt{16} < 4.25$ , $\sqrt{15} < 4.25$	

**CLAST EXAMPLES****Example**

4. Identify the symbol that should be placed in the box to form a true statement.

$$\frac{1}{5} \quad \square \quad -\frac{5}{9}$$

A. =      B. <      C. >

**Solution**

A positive number is always greater than a negative number, so  $\frac{1}{5}$  must be greater than  $-\frac{5}{9}$ . The correct answer is C.

**Example**

5. Identify the symbol that should be placed in the box to form a true statement.

$$\frac{5}{26} \square \frac{11}{20}$$

- A. =      B. <      C. >

**Solution**

We use the "cross multiplication" rule.

$$5 \times 20 = 100 \quad \text{and} \quad 26 \times 11 = 286$$

Since  $5 \times 20 < 26 \times 11$

$$\frac{5}{26} < \frac{11}{20}$$

The correct answer is **B**.

**Example**

6. Identify the symbol that should be placed in the space to form a true statement.

$$8.\overline{61} \quad \square \quad 8.6\overline{1}$$

- A. =      B. <      C. >

**Solution**

Write  $8.\overline{61} = 8.616161\dots$  (61 repeats)

and  $8.6\overline{1} = 8.611111\dots$  (1 repeats)

The first three digits from the left are the same in both numbers: 8, 6 and 1. The fourth digit in the first number is 6, in the second number it is 1. Thus, the first number is greater, that is,

$8.\overline{61} > 8.6\overline{1}$ . The correct answer is **C**.

**Example**

7. Identify the symbol that should be placed in the box to form a true statement.

$$-7\frac{1}{5} \square -6\frac{2}{5}$$

- A. =      B. <      C. >

**Solution**

Since  $-7 < -6$ ,  $-7\frac{1}{5} < -6\frac{2}{5}$ .

The correct answer is **B**.

**Example**

8. Identify the symbol that should be placed in the box to form a true statement.

$$0.82 \square \frac{17}{20}$$

- A. =      B. <      C. >

**Solution**

To compare 0.82 and  $\frac{17}{20}$  we write both numbers as decimals. Either divide 17 by 20 or note that:

$$\frac{17}{20} = \frac{17 \times 5}{20 \times 5} = \frac{85}{100} = 0.85$$

Since  $0.82 < 0.85$ , the correct answer is **B**.

**Example**

9. Identify the symbol that should be placed in the box to form a true statement.

$$\sqrt{30} \quad \square \quad 7.\bar{2}$$

- A. =      B. <      C. >

**Solution**

Since  $\sqrt{36} = 6 < 7.\bar{2}$  and

$$\begin{aligned} \sqrt{30} &< \sqrt{36} < 7.\bar{2} \\ \sqrt{30} &< 7.\bar{2} \end{aligned}$$

Thus, the correct answer is **B**.

**C. Number Sequences****Objective IIIA1****CLAST SAMPLE PROBLEMS**

Look for a common linear relationship, then find the missing term:

1. (4, 2) (0.16, 0.08) (-20, -10)  $(\frac{1}{5}, \frac{1}{10})$   $(\frac{1}{3}, ?)$       2. (13, 10) (7, 4) (-2, -5) (0, -3) (-6, ?)

3. Look for a common quadratic relationship, then find the missing term:

$$(36, 6) (64, 8) (1, 1) (\frac{1}{9}, \frac{1}{3}) (81, ?)$$

4. Identify the missing term in the arithmetic progression: 4, 9, 14, 19, 24, \_\_\_\_\_

5. Identify the missing term in the geometric progression: 8, -4, 2, -1,  $\frac{1}{2}$ , \_\_\_\_\_

6. Identify the missing term in the harmonic progression:  $\frac{1}{3}, \frac{1}{7}, \frac{1}{11}, \frac{1}{15}, \frac{1}{19},$  \_\_\_\_\_

A **sequence** or **progression** is a sequence of numbers arranged according to some given law. We shall discuss three types of sequences next.

TERMINOLOGY--SEQUENCES	
SEQUENCE	EXAMPLES
An <i>arithmetic progression</i> is a sequence in which each term after the first is obtained by adding a constant <b>c</b> , the <b>common difference</b> , to the preceding term.	1, 3, 5, ... (Add <b>2</b> to the preceding term.) -4, -1, 2, ... (Add <b>3</b> to the preceding term.)
A <i>geometric progression</i> is a sequence in which each term after the first is obtained by multiplying the preceding term by a constant <b>r</b> , the <b>common ratio</b> .	2, 6, 18, ... (Multiply the preceding term by <b>3</b> .) -10, 20, -40, ... (Multiply the preceding term by <b>-2</b> .)
A <i>harmonic progression</i> is a sequence in which each term after the first is obtained by adding a constant value to the denominator of each term.	$1, \frac{1}{2}, \frac{1}{3}, \dots$ (Add <b>1</b> to the denominator.) $\frac{4}{2}, \frac{4}{3}, \frac{4}{4}, \frac{4}{5}, \dots$ (Add <b>1</b> to the denominator.) This sequence is usually written as: $2, \frac{4}{3}, 1, \frac{4}{5}, \dots$

**ANSWERS**

1.  $\frac{1}{6}$

2. -9

3. 9

4. 29

5.  $-\frac{1}{4}$

6.  $\frac{1}{23}$

## CLAST EXAMPLES

## Example

10. Identify the missing term in the following geometric progression

$$-1, \frac{1}{4}, -\frac{1}{16}, \frac{1}{64}, -\frac{1}{256}, \text{---}$$

- A.  $\frac{1}{2048}$                       B.  $\frac{1}{1024}$   
 C.  $-\frac{1}{1024}$                       D.  $\frac{1}{4}$

## Example

11. Identify the missing term in the following arithmetic progression  
 6, 10, 14, 18, 22, ---

- A. 18              B. 25              C. 26              D. 88

## Example

12. Identify the missing term in the following harmonic progression

$$\frac{1}{2}, \frac{1}{5}, \frac{1}{8}, \frac{1}{11}, \frac{1}{14}, \text{---}$$

- A.  $\frac{1}{42}$               B.  $\frac{1}{17}$               C.  $\frac{1}{2}$               D. 17

## Solution

Since we have a geometric progression, we have to **multiply** by a constant to get the next term. What do we have to multiply -1 by in order to get the second term,  $\frac{1}{4}$ ?

The answer is  $-\frac{1}{4}$  (Check:  $-1 \times \left(-\frac{1}{4}\right) = \frac{1}{4}$ )

Thus, the missing term is  $\left(\frac{1}{4}\right) \times \left(-\frac{1}{256}\right)$

or  $\frac{1}{1024}$ . The answer is **B**.

