CHAPTER 1
SETS AND PROBLEM SOLVING

EXERCISE 1.l

STUDY TIPS Word problems, sometimes called "story problems" or "statement problems" are at the heart of math anxiety. So says Sheila Tobias, author of *Overcoming Math Anxiety*. Her solution? Figure out some way to help people conquer their fear and disability in solving word problems. Let us do this together and do it now!
We start this book with a discussion of how to solve problems. This section is very important because the procedures and techniques discussed here will be used in the rest of the book. Why should you do this? Read the Getting Started again to remind you. The Exercises at the end of this section are designed to train and help you solve problems. Here are the answers to the problems in Exercise 1.1. If you have access to the computer, we also have the Bello Website at **http://college.hmco.com/mathematics.** This site offers links where the topics in the text are discussed, amplified and expanded. For example, this very section contains **six** different links where Problem Solving is studied. Take advantage of these links!

1. **Step 1.** Understand the problem.
   Step 2. Devise a plan.
   Step 3. Carry out the plan.
   Step 4. Look Back.

3. What does the problem ask for? What is the unknown? After all, if you don't know the question, how can you find the answer?

5. Go to the "Occasional Plan" row and look at the last column. It says that the transaction fee is 20 cents. Since you made 15 transactions, the answer is 15 × 20 cents or **$3.00**

7. If you are planning to make 15 transactions per month, the Light Use plan will cost 15 × 15 cents or $2.25 while the Occasional Plan is $3.00, thus the **Light Use Plan** is **less expensive**.

9. To answer this question find the cost of each plan when you make 20, 40 and 60 calls. For 60 calls the cost for the Light Use Plan is

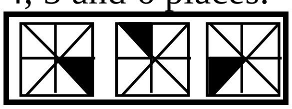
Monthly Fee	Number of paid calls	Cost per call

$1.95 + (60 - 10)0.15 = $1.95 + $7.50 = $9.45
For **63** calls it will be 45 cents more or $9.90. After that the **Standard Use Plan** ($9.95 for 100 free calls) is less expensive.

11. To get the 2nd term (**2**), you add 1 to the 1st term.
To get the 3rd term (**4**), you add 2 to the 2nd term.
To get the 4th term (**7**), you add 3 to the 3rd term.
To get the 5th term (**11**), you add 4 to the 4th term.
The 6th and 7th terms are 11 + 5 = **16** and 16 + 6 = **22.**

13. Note that the odd numbered terms are always 1's and the even numbered terms are multiples of 5. Thus, the 7th and 9th terms are 1's and the eighth term is the next multiple of 5 after 15, that is 20. Hence, the next three terms are **1, 20 and 1**.

15. Going clockwise, the shaded region is moved 1 place, 2 places, 3 places and so on. The next three moves will move the shaded region 4, 5 and 6 places. The answer is shown.



17. The numbers in the denominator are obtained by doubling. Thus, the next three terms are $\frac{1}{\mathbf{16}}$, $\frac{1}{\mathbf{32}}$ and $\frac{1}{\mathbf{64}}$. Note that each term is half the preceding term.

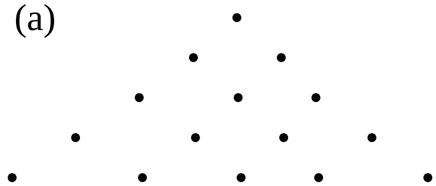
19. The odd numbered terms are 1, 2, 3, 4, 5, ... and the even numbered terms are 5, 6, 7, 8, 9, ... The next three terms are **7, 4, 8** as shown.

↓ ↓ ↓

1, 5, 2, 6, 3, **7**, **4**, **8**

↑ ↑ ↑ ↑

21. (a)



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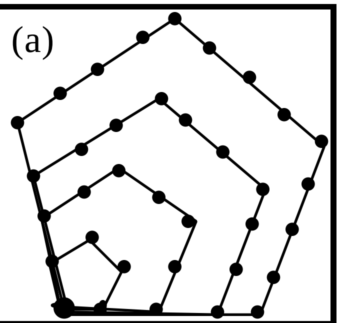
(b) The rows are constructed by adding one more dot than on the preceding row. The next triangular numbers after 10 are 10 + **5** = $\boxed{\mathbf{15}}$ 15 + **6** = $\boxed{\mathbf{21}}$ and 21 + **7** = $\boxed{\mathbf{28}}$.