**Note:** Another way to find out the number that each term must be multiplied by to get the next term, is to **divide** one term by the preceding term. Thus, if we divide  $\frac{1}{4}$  by -1, we obtain

$-\frac{1}{4}$  as before.

## Solution

Since we have an arithmetic progression, we have to **add** a constant to get the next term. What do we have to add to 6 in order to get the second term, 10? The answer is **4**

(Check:  $6 + 4 = 10$ .)

Thus, the missing term is  $22 + 4 = 26$

The answer is **C**.

## Solution

Since we have a harmonic progression, where all numerators are 1. What do we have to **add** to the denominator 2 in order to get the second denominator, 5. The answer is **3**

(Check:  $2 + 3 = 5$ )

Thus, the missing term is  $\frac{1}{14 + 3} = \frac{1}{17}$

The answer is **B**.

**Example**

- 13.** Look for a common linear relationship between the numbers in each pair. Then identify the missing term

$$(3,1), (0.6, 0.2), (-6, -2) \left(\frac{3}{2}, \frac{1}{2}\right) \left(\frac{1}{3}, \underline{\quad}\right)$$

- A.  $\frac{2}{3}$       B. 1      C.  $\frac{1}{9}$       D.  $\frac{3}{2}$

Note: *Linear relationships* involve adding, subtracting, multiplying or dividing one of the numbers in the pair to obtain the other number in the pair.

**Solution**

Look at the first ordered pair (3, 1). Note that the first number is **3** times the second number, or equivalently, that the second number is  $\frac{1}{3}$  of the first number. This is true in every pair. Thus, if we are given the pair  $(\frac{1}{3}, \underline{\quad})$ , the second number must be  $\frac{1}{3}$  of the first number, that is,  $\frac{1}{3}$  of  $\frac{1}{3}$  or  $\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$ . The correct answer is **C**.

## Section 1.4 Exercises

**WARM-UPS A***WRITE AS A FRACTION*

- |         |         |
|---------|---------|
| 1. 0.27 | 2. 0.46 |
| 3. 2.2  | 4. 3    |

*WRITE AS A DECIMAL*

- |                      |                      |
|----------------------|----------------------|
| 5. $\frac{3}{4}$     | 6. $\frac{3}{25}$    |
| 7. $\frac{9}{20}$    | 8. $\frac{7}{4}$     |
| 9. 150%              | 10. 450%             |
| 11. 18%              | 12. 49%              |
| 13. 3.2%             | 14. 8.9%             |
| 15. $8\frac{1}{4}\%$ | 16. $9\frac{3}{4}\%$ |



*WRITE AS A PERCENT*

- |     |                   |     |                   |
|-----|-------------------|-----|-------------------|
| 17. | 0.07              | 18. | 0.01              |
| 19. | 0.36              | 20. | 0.85              |
| 21. | 0.139             | 22. | 0.384             |
| 23. | 3.12              | 24. | 5.06              |
| 25. | $3\frac{1}{4}$    | 26. | $7\frac{1}{2}$    |
| 27. | $0.18\frac{1}{5}$ | 28. | $0.21\frac{2}{5}$ |
| 29. | $0.02\frac{4}{5}$ | 30. | $0.09\frac{1}{2}$ |
| 31. | $\frac{3}{20}$    | 32. | $\frac{5}{25}$    |
| 33. | $\frac{7}{8}$     | 34. | $\frac{1}{2}$     |
| 35. | $\frac{3}{8}$     | 36. | $\frac{6}{25}$    |

*WRITE AS A FRACTION*

- |     |                   |     |                   |
|-----|-------------------|-----|-------------------|
| 37. | 39%               | 38. | 12%               |
| 39. | $14\frac{1}{2}\%$ | 40. | $28\frac{3}{4}\%$ |

**CLAST PRACTICE A****PRACTICE PROBLEMS: Chapter 1, # 22-25**

41.  $0.94 =$
- |                    |         |                   |                      |
|--------------------|---------|-------------------|----------------------|
| A. $\frac{47}{50}$ | B. 9.4% | C. $\frac{47}{5}$ | D. $\frac{47}{50}\%$ |
|--------------------|---------|-------------------|----------------------|
42.  $0.24 =$
- |                   |         |                     |                   |
|-------------------|---------|---------------------|-------------------|
| A. $\frac{12}{5}$ | B. 2.4% | C. $\frac{6}{25}\%$ | D. $\frac{6}{25}$ |
|-------------------|---------|---------------------|-------------------|
43.  $387\% =$
- |          |           |         |          |
|----------|-----------|---------|----------|
| A. 0.387 | B. 3870.0 | C. 3.87 | D. 387.0 |
|----------|-----------|---------|----------|

44.  $305\% =$

- A. 3.05      B. 0.305      C. 3050.0      D. 305.0

45.  $\frac{13}{25} =$

- A. 5.2%      B. 0.52%      C. 0.052      D. 0.52

46.  $\frac{13}{20} =$

- A. 0.65      B. 6.5%      C. 0.65%      D. 0.065

**WARM-UPS B**

Which of the symbols =, < or > should be placed in the box to form a true statement?

47.  $-\frac{3}{8} \square \frac{33}{100}$

48.  $\frac{19}{100} \square -\frac{1}{5}$

49.  $\frac{3}{4} \square \frac{24}{37}$

50.  $\frac{3}{34} \square \frac{2}{21}$

51.  $6.82 \square 6.872$

52.  $5.74 \square 5.723$

53.  $3.1\bar{7} \square 3.1\bar{7}$

54.  $9.9\bar{6} \square 9.9\bar{6}$

55.  $0.28 \square \frac{7}{25}$

56.  $0.30 \square \frac{3}{10}$

57.  $\sqrt{50} \square 6.98$

58.  $\sqrt{20} \square 5.2$

59.  $9.8 \square \sqrt{80}$

60.  $6.71 \square \sqrt{50}$

**CLAST PRACTICE B**

**PRACTICE PROBLEMS: Chapter 1, #26-30**

IN PROBLEMS 61-70 IDENTIFY THE SYMBOL THAT SHOULD BE PLACED IN THE BOX TO FORM A TRUE STATEMENT.