(c) Following the pattern after the 7th triangular number which is 28, the 10th triangular number is: 28 + 8 + 9 + **10** = $\boxed{\mathbf{55}}$.

23. (a)



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(b) At each step, increase the length of the bottom and left lower side of the pentagon by one unit. The number of dots on each side is increased by one unit.

(c) The 6th pentagonal number is **51**.

25. (a) Here is a summary of the information shown in the figure:

Sides	**4**	**5**	**6**	**7**
Diagonals	**1**	**2**	**3**	**4**

The number of diagonals is **three** less than the number of sides. Thus, 10 - **3** = 7 diagonals can be drawn from one vertex of a decagon.

27. (a) It is always **4**.
    (b) If you pick any number and follow the instructions you eventually get to a number less than or equal to 10. For any of these numbers the pattern leads to the number 4.

29. (a)
$$(1 + 2 + 3 + 4)^2 = 1^3 + 2^3 + 3^3 + 4^3$$
$$(1 + 2 + 3 + 4 + 5)^2 = 1^3 + 2^3 + 3^3 + 4^3 + 5^3$$
$$(1 + 2 + 3 + 4 + 5 + 6)^2 = 1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3$$
    (b) The square of the sum of the first n counting numbers equals the sum of the cubes of these numbers.

31. The number of units of length of the pendulum is always the square of the number of seconds in the time of the swing.

33. (a) **12, 15, 18**
    (b) We can see from the table that each unit increase in size

corresponds to a $\frac{1}{3}$ of an inch increase in length. Thus, a 2 unit increase in size (from 6 to 8) corresponds to a $\frac{2}{3}$ increase in length, from 9 to **$9\frac{2}{3}$ in.**

39. 1 + 1 + 2 + 3 + 5 = 12. The sixth term is 3 + 5 = 8, so the seventh term is 5 + 8 = **13**, which is 1 more than the sum of the first five terms.

41. The fourteenth term is 377 (check this!), so the sum of the first twelve terms is one less or **376**.

EXERCISE 1.2

STUDY TIPS Most mathematics textbooks start with a discussion about sets, so this section can help you later. The word "set" is an undefined term but at the same time it is listed in the Guinness Book of Records as the "word with the most meanings" (194 in all!) In this section you learn:

(1) How to determine if a set is well defined (Hint: Stay away from subjective words like "good", "bad" and "beautiful")

(2) How to describe a set (There are three ways: writing the set in words, listing the elements separated by commas between braces and using set builder notation.)

Special caution: the empty set is denoted by or { }, but not { }

(3) Determine if two sets are equal. (When two sets are equal, they have exactly the same elements, but these elements do not have to be written in the same order.)

(4) Find the subsets of a set.

Definitions 1.4 and 1.5 mean the same thing but Definition 1.5 is used to convince you that the empty set is a subset of every set A (there is no element in the empty set that is not in A.) To find the subsets of a set start by finding the subsets with no elements, then the subsets with one element and so on. How do you know when you are finished? If you have 1 element, you need $2^1 = 2$ subsets, if you have 2 elements you need $2^2 = 4$ subsets and so on.

1. People do not agree on the meaning of “grouchy”, so this description does **not** define a set.

3. A set

5. A set

7. People do not agree on what is “good”, so this description does **not** define a set.

9. (a) ***Incorrect***. The letter D is not an element of A.
   (b) ***Correct***. Desi is an element of A.
   (c) ***Incorrect***. Jane is an element of A, not the other way around.
   (d) ***Correct***. The letter D is not an element of A.
   (e) ***Incorrect***. Jane is an element of A

11. x is an element of the set X. Fill the blank with .

13. A is not an element of the set X. Fill the blank with .

15. The set consisting of the first and last letters of the English alphabet.

17. The set consisting of the names of the first biblical man and woman.

19. The set of counting numbers from l to 7. Note that the numbers do not have to be in any specific order.

21. The set of odd counting numbers from 1 to 51.

23. The set of counting numbers starting with 1 and then adding 3 successively until the number 25 is obtained.

25. {Dioxin, Xylene} is the set that was found in everybody's tissue.