61.  $\frac{6}{17} \square -\frac{14}{15}$

62.  $-\frac{3}{4} \square \frac{4}{7}$

- A. =      B. <      C. >

- A. =      B. <      C. >

63.  $\frac{9}{15} \square \frac{15}{25}$

64.  $\frac{3}{4} \square \frac{24}{29}$

- A. =      B. <      C. >

- A. =      B. <      C. >

65.  $3.283 \square 3.28$

A. =      B. &lt;      C. &gt;

66.  $2.84 \square 2.877$

A. =      B. &lt;      C. &gt;

67.  $1.\overline{68} \square 1.\overline{68}$

A. =      B. &lt;      C. &gt;

68.  $2.\overline{96} \square 2.\overline{96}$

A. =      B. &lt;      C. &gt;

69.  $0.55 \square \frac{11}{20}$

A. =      B. &lt;      C. &gt;

70.  $0.34 \square \frac{7}{20}$

A. =      B. &lt;      C. &gt;

71.  $\sqrt{37} \square 5.3$

A. =      B. &lt;      C. &gt;

72.  $5.9 \square \sqrt{40}$

A. =      B. &lt;      C. &gt;

**WARM-UPS C**

73. Identify the missing term in the following geometric progression:

$2, -\frac{2}{3}, \frac{2}{9}, -\frac{2}{27}, \frac{2}{81}, \text{---}$

74. Identify the missing term in the following geometric progression:

$4, -\frac{4}{3}, \frac{4}{9}, -\frac{4}{27}, \frac{4}{81}, \text{---}$

75. Identify the missing term in the following arithmetic progression:

$-1, 1, 3, 5, 7, \text{---}$

76. Identify the missing term in the following arithmetic progression:

$-10, -12, -14, -16, -18, \text{---}$

77. Identify the missing term in the following harmonic progression:

$\frac{1}{6}, \frac{1}{15}, \frac{1}{24}, \frac{1}{33}, \frac{1}{42}, \text{---}$

78. Identify the missing term in the following harmonic progression:

$\frac{1}{10}, \frac{1}{20}, \frac{1}{30}, \frac{1}{40}, \frac{1}{50}, \text{---}$

*In Problems 79 and 80, look for a common linear relationship between the numbers in each pair. Then identify the missing term.*

79.  $(20, 5), (1.2, 0.3), (-8, -2), (\frac{1}{10}, \frac{1}{40}), (44, 11), (\frac{1}{3}, \text{---})$

80.  $(12, 4), (0.6, 0.2), (-3, -1), (\frac{1}{5}, \frac{1}{15}), (24, 8), (\frac{1}{2}, \text{---})$

**CLAST PRACTICE C****PRACTICE PROBLEMS: Chapter 1, #31-33**

81. Look for a common linear relationship between the numbers in each pair. Then identify the missing term.

$$(12, 3), (0.4, 0.1), (-20, -5), \left(\frac{1}{5}, \frac{1}{20}\right), (24, 6), \left(\frac{1}{3}, \underline{\hspace{1cm}}\right)$$

- A.  $\frac{3}{2}$                       B.  $\frac{2}{3}$                       C. 12                      D.  $\frac{1}{12}$

- 82.** Identify the missing term in the following geometric progression:

$$5, -\frac{5}{3}, \frac{5}{9}, -\frac{5}{27}, \frac{5}{81}, \underline{\hspace{1cm}}$$

- A.  $-\frac{5}{243}$                       B.  $\frac{5}{243}$                       C.  $-\frac{5}{486}$                       D.  $\frac{1}{3}$

**EXTRA CLAST PRACTICE**

83. Identify the symbol that should be placed in the box to form a true statement.

$$\frac{5}{7} \square \frac{2}{3}$$

- A. =                      B. <                      C. >

84. Identify the symbol that should be placed in the box to form a true statement.

$$-\frac{5}{7} \square -\frac{2}{3}$$

- A. =                      B. <                      C. >

85.  $5.3 \square 5\frac{3}{10}$

- A. =                      B. <                      C. >

86.  $2\frac{4}{5} =$

- A. 0.3771                      B. 2.08                      C. 2.8%                      D. 2.8

## 1.5 PERCENTS

In this section we shall use the information we have learned about fractions, decimals and percents to solve percent problems. We begin by studying percent increase and decrease, then discussing three types of percent problems, and we end the section with word problems involving percents.

### A. Calculating Percent Increase or Decrease

#### Objective IA3

#### CLAST SAMPLE PROBLEMS

1. If you increase 600 by 25% of itself, what is the result?
2. A computer that costs \$800 is discounted 20%. What is the new cost of this computer?
3. If 400 is decreased to 100, what is the percent of decrease?
4. A worker's salary changes from \$1000 to \$1200 per week. What is the percent increase?

When you are given a number and you are asked to find the percent increase or decrease, you can use the following rules:

1	FINDING PERCENT INCREASE OR DECREASE	
	<i>RULE</i>	<i>EXAMPLES</i>
	<p><b>Percent change</b> = <math>\frac{\text{change}}{\text{original number}}</math></p> <p>Note that the answer <b>must be</b> written as a percent, so look at the denominator of the fraction (the original number) and try to multiply numerator and denominator by a number that will make the denominator 100. If you can not do this, divide the numerator (<b>change</b>) by the denominator (<b>original number</b>) and write the resulting decimal as a percent.</p>	<p>If 20 is decreased to 15, what is the percent decrease?</p> <p>The <b>change</b> is <math>20 - 15 = 5</math> and the <b>original number</b> is 20</p> <p><b>Percent change</b> = <math>\frac{5}{20} = \frac{25}{100} = 25\%</math></p> <p>Note that we multiplied the numerator and denominator of <math>\frac{5}{20}</math> by 5 to make the denominator 100. This is easier than dividing 5 by 20 and changing the answer to a percent!</p>

#### ANSWERS

1. 750

2. 640

3. 75%

4. 20%

1	FINDING PERCENT INCREASE OR DECREASE (CONT.)
<p>In some problems, an increase or decrease is given as a <b>percent</b> of a number. Here is the rule you need:</p> <p><b>To find a percent of a number, write the percent as a decimal and multiply by the number.</b></p>	<p>If you increase 50 by 60% of itself, what is the result? Let us translate the problem:</p> $\begin{array}{rcccl} & \text{Increase 50 by} & & \text{60\% of itself} & \\ & \downarrow & & \downarrow & \\ = & 50 & + & (60\% \text{ of } 50) & \\ = & 50 & + & (0.60 \times 50) & \\ = & 50 & + & 30 & \\ = & 80 & & & \end{array}$ <p>Thus, when we <b>increase</b> (add to) 50 by 60% of itself (<math>0.60 \times 50 = 30</math>), the result is <math>50 + 30 = 80</math>.</p>

**CLAST EXAMPLES****Video Example**

1. If 30 is decreased to 6, what is the percent decrease?

A. 8%    B. 24%    C. 20%    D. 80%

Remember to reduce your fraction and write it with a denominator of 100 (if possible) to make the percent conversion easier.

**Solution**

If the original number **30** is decreased to 6, the **change** is  $30 - 6$  or 24.

$$\text{Now, \% change} = \frac{\text{change}}{\text{original number}}$$

$$= \frac{24}{30} = \frac{4}{5} = \frac{80}{100}$$

Thus, the percent decrease is 80%, **D**.