27. {1, 2, 3, 4, 5, 6, 7}

29. {0, 1, 2, 3, 4, 5, 6, 7}

31. The word “between” is to be taken literally. The 3 and the 8 are not elements of this set. The answer is **{4, 5, 6, 7}**.

33. There are no counting numbers between 6 and 7, so this set is empty. The answer may be written as or as { }.

35. {4, 5, 6, . . .}

37. {WangB, Gull}

39. {ENSCO, WDigitl, TexAir}

41. {WhrEnf, Ech Bg}

43. {TexAir}

45. The set A = {4, 8, 12, 16, ...} and the set B = {2, 4, 6, 8, ...}, so these sets are **not** equal.

47. Both A and B are empty, so these sets **are** equal.

49. (a) Every element of A is an element of B, and every element of B is an element of A. Thus, A = B.
(b) ≠ is correct because 0 is an element of C but not of A.
(c) ≠ is correct because 0 is an element of C but not of B.

51. , {a}, {b}, {a, b} The first three of these are proper subsets of the given set.

53. , {l}, {2}, {3}, {4}, {1, 2}, {1, 3},{1, 4}, {2, 3}, {2, 4}, {3, 4}, {l, 2, 3}, {1, 2, 4}, {1, 3, 4}, {2, 3, 4}, {l, 2, 3, 4} All but the last one of these are proper subsets of the given set.

55. , {l}, {2}, {1, 2} The first three of these are proper subsets of the given set.

57. Since there are **4** elements in this set, there are 2^4 or 16 subsets.

59. There are **10** elements in A, so A has 2^{10} or 1024 subsets.

61. Note that $32 = 2^5$, so the answer is **5**.

63. Since $64 = 2^6$, the answer is **6**.

65. **Yes.** Since has no elements, there is no element of that is not in . Furthermore, every set is a subset of itself.

67. B A because every counting number that is divisible by 4 is also divisible by 2. (A is not a subset of B because a number can be divisible by 2 and not by 4. For instance, 6 is divisible by 2, but not by 4.)

69. (a) There are 5 toppings, so you have **5** choices.
(b) There are 10 subsets with 2 elements in each. So you have **10** choices. (Try writing these out.)
(c) You can choose which two toppings you don't want. So the answer is **10** as in (b).

71. Since $256 = 2^8$, you would need **8** different condiments.

77. (a) If g S, then Gepetto shaves himself, which contradicts the statement that Gepetto shaves all those men and only those men of the village who do not shave themselves. Hence, **g S.**
(b) If g D, then Gepetto does not shave himself, and so by the same statement, he does shave himself. Thus, there is again a contradiction and **g D.**

79. The word "*non-self-descriptive*" cannot be classified in either way without having a contradiction. If it is an element of S, then it is a self-descriptive word, which contradicts the definition, "non-self-descriptive is a non-self-descriptive word". On the other hand if non-self-descriptive is a non-self-descriptive word, then it is an element of S, which is again a contradiction.

EXERCISE 1.3

STUDY TIPS You have to learn how to do three operations with sets: form the **intersection** of sets A and B (make sure all the elements are in A **and** also in B), form the **union** of two sets (the elements in the union can be in A **or** in B **or** in both), find the **difference** of sets A and B (the elements have to be in A but not in B.) In addition, you have to learn how to find the complement of a set. Hint: To find the complement A' of a set, A you have to know the universal set **U.** The complement consists of all the elements that are in **U** and **not** in A, that is, the difference of **U** and A.

1. (a) A ∩ B, the set of all elements in both A and B, is **{1, 3, 4}.**
   (b) A ∩ C, the set of all elements in both A and C**,** is **{1}**.
   (c) B ∩ C**,** the set of all elements in both B and C**,** is **{1, 6}**.

3. (a) A ∩ (B   C), the set of all elements in both A and (B   C), is **{1, 3, 4}**.
   (b) A   (B ∩ C)**,** the set of all elements in A or in (B ∩ C)**,** is **{1, 2, 3, 4, 5, 6}.**

5. A   (B   C), the set of all elements in A or in (B   C), is **{1, 2, 3, 4, 5, 6, 7}**.

7. A ∩ (B ∩ C), the set of all elements in A and in (B ∩ C), is **{1}**.

9. (a) A ∩ B, the set of all elements in both A and B, is **{c}**.
   (b) A ∩ C**,** the set of all elements in both A and C, is . Note: the set {a, b} is an element of A, but a and b, separately, are not elements of A.

11. (a) **Correct**. The set {b} is a subset of the set A ∩ B.
    (b) **Incorrect**. The set {b} is not an element of the set A ∩ B.

13. (a) **Correct.** The set {a, b, c} is a subset of the set A   B.
    (b) **Correct**. The set {a, b, c} is an element of the set A, so it is an element of the set A   B.

15. (a) A', the set of elements in **U** but not in A, is **{b, d, f}**.
    (b) B', the set of elements in **U** but not in B, is **{a, c}**.

17. (a) (A   B)', the set of elements in **U** but not in (A   B), is . Note

that (A B) includes all the elements in **U.**

(b) A' B', the set of elements in A' or in B' is **{a, b, c, d, f}**.

19. (a) (A ∩ B) C', the set of elements in (A ∩ B) or in C', is **{c, e}**. Note that (A ∩ B) is {e}.
(b) C (A ∩ B)', the set of elements in C or not in (A ∩ B) is **{a, b, c, d, f}**.

21. (a) A' ∩ B, the set of elements not in A but in B, is **{b, d, f}.**
(b) A ∩ B', the set of elements in A but not in B, is **{a, c}**.

23. (a) The elements in C' are c and e and those in (A ∩ B)' are a, b, c, d, and f. Thus, C' (A ∩ B)' = **{a, b, c, d, e, f}**
(b) C' = {c, e}, A B = **U,** so (A B)' = . Thus, C' (A B)' = **{c, e}**.

25. (a) This is the set **U** with the elements in A taken out, that is, **{b, d, f}**.
(b) This is the set **U** with the elements in B taken out, that is, **{a, c}**.

27. (a) This is the set in **U** and not in B. The answer is **{2, 3}**
(b) This is the set **U** with the elements in B taken out. The answer is the same as in Part (a), **{2, 3}**.

29. ', the set of all elements in **U** that are not in the empty set, is **U.**

31. The set of all elements that are in both A and the empty set is .

33. This is the set of all elements that are in both A and **U** , that is, **A**.

35. This is the set of elements that are both in A and not in A, that is, .

37. This is a double negative; the elements that are not in “not A” are, of course, in A. Thus, the answer is **A**.

39. To include A, you must have the elements 1, 2, 3. Then, to include B, you must have the element 4, and to include C, you must have the element 5. Thus, the smallest set that can be used for **U** is **{1, 2, 3, 4, 5}**.

41. {Beauty, Consideration, Kindliness, Friendliness, Helpfulness, Loyalty}

43. This is the set of traits that are in both M_w and M_m, so the answer is

{Intelligence, Cheerfulness, Congeniality}.

45. {Intelligence, Cheerfulness}

47. {Is aware of others, Follows up on action}

49. {Follows up on action}

51. (a) M', the set of employees who are not male, is **F**.
(b) F', the set of employees who are not female, is **M**.

53. (a) Male employees who work in the data processing department.
(b) Female employees who are under 21.

55. These employees would have to be in sets D and S, both. Thus, the answer is $\mathbf{D} \cap \mathbf{S}$.

57. These employees would have to be in sets M and D, both. Thus, the answer is $\mathbf{M} \cap \mathbf{D}$.

59. Male employees or employees who are 21 or over.

61. (a) $F \cap S = \{04, 08\}$ is the set of full-time employees who do shop work.
(b) $P \cap (O \quad I) = \{02, 05, 07\}$ is the set of part-time employees who do outdoor field work or indoor office work.

63. A and B have no elements in common.

65. All the elements of A are elements of B, and all the elements of B are elements of A.

67. (a) This is the set of characteristics that are in both columns of the table: {Long tongue, Skin-covered horns, Native to Africa}
(b) The same answer as in Part (a).
(c) This is the set of characteristics that occur in either column of the table:{Tall, Short, Long neck, Short Neck, Skin-covered horns, Native to Africa}
(d) G' = {Short, Short neck} (e) O' = {Tall, Long neck}