**Example**

2. If you increase 30 to 36, what is the percent increase?

A. 600%    B. 2%    C. 20%    D. 83%

**Solution**

Here the change is  $36 - 30 = 6$ , and the original number is 30. We have:

$$\begin{aligned} \% \text{ change} &= \frac{\text{change}}{\text{original number}} \\ &= \frac{6}{30} = \frac{1}{5} = \frac{20}{100} \end{aligned}$$

The correct answer is **C**.

**Example**

3. If you increase 10 by 20% of itself, what is the result?

- A. 8      B. 2      C. 12      D. 30

Note that the answer has to be more than 10 (**C** or **D**) since 10 is being *increased* by 20% of itself.

**Solution**

Note that you do **not** use the formula here. We must **increase** 10 by 20% of itself. Here is the translation:

$$\begin{array}{rcll}
 \text{Increase 10 by} & \text{20\%} & \text{of itself} & \\
 \downarrow & & \downarrow & \downarrow \\
 10 & + & 20\% & \text{of 10} \\
 = 10 & + & 0.20 \times 10 & \\
 = 10 & + & 2 & \\
 = 12 & & & 
 \end{array}$$

The answer is **C**.

**B. Percent Problems****Objective IA4****CLAST SAMPLE PROBLEMS**

- |                              |                              |                              |
|------------------------------|------------------------------|------------------------------|
| 1. What is 3% of 200?        | 2. What is 120% of 26?       | 3. 30 is 15% of what number? |
| 4. 6 is 120% of what number? | 5. 12 is what percent of 60? | 6. 64 is what percent of 32? |

Percent problems require finding the **base** or **total (B)**, the **percentage (P)** or the **percent** or **rate (R)** by using the formula  $P = B \times R$  or  $R = \frac{P}{B}$  or  $B = \frac{P}{R}$ . Naturally, there are three types of percent problems in the CLAST. The definitions we need are next.

T	TERMINOLOGY--BASE, PERCENTAGE AND RATE	
	<i><b>BASE, PERCENTAGE AND RATE</b></i>	<i><b>EXAMPLES</b></i>
	1. The <b>base</b> or <b>total</b> is the standard used for comparison purposes.	In the expression 30% of <b>50</b> = 15, the <b>base</b> is <b>50</b> (the total)
	2. The <b>percentage</b> is the part being compared with the base or total.	In the expression 30% of 50 = <b>15</b> , the number <b>15</b> is the <b>percentage</b> .
	3. The <b>percent</b> or <b>rate</b> is the part indicating the ratio of the percentage to the base.	In the expression <b>30%</b> of 50 = 15, the <b>percent</b> or <b>rate</b> is <b>30%</b> .
		Note that $\frac{15}{50} = \frac{30}{100} = 30\%$

<b>ANSWERS</b>	1. 6	2. 31.2	3. 200	4. 5	5. 20%	6. 200%
----------------	------	---------	--------	------	--------	---------

**CLAST EXAMPLES****Example**

4. What is 10% of 90?
- A.  $\frac{1}{9}$       B. 9      C. 90      D. 9%

Stop and think!  $10\% = \frac{10}{100} = \frac{1}{10}$ , so you need only find  $\frac{1}{10}$  of 90 or 9. The answer must be **B**. Now you can look at the solution.

**Example**

5. What is 120% of 30?
- A. 0.25      B. 25      C. 36      D. 3.6

Remember your estimation! 100% of 30 is 30, so 120% of 30 must be **more** than 30. The only possible answer is 36. Look at the solution to see why.

**Example**

6. Find  $50\frac{1}{2}\%$  of 90.
- A. 454.5      B. 4.545
- C. 45.45      D. 0.18

Since 50% of 90 is 45, the answer must be a little more than 45. It must be **C**. The solution shows you why, but be careful with the decimal point when multiplying decimals.

**Solution**

Let us translate the problem:

$$\begin{array}{ccccccc} \text{What} & \text{is} & \text{10\% of 90} & & & & \\ \downarrow & \downarrow & \downarrow & & & & \\ n & = & 0.10 \times 90 & & & & \\ & & = 9 & & & & \end{array}$$

The answer is **B**.

Note: This **same** question, can be asked in **3** different ways:

1. What is 10% of 90
2. 10% of 90 =
3. Find 10% of 90

Don't be fooled by the wording, all these problems have the same answer.

**Solution**

As usual, let us translate the problem

$$\begin{array}{ccccccccc} \text{What} & \text{is} & \text{120\%} & \text{of} & \text{30} & ? & & & \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & & & \\ P & = & 1.2 & \times & 30 & & & & \\ P & = & 36 & & & & & & \end{array}$$

Now you can see that the answer is **C**

**Solution**

Find  $50\frac{1}{2}\%$  of 90 is translated as:

$$\begin{aligned} P &= 50\frac{1}{2}\% \text{ of } 90, \text{ that is,} \\ P &= 50.5\% \times 90 \text{ (Since } \frac{1}{2} = 0.5) \\ &= 0.505 \times 90 \\ &= 45.450 \\ &= 45.45 \end{aligned}$$

The answer is **C** as expected.



**Example**

7. 5 is what percent of 20?

- A. 0.25    B. 4%    C.  $\frac{1}{4}$     D. 25%

Since you are asked for the **percent** your answer must be written as a **percent**. (B or D).

This question can also be written as:

1. What percent of 20 is 5?
2. Find what percent of 20 is 5.

**Example**

8. 60 is what percent of 40?

- A. 200%    B. 150%    C. 1.5%    D.  $66\frac{2}{3}\%$

Note that since 60 is greater than 40, the answer must be more than 100% (**A** or **B**).

**Example**

9. 12 is 40% of what number?

- A. 0.3    B. 333    C. 4.8    D. 30

Hopefully, you can see that 12 is 50% of 24, so answers A, B and C are not plausible. It must be **D**.

The solution on the right hand side relies on the fact that both sides of an equation can be divided by the same non-zero number or on the fact that

$$B = \frac{P}{R} = \frac{12}{0.40}.$$

Also, you have to know how to divide by a decimal. (Remember, when dividing by a decimal, we start by making the divisor a whole number.)

**Solution**

$$\begin{array}{ccccccc} \underline{5} & \text{is} & \underline{\text{what percent}} & \underline{\text{of}} & \underline{20} & \text{means} & \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & \\ 5 & = & R & \times & 20 & & \end{array}$$

Since we need to find R, we divide both sides of the equation by **20** obtaining

$$\frac{5}{20} = \frac{R \times 20}{20} = R$$

(Remember that  $R = \frac{P}{B}$ , so  $R = \frac{5}{20}$ .)