69. This is the second item in the 35 and older column: **685,000**.

71. This is the set of 12 - 17 year old females: $\mathbf{F} \cap \mathbf{A}$.

73. There are no persons that are both male and female. This set is empty.

75. **$41,339**

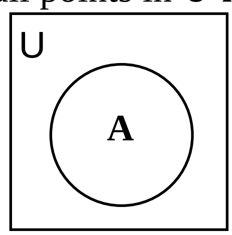
77. **$28,403**

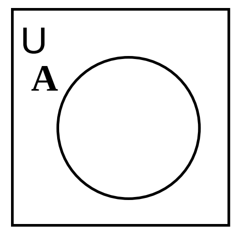
79. Average earnings of males with a High School degree; **$32,521**

EXERCISE 1.4

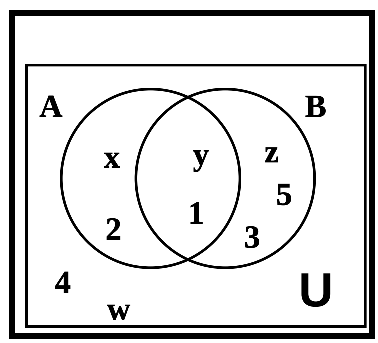
STUDY TIPS In this section you will learn to make pictures associated with the operations of intersections and unions. These pictures can be used to verify the equality of certain sets. Think of sets A and B as sets of points inside two circles. To make a picture (called a Venn Diagram) of an intersection, draw vertical lines inside of A and horizontal lines inside of B. Where the lines **intersect** is the intersection. If there is no intersection, the sets are called **disjoint.** To form the **union**, draw vertical lines inside of A and continue drawing vertical lines inside B, the result is the **union** of A and B. What about the complement of A? As before, you need the universal set **U**, represented by a rectangle outside the circles A and B. The complement of A consists, as before, of all points in **U** **not** in A.

A word of warning: Sometimes students get confused by the placing of the name of the set on the diagram.. Both diagrams show a set **A** inside a universal set **U.** It does not matter if the set **A** is labeled outside or inside the circle. In both cases, we have a set A inside a universal set **U.**





l. **Step I**: Draw a diagram with two circles and label the regions w, x, y, z, as shown.



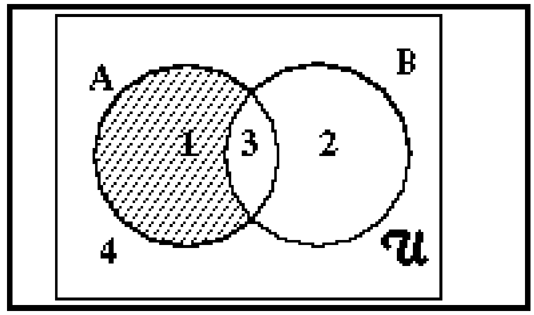
Step 2: Select the element that is in both A and B, the number l. Write l in Region y.

Step 3: Select the element that is in A but not in B, the number 2. Write 2 in Region x.

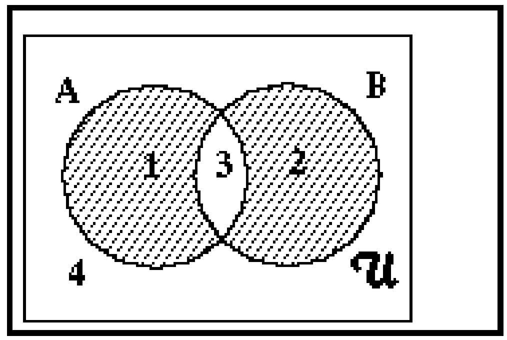
Step 4: Select the elements that are in B but not in A, the numbers 3 and 5. Write 3 and 5 in Region z.

Step 5: Select the element that is not in A or B, the number 4, and write 4 in Region w. This completes the diagram.

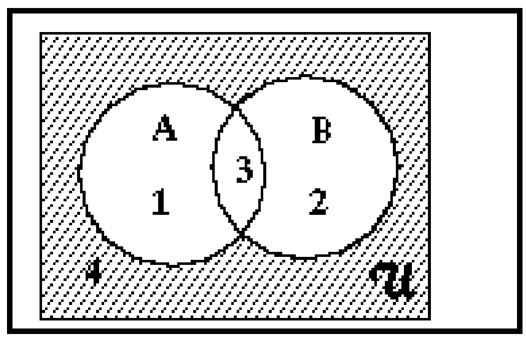
3. Draw a Venn diagram labeled as in the figure Then look at the regions corresponding to the various sets. A: Regions 1, 3; B' (regions outside of B) Regions 1, 4; A ∩ B' (regions in both A and B'): Region l. Shade Region 1 for the desired diagram.



5. Draw a diagram as in Problem 3. Then find the regions corresponding to the various sets. (A   B) (regions in A or B): Regions 1, 2, 3; (A ∩ B) (regions in both A and B): Region 3; (A   B) - (A ∩ B): Take the region in (A ∩ B) away from the regions in (A   B), leaving Regions l, 2. Shade Regions l, 2 for the desired diagram.



7. Draw a diagram as in Problem 3. The region corresponding to A' is the entire region outside of circle A. The region corresponding to B' is the entire region outside of circle B. Thus, the region corresponding to A' ∩ B' is the region outside both circles, the shaded region.



9. A: Regions 1, 3, 5, 7; (B   C) (regions in B or C)**:** Regions 2, 3, 4, 5, 6, 7; A - (B   C)**:** Take the regions common to A and (B   C) away from the regions in A, leaving the answer, **Region 1.**

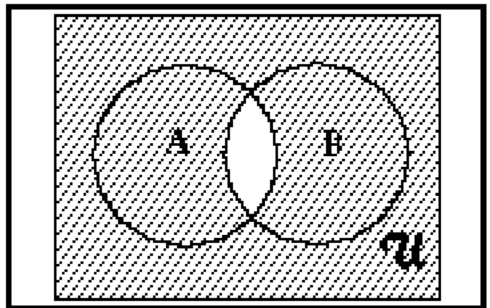
11. (A ∩ B ∩ C) (regions common to A, B, C): Region 7; (A ∩ B) (regions in both A and B): Regions 3, 7; (A ∩ B ∩ C) - (A ∩ B): Take the regions common to (A ∩ B ∩ C) and (A ∩ B) away from those in (A ∩ B ∩ C). This leaves no regions, so we get the empty set,   .

13. (A   B') (regions in A or outside of B): Regions 1, 3, 4, 5, 7, 8; C: Regions 4, 5, 7; (A   B') ∩ C (regions in both (A   B') and C): Regions **4, 5, 7**.

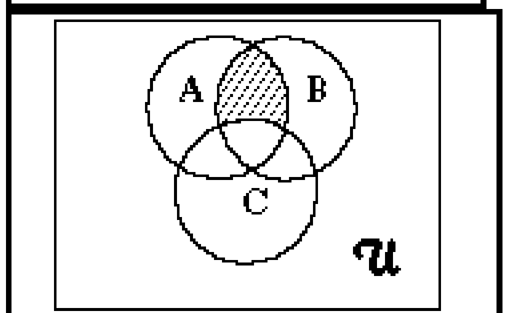
15. (A ∩ B') (regions in A and not in B): Regions 1, 5. (A ∩ B')   C (regions in either (A ∩ B') or C: Regions **1, 4, 5, 6, 7.**

17. This consists of the region that is outside all three of A, B, C: **Region 8**

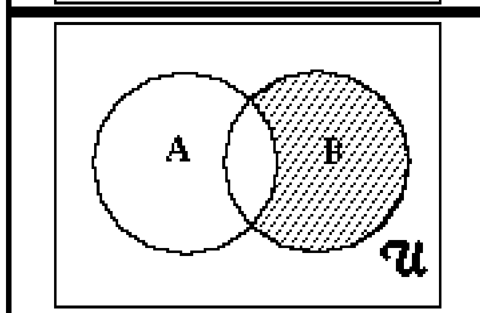
19. This includes all the elements that are outside of A or outside of B. Thus, everything except the region common to the two circles is shaded.



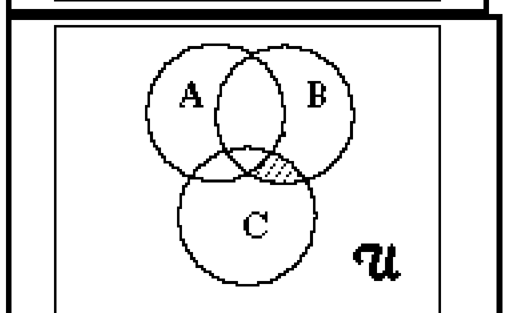
21. This includes all the elements that are in A and also in B, but not in C. Thus, the region that is common to A and B but is outside of C is shaded.