Now, the answer must be written as a percent, so we write

$$R = \frac{5}{20} = \frac{25}{100} = 25\% \quad \text{The answer is } \mathbf{D}.$$

**Solution**

We translate the problem as:  $60 = R \times 40$

or, dividing by 40,  $\frac{60}{40} = R$

Thus,  $R = \frac{3}{2} = 1.5 = 150\%$ .

The answer **has** to be a percent, and it is **B**.

**Solution**

The translation is:

$$\begin{array}{ccccccc} \underline{12} & \text{is} & \underline{40\%} & \underline{\text{of}} & \underline{\text{what number}} & ? & \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & \\ 12 & = & 0.40 & \times & B & & \end{array}$$

Since we need to find B, which is multiplied by 0.40, we **undo** this multiplication by dividing both sides by 0.40. Here is the original equation:

$$\begin{array}{l} 12 = 0.40 \times B \\ \text{Divide by } \mathbf{0.40} \quad \frac{12}{\mathbf{0.40}} = \frac{0.40}{\mathbf{0.40}} \times B = B \end{array}$$

Thus,  $B = \frac{12}{0.40} = \frac{12 \times 100}{0.40 \times 100} = \frac{1200}{40}$  or 30.

The answer is **D**.

## C. Real-World Problems Involving Percent

### Objective IVA2

### CLAST SAMPLE PROBLEMS

1. A company produced 500 computers at a plant in California and 800 computers at a plant in Florida. 20% of the computers from the California plant were lime colored and 10% of the Florida computers were lime colored. How many lime colored computers were produced?
2. Pedro was told that if he used the ABC phone card he would only pay 60% as much as with his current phone card. The amount he paid for his current phone card was \$90. How much money would Pedro save by using the ABC card?
3. A computer regularly sells for \$800. If the computer is purchased at the University bookstore, students receive a 5% discount. The tax rate is 7%. The student decides to buy the computer at the University bookstore. How much will the student have to pay for the computer including the tax?

Now that you know how to solve percent problems you need to know how to apply this knowledge to solve real-world problems involving percents. This is a CLAST area in which many students encounter difficulties, but don't panic; we will give you a proven method for tackling any word problem. The procedure is as easy as 1-2-3-4-5!

Here are the steps you need.

1. **R**ead the problem. Not once or twice but until you understand it.
2. **S**elect the unknown, that is, find out what the problem asks for.
3. **T**ranslate the problem and think of a plan to solve it
4. **U**se the techniques you are studying to carry out the plan.
5. **V**erify the answer.

If you look at the first letter in each of the steps, you will realize why we call this the **R-S-T-U-V** procedure. Use it from here on!

### ANSWERS

1. 180

2. \$36

3. \$813.20

## CLAST EXAMPLE

## Example

**10.** Gladys was told that if she tuned her car she would use only 80% as much gasoline. If she was using 120 gallons of gas every month, how many gallons did Gladys save by having her car tuned?

- A. 96 gallons                      B. 16.6 gallons  
C. 24 gallons                      D. 40 gallons

## Solution

- 1. Read the problem carefully.**
- 2. Select the unknown.**  
We want to find the amount of gallons **g** saved by tuning the car.
- 3. Translate the problem and think of a plan to solve it.**  
Since she uses 80% as much gas, Gladys **saves** 20% (100% - 80%) of the 120 gallons. Thus,  
The gallons **g** saved are 20% of 120
- 4. Use the techniques we studied to carry out the plan.**  
 $g = 20\%$  of 120  
 $= 0.20 \times 120$   
 $= 24.0$  gallons

The answer is **C**.

- 5. Verify the answer.**  
If 24 gallons out of 120 are saved,  
the percent saved is  $\frac{24}{120} = \frac{1}{5} = 20\%$

## Example

**11.** During a certain week, 3% of the computers manufactured at plant A were defective, and 10% of the computers manufactured at plant B were defective. Plant A produced 300 computers and plant B produced 250. How many defective computers were produced?

- A. 9              B. 25              C. 34              D. 72

## Solution

- 1. Read the problem carefully**
- 2. Select the unknown, the number of defective computers**
- 3. Translate the problem.**  
3% of 300 and 5% of 250 are defective
- 4. Use the techniques studied.**  
3% of 300 =  $0.03 \times 300 = 9$   
10% of 250 =  $0.10 \times 250 = 25$

The answer is  $25 + 9 = 34$ , or **C**

## Section 1.5 Exercises

### WARM-UPS A

1. If 20 is decreased to 8 what is the percent decrease?
2. If 30 is decreased to 12, what is the percent decrease?
3. If you decrease 32 to 24, what is the percent decrease?
4. If 30 is increased to 36, what is the percent increase?
5. If 50 is increased to 70, what is the percent increase?
6. If you increase 50 to 80, what is the percent increase?
7. If you increase 80 by 30% of itself, what is the result?
8. If you increase 20 by 150% of itself, what is the result?
9. If you decrease 80 by 40% of itself, what is the result?
10. If you decrease 32 by 25% of itself, what is the result?

### CLAST PRACTICE A

### PRACTICE PROBLEMS: Chapter 1, # 34-37

11. If 20 is decreased to 12, what is the percent decrease?  
A. 60%            B. 8%            C. 40%            D. 4%
12. If you increase 50 to 60, what is the percent increase?  
A. 2%            B. 20%            C. 1000%            D. 83%
13. If you increase 50 by 80% of itself, what is the result?  
A. 140            B. 90            C. 48            D. 10
14. If you decrease 40 by 75% of itself, what is the result?  
A. 35            B. 30            C. 10            D. 70

**WARM-UPS B**

- |                                   |                                    |
|-----------------------------------|------------------------------------|
| 15. What is 20% of 70?            | 16. What is 35% of 72?             |
| 17. What is 120% of 85?           | 18. What is 140% of 95?            |
| 19. 150% of 75 is _____           | 20. 140% of 92 is _____            |
| 21. Find $70\frac{1}{2}\%$ of 70. | 22. Find $20\frac{1}{4}\%$ of 40.  |
| 23. Find $20\frac{3}{4}\%$ of 80. | 24. Find $50\frac{1}{4}\%$ of 300. |
| 25. 7 is what percent of 35?      | 26. 9 is what percent of 18?       |
| 27. 24 is what percent of 6?      | 28. 36 is what percent of 24?      |
| 29. 12 is 40% of what number?     | 30. 18 is 30% of what number?      |
| 31. 6 is 60% of what number?      | 32. 42 is 84% of what number?      |
| 33. 0.8 is 40% of what number?    | 34. 16 is 16% of what number?      |