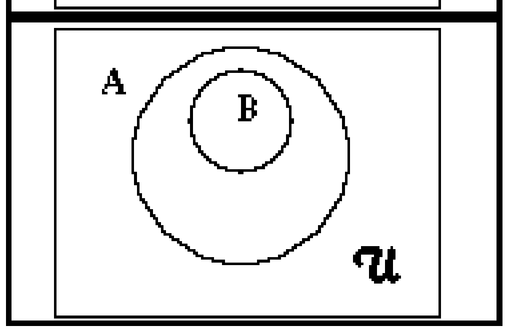
23. This includes all the elements of B that are not elements of A. Thus, the region in B that is outside of A is shaded.



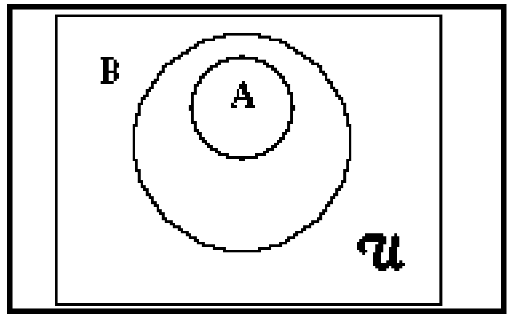
25. This includes all the elements that are in both B and C, but not in A. Thus, the region common to B and C, but outside of A is shaded.



27. Because the intersection of A and B is given equal to B, all of B is contained in A, as shown in the diagram.



29. Note that A ∩ B contains all the elements common to A and B, so that A ∩ (A ∩ B) is the same as (A ∩ B). Because A ∩ (A ∩ B) = A, all the elements of A must be in B. Thus, the circle A was drawn inside the circle B.

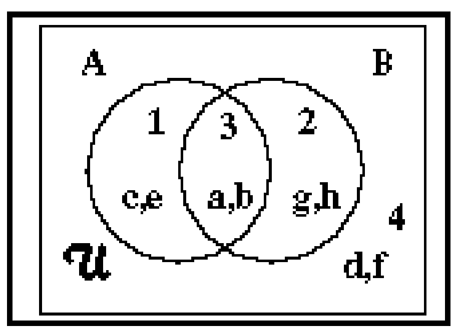


31. (a) A: Regions l, 3, 5, 7; (B C): Regions 2, 3, 4, 5, 6, 7. Thus, for A (B C), we have Regions 1, 2, 3, 4, 5, 6, 7. Similarly, we have the following correspondences. A B: Regions l, 2, 3, 5, 6, 7; C: Regions 4, 5, 6, 7. So for (A B) C, we have Regions 1, 2, 3, 4, 5, 6, 7. This verifies the given equality.
    (b) A: Regions 1, 3, 5, 7; (B ∩ C): Regions 6, 7. Thus, for A ∩ (B ∩ C), we have Region 7. Also, (A ∩ B): Regions 3, 7; C: Regions 4, 5, 6, 7. Thus, for (A ∩ B) ∩ C, we have Region 7. This verifies the given equality.

33. (a) A: Regions 1, 3, 5, 7: A': Regions 2, 4, 6, 8. Thus, for A A', we have Regions 1, 2, 3, 4, 5, 6, 7, 8, the same set of regions that represents **U**. This verifies the given equality.
    (b) From Part a, we see that A and A' have no elements in common. Therefore, A ∩ A' = .
    (c) A - B: Regions 1, 5; A ∩ B': Regions **1, 5**. This verifies the equality.