**CLAST PRACTICE B****PRACTICE PROBLEMS: Chapter 1, # 38-41**

35. What is 90% of 70?  
 A. 630      B. 63%      C. 63      D.  $\frac{9}{7}$
36. What is 125% of 92?  
 A. 0.736      B. 115      C. 73.6      D. 11.5
37. Find  $60\frac{1}{2}\%$  of 40  
 A. 242      B. 2420      C. 2.420      D. 24.20
38. 6 is what percent of 24?  
 A. 0.25      B.  $\frac{1}{4}$       C. 4%      D. 25%
- 39.** 90 is what percent of 60?  
 A. 1.5%      B.  $66\frac{2}{3}\%$       C. 150%      D. 200%
40. 30 is 60% of what number?  
 A. 200      B. 50      C. 18      D. 0.5

**WARM-UPS C**

41. This year Pedro made 80% as much money as last year. If Pedro made \$12,500 last year, what was the decrease in Pedro's pay?
42. Maria was told that if she turned her air conditioner off, her electric bill would be 80% of last month's bill. If her bill last month amounted to \$120, how much would Maria save?
43. An item was being sold for 70% of its regular price of \$40. How much would you save if you bought this item?
44. An article that regularly sells for \$25 is on sale at 20% off. How much do you save by buying this article, and what is the sale price?
45. Twanda was told that by insulating her hot water heater, she would only use 90% as much electricity. If her previous bill was for \$98, how much did Twanda save by insulating her heater?
46. V. Tran was told that if he shopped at the Mart, he would only spend 75% as much. If Tran's previous bill was for \$128, how much would he save if he shopped at the Mart?
47. A discount store sells merchandise at 60% of its regular price. If an item regularly sold for \$120, how much would you save by buying it at the discount store?
48. An airplane ticket can be bought for 80% of its regular price. If the ticket is regularly sold for \$290, how much would you save by buying the ticket at the discounted price?

**CLAST PRACTICE C****PRACTICE PROBLEMS: Chapter 1, # 42-44**

49. Tyrone bought an economy car that used only 65% as much gas as his previous car. If Tyrone's old car used 80 gallons of gas last month, how much gas did Tyrone save by buying the economy car?
- A. 52 gallons      B. 28 gallons      C. 43.75 gallons      D. 30 gallons
50. Jane was told that if she bought her appliances at a discount store, she would only pay 75% as much. If the regular price for a refrigerator was \$588, how much did Jane save by buying at the discount store?
- A. \$441      B. \$161      C. \$147      D. \$133

**EXTRA CLAST PRACTICE**

51. An item that sells for \$250 is put on sale at \$200. What is the percent of decrease?
- A. 20%                      B. 25%                      C. 50%                      D. 80%
52. Maria weighs 120 pounds. Her weight increased by 15%. What is her new weight?
- A. 18 pounds              B. 138 pounds              C. 102 pounds              D. 135 pounds
53. The main ingredient in a popular drink is sugar. In fact, the drink is 30% sugar. A person consumes 20 ounces of the drink. What amount of non-sugar ingredients was consumed by the person?
- A. 24 ounces              B. 60 ounces              C. 6 ounces                      D. 14 ounces
54. There are 265 students in a class. 190 are females and the rest are males. If 76 females and 25 males receive A's or B's, what percent of the males will receive an A or B?
- A. 29%                      B.  $33\frac{1}{3}\%$                       C. 38%                      D. 40%

## 1.6 WORD PROBLEMS

In this section, we shall continue the study of word problems. The difference between this section and the previous one is that the problems here **do not** involve percents but rather concentrate on applications involving decimals, fractions and a little number theory.

We shall again use the R-S-T-U-V procedure to solve these problems.

### A. Real-World Problems Not Involving Percent

#### Objective IVA1

#### CLAST SAMPLE PROBLEMS

1. A plumber charges \$35 for a house call plus \$25 an hour for labor. Find the cost of a plumbing job that took two hours to complete.
2. Each of two plants produce 240 VCR's per day. On a certain day,  $\frac{5}{6}$  of the production of one plant and  $\frac{7}{8}$  of the other are grape colored. How many are **not** grape-colored?
3. The scale on a map is  $1\frac{1}{2}$  inches equals 30 miles. What is the distance between two points that are 16 inches apart on the map?

#### CLAST EXAMPLES

- | Example                                                                                                                                       |          |
|-----------------------------------------------------------------------------------------------------------------------------------------------|----------|
| 1. A car rents for \$180 per week plus \$0.25 per mile. Find the cost of renting this car for a two-week trip of 400 miles for a family of 4. |          |
| A. \$280                                                                                                                                      | B. \$380 |
| C. \$460                                                                                                                                      | D. \$760 |

Before you go on, note that the fact that the family consists of 4 members is immaterial! The cost would be the same regardless of the number of family members.

#### Solution

1. **Read the problem.**
2. **Select the unknown.**  
We want to find the cost of renting the car for a 400 mile, two-week trip.
3. **Translate the problem and think of a plan.**  
The cost C for the car consists of two parts: the weekly fee (\$180 per week) and the mileage charge (\$0.25 per mile) so we have to find the cost for the 2 weeks and add the mileage charge.
4. **Use arithmetic to solve.**  
Weekly fee:  $2 \times \$180 = \$360$   
Mileage charge:  $\$0.25 \times 400 = \$100$   
Total Cost =  $\$360 + \$100 = \$460$ .  
The correct answer is **C**.
5. **The verification is left for you to do.**

#### ANSWERS

1. \$85

2. 70

3. 320



**Example**

2. A 4-ounce can of orange juice concentrate costs \$0.40 and a 10-ounce can costs \$1.20. How much money can be saved by buying 120 ounces of the more economical size?

- A. \$0.80                      B. \$12.00  
C. \$2.40                      D. \$14.40

You can do this problem by finding out the cost per ounce for the 4-ounce can ( $\frac{\$0.40}{4} = \$0.10$ )

and the 10-ounce ( $\frac{\$1.20}{10} = \$0.12$ ), and then find the cost of 120 ounces of each, but it is easier to start the computation by comparing the cost of 120 ounces of each.

**Solution**

1. **Read the problem.**

2. **Select the unknown.**

We want to know the price of 120 ounces costing \$0.40 for 4-ounces and the price of 120 ounces costing \$1.20 for 10-ounces

3. **Translate the problem and think of a plan.**

We can buy 4-ounces of concentrate for \$0.40, so to have 120 ounces we need to buy  $\frac{120}{4} = 30$

of the 4-ounce cans or  $\frac{120}{10} = 12$  of the 10-ounce cans. We can then compare prices and find out how much is **saved**.

4. **Use arithmetic to solve.**

4-ounce cans:  $30 \times \$0.40 = \$12.00$

10-ounce cans:  $12 \times \$1.20 = \$14.40$

Savings:  $\$14.40 - \$12.00 = \$2.40$

The correct answer is **C**.