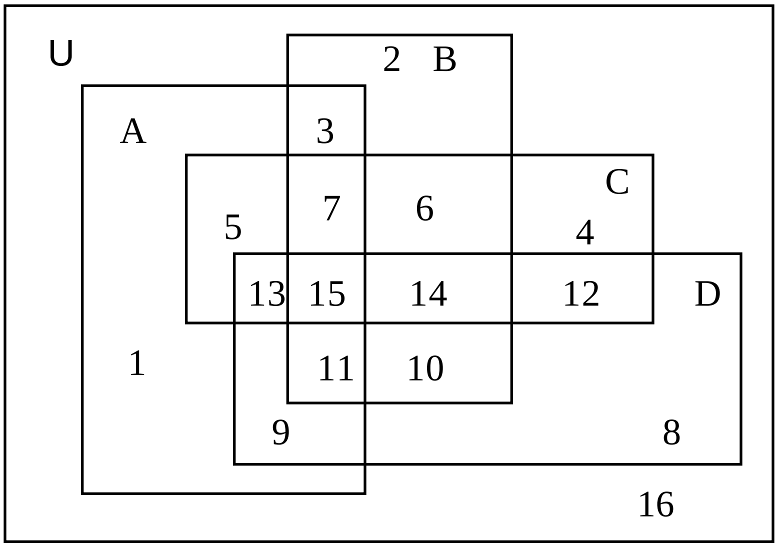
35. A ∩ B (The regions that are in both A and B): Regions **3, 7**.

37. Draw a Venn diagram with the regions numbered as shown. A ∩ B: The region common to A and B is Region 3, so write a, b in Region 3. A ∩ B': The region in A and not in B is Region 1, so write c, e in Region l. A'∩ B: The region in B but not in A is region 2, so write g, h in Region 2 (A B)': The region that is outside the union of A and B is Region 4. Write d, f in Region 4. Now, you can read off each of the required sets:



    (a) A = {a, b, c, e}, B = {a, b, g, h}, **U** = {a, b, c, d, e, f, g, h}
    (b) A B = {a, b, c, e, g, h}
    (c) (A ∩ B)' = {c, d, e, f, g, h}

39.



41. Arizona, California, Florida, Texas

43. The set of elements common to A and B.

45. The set of elements in **U** and not in either A or C.

47. **False**.

49. **False**.

51. An AB^+ person has all three antigens and thus may receive blood from any donor.

53. **No**, because the B^- person does not have the A antigen.

55. **No**, because the O^- person does not have the Rh antigen.

57. Look at the diagram. It gives the answer, 16 or $\mathbf{2^4}$.

59. (a) This requires everything common to A, B, and D, that is not in C. Thus, the answer is **Region 11.**
(b) This requires everything that is outside of the union of A, B, and C. Thus, the answer is **Regions 8** and **16**.

EXERCISE 1.5

STUDY TIPS To find the number of elements in set A, denoted by n(A), and called the **cardinal number of A,** simply count the elements in set A. To find the number of elements in the union of A and B, you need the formula **n(A B) = n(A) + n(B) - n(A ∩ B)** If A has 15 elements and B has 20 (as in Exercise 1), you can not simply add 20 and 15, you have to use the formula and subtract the number of elements in the intersection so that these are not counted twice.

Hint: When doing survey problems like Example 3, try to diagram the data given at the end of the problem (**3** like rock, jazz and classical) first.

1. Use Equation (1): **n(A B) = n(A) + n(B) - n(A ∩ B)**. Then, since n(A) = 15, n(B) = 20, and n(A ∩ B) = 5, it follows that

   n(A B) = 15 + 20 - 5 = **30.**

3. Use the same equation as in Problem 1. Thus, n(A) = 15, n(A ∩ B) = 5, and n(A B) = 30, which we substitute into Equation (1) to get

   30 = 15 + n(B) - 5, that is, 30 = 10 + n(B). Thus, n(B) = **20.**

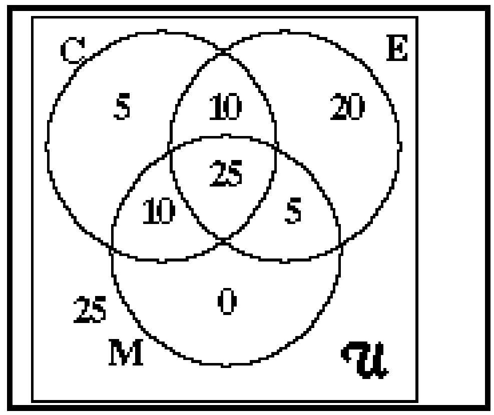
5. Let T be the set of families who subscribe to Time and N be the set who subscribe to Newsweek. Since 100 families were surveyed and 10 subscribe to neither magazine, 90 subscribe to one or both. Hence, we can use Equation (1) in