5. **Verify this!**

**Example**

3. The fine for speeding in Florida is \$54 plus \$4 per each mile over the speed limit. A person paid a \$102 fine, how many miles over the speed limit was this person driving?

- A. 25.5    B. 26    C. 12    D. 13

Note that you have to pay the initial \$54 regardless of your speed. You can also do this problem by multiplying each answer by \$4 and adding \$54 until you get \$102. Note that A and B will not work!

**Solution**

1. **Read the problem.**

2. **Select the unknown.**

We want to find the number of miles over the speed limit.

3. **Translate the problem and think of a plan.**

The fine is \$54 initially **plus** the \$4 per mile over the speed limit.

4. **Use arithmetic to solve.**

Since the person paid \$102 - \$54 or \$48 for the miles over the speed limit the person was going  $\frac{48}{4} = 12$  miles over the speed limit. The

answer is **C**.

5. **Verify this!**

**Example**

4. A bookstore ordered 30 books for a class of 25 students. Each book cost the store \$15 and was to be sold for \$20. The bookstore must pay a service charge of \$2 for each unsold book it returns to the publisher. If the bookstore had to return 5 books, how much profit did the bookstore make?

- A. \$115                      B. \$490  
C. \$140                      D. \$125

Note that the cost of returns must be subtracted from the initial profit.

**Example**

5. A test consisted of 40 multiple choice and 30 essay questions. If Allyson got  $\frac{3}{10}$  of the multiple choice and  $\frac{3}{5}$  of the essay questions correct, how many questions did Allyson miss?

- A. 28    B. 41    C. 12    D. 40

**Important:** We want to find the number of questions missed, not the number of correct questions.

**Solution**

1. **Read the problem.**

2. **Select the unknown.**

We want to find how much profit the bookstore made.

3. **Translate the problem and think of a plan.**

We have to find the profit per book, the profit for 25 books, and subtract the service charge for returning 5 books.

4. **Use arithmetic to solve.**

$$\text{Profit per book: } \$20 - \$15 = \$5$$

$$\text{Profit for 25 books: } \$5 \times 25 = \$125$$

$$\text{Less service charge: } \$2 \times 5 = -\$10$$

$$\text{Total profit} = \$115$$

The correct answer is **A**.

5. **Verify this!**

**Solution**

1. **Read the problem.**

2. **Select the unknown.**

We want the number of questions missed.

3. **Translate the problem and think of a plan.**

Since Allyson got  $\frac{3}{10}$  of the multiple choice questions, he missed  $\frac{7}{10}$  of 40.

He also missed  $\frac{2}{5}$  of the 30 essay questions.

4. **Use arithmetic to solve.**

$$\text{Missed multiple choice: } \frac{7}{10} \times 40 = 28$$

$$\text{Missed essay: } \frac{2}{5} \times 30 = 12$$

$$\text{Total missed: } 28 + 12 = 40$$

The answer is **D**.

5. **The verification is left for you.**

## B. Number Theory

### Objective IVA3

### CLAST SAMPLE PROBLEMS

- How many whole numbers leave a remainder of 2 when divided into 56 and a remainder of 1 when divided into 28?
- Find the smallest positive multiple of 3 that leaves a remainder of 3 when divided by 5 and a remainder of 5 when divided by 7.

The problems here will deal with the structure and logic of arithmetic as it pertains to number theory. Most of the problems deal with multiples, factors, remainders and divisibility. Here is the terminology we need.

TERMINOLOGY--MULTIPLES AND DIVISIBILITY	
<b>MULTIPLES</b>	<b>EXAMPLES</b>
<p>A number <math>m</math> is a <b>multiple</b> of <math>n</math> if:  <math display="block">m = n \times k</math>           where <math>k</math> is a natural number.            We also say that <math>n</math> is <b>factor</b> of <math>m</math>.</p> <p>The factors of a number <math>n</math> always include <b>1</b> and <math>n</math>.</p>	<p>8 is a multiple of <b>2</b> because <math>8 = 2 \times 4</math>            27 is a multiple of <b>3</b> since <math>27 = 3 \times 9</math>            42 is a multiple of <b>7</b> since <math>42 = 7 \times 6</math>  <b>2</b> and <b>4</b> are factors of 8  <b>3</b> and <b>9</b> are factors of 27  <b>7</b> and <b>6</b> are factors of 42.            The factors of 2 include <b>1</b> and <b>2</b>.            The factors of 42 include <b>1</b> and <b>42</b>.</p>
<b>DIVISIBILITY</b>	<b>EXAMPLES</b>
<p>A number <math>n</math> is <b>divisible</b> by a number <math>m</math> if  <math>n = m \times k</math> for some natural number <math>k</math>. Note            that this definition means that <math>n</math> is divisible by  <math>m</math> if <math>n</math> is a <b>multiple</b> of <math>m</math>.            If <math>n</math> is not divisible by <math>m</math>, then there is a            remainder <math>r</math> and we can write:  <math display="block">n \div m = q \text{ R } r</math>           This means that <math>n</math> divided by <math>m</math> yields a            quotient <math>q</math> with a remainder <math>r</math>. In this case  <math display="block">n = m \times q + r</math></p>	<p>24 is divisible by <b>6</b> since <math>24 = 6 \times 4</math>            Note that 24 is a multiple of <b>6</b>.            49 is divisible by 7 since <math>49 = 7 \times 7</math>            Note that 49 is a multiple of <b>7</b>.            On the other hand, 49 is not divisible by 3            (try it!)  <math>49 \div 3 = 16 \text{ R } 1</math> (since <math>49 = 3 \times 16 + 1</math>)  <math>18 \div 5 = 3 \text{ R } 3</math> (since <math>18 = 5 \times 3 + 3</math>)  <math>29 \div 4 = 7 \text{ R } 1</math> (since <math>29 = 4 \times 7 + 1</math>)            and <math>53 \div 28 = 1 \text{ R } 25</math> (<math>53 = 28 \times 1 + 25</math>)</p>

#### CLAST EXAMPLE

Example	Solution
<p>6. Find the smallest positive multiple of 6 which leaves a remainder of 6 when divided by 10 and a remainder of 8 when divided by 14.</p> <p>A. 36    B. 18    C. 48    D. 53</p>	<p>The best approach is to determine which of the choices satisfies the given conditions. Since we want a multiple of 6, choice <b>D</b> is eliminated. When 36 is divided by 10 the remainder is 6. Also, when 36 is divided by 14 the remainder is 8 as desired. Thus, <b>A</b> is the correct answer. (Try <b>B</b> and <b>C</b>.)</p>

#### ANSWERS

1. Three (3, 9 and 27)

2. 33

**CLAST EXAMPLE****Example**

7. Find the largest positive integer which is a factor of both 18 and 30 and is also a factor of the difference of 18 and 30.
- A. 2      B. 3      C. 6      D. 12

**Solution**

We start by listing the factors of 18, 30 and the *difference* of 18 and 30, which is 12.

Factors of 18: 1, 2, 3, 6, 9, 18

Factors of 30: 1, 2, 3, 5, 6, 10, 15, 30

Factors of 12: 1, 2, 3, 6, 12

The largest positive integer appearing on all of the three lists is 6. The answer is **C**.

**Example**

8. How many whole numbers leave a remainder of 3 when divided into 31 and a remainder of 4 when divided into 25?
- A. 0      B. 1      C. 2      D. 3

**Note:** This time, we are not looking for a particular number. We are looking for *how many* numbers will satisfy the two given conditions.

**Solution**

Start with the 25 (the smaller of the two given numbers). Try dividing by 2 (the remainder is 1), by 3 (the remainder is 1) and so on. The numbers that leave a remainder of 4 when divided into 25 are **7** and **21**. When the **7** is divided into 31, the remainder is indeed 3. When the **21** is divided into 31, the remainder is 10. This means that there is only one (1) number satisfying both conditions: the number 7, so the answer is **B**.

## Section 1.6 Exercises

**WARM-UPS A**

- A car rents for \$210 per week plus \$0.20 per mile. Find the cost of renting this car for a three-week trip of 500 miles for a family of 5.
- A checking account charges \$10 per month, plus \$0.20 per check. Find the monthly charge for a month in which 53 checks were written to 47 individuals.
- A 4-ounce can of tuna costs \$0.95 while a 10-ounce can costs \$2.50. How much would you save by buying 5 pounds (80 ounces) of the more economical size?
- A 4-ounce can of sardines costs \$0.48 and a 6 ounce can costs \$0.66. How much would you save by buying 6 pounds (96 ounces) of the more economical size?

5. A checking account charges \$8 per month, plus \$0.15 per check. How many checks were written in a month in which the total charges amounted to \$14.45?
6. The library fines for overdue books are \$0.50 for the first day plus \$0.20 for each additional day. If the fine for a book amounted to \$2.90, how many days was the book overdue?
7. A frame store ordered 20 special frames at a cost of \$35 each. If the store sold 16 of the frames for \$40 each and returned the other 4 to the manufacturer at a service charge of \$2 each, how much profit did the store make?
8. A software store ordered from the manufacturer 22 virus-detecting programs costing \$20 each and selling for \$30. After giving 7 of the programs to their best customers and selling the other 15, how much did the store have to send the manufacturer?
9. There were 30 multiple choice and 10 essay questions on Rhonda's test. If she got  $\frac{4}{5}$  of the multiple-choice and  $\frac{3}{10}$  of the essay questions correct, how many questions did Rhonda miss?
10. Two classes each have an enrollment of 24 students. On a certain day  $\frac{3}{4}$  of one class and  $\frac{5}{8}$  of the other are present. How many students are absent from the two classes?

**CLAST PRACTICE A****PRACTICE PROBLEMS: Chapter 1, # 45-47**

11. A caterer charges \$250 per banquet plus \$6.50 a plate. Find the cost of two banquets in which the attendance was 25 and 30 persons, respectively.
  - A. \$357.50
  - B. \$607.50
  - C. \$506.50
  - D. \$857.50
12. An 8-ounce can of tomato sauce costs \$0.80 and a 20 ounce can costs \$2.40. How much money can be saved by buying 120 ounces of the more economical size?
  - A. \$2.40
  - B. \$14.40
  - C. \$12.00
  - D. \$0.80
13. The library fine for overdue books is 20¢ for the first day and 10¢ for each additional day. If a borrower paid \$1.60 for an overdue book, how many days was the book overdue?
  - A. 16
  - B. 14
  - C. 15
  - D. 18

14. A jewelry store ordered 30 graduation rings for a class of 25 students. Each ring cost the store \$150 and was to be sold for \$200. The store must pay a \$20 service charge for each returned ring. If the store had to return 5 rings, how much profit did the store make?
- A. \$1400            B. \$1150            C. \$4900            D. \$1250
15. There were 40 multiple choice and 30 essay questions on Becky's test. If Becky got  $\frac{7}{10}$  of the multiple-choice and  $\frac{9}{10}$  of the essay questions correct, how many questions did Becky miss?
- A. 16                B. 12                C. 3                D. 15

**WARM-UPS B**

16. Find the smallest positive multiple of 5 which leaves a remainder of 3 when divided by 7 and a remainder of 5 when divided by 8.
17. Find the smallest positive multiple of 9 which leaves a remainder of 3 when divided by 6 and a remainder of 1 when divided by 4.
18. How many whole numbers leave a remainder of 5 when divided into 29 and a remainder of 5 when divided into 137?
19. How many whole numbers leave a remainder of 2 when divided into 34 and a remainder of 3 when divided into 19?

**CLAST PRACTICE B****PRACTICE PROBLEMS: Chapter 1, # 48, 49**

20. Find the smallest positive multiple of 8 which leaves a remainder of 8 when divided by 10 and a remainder of 6 when divided by 14.
- A. 108            B. 98            C. 88            D. 48
21. How many whole numbers leave a remainder of 2 when divided into 56 and a remainder of 1 when divided into 28.
- A. 3                B. 4                C. 2                D. 5

**EXTRA CLAST PRACTICE**

22. How many whole numbers leave a remainder of 3 when divided into 31 and a remainder of 4 when divided into 25?
- A. 0                      B. 1                      C. 2                      D. 3
23. A worker is paid \$6.00 per hour for the first 40 hours she works. She is paid an additional \$2 per hour for each hour over 40 that she works. How much will she earn if she works 52 hours in one week?
- A. \$312                      B. \$248                      C. \$224                      D. \$376
24. Find the largest positive integer which is a factor of both 18 and 30 and is also a factor of the sum of 18 and 30.
- A. 2                      B. 3                      C. 6                      D. 54
25. Which is the largest whole number divisible by 7 and a factor of 84?
- A. 14                      B. 28                      C. 42                      D. 84
26. What is the smallest number which leaves a remainder of 3 when divided by 4, a remainder of 2 when divided by 3 and a remainder of 1 when divided by 2?
- A. 7                      B. 11                      C. 17                      D. 21
27. The three digits of a number are X36. Find the largest digit X so that the number is divisible by 9.
- A. 3                      B. 6                      C. 9                      D. 6
28. What is the largest even number that is also prime?
- A. 1                      B. 2                      C. 3                      D. There is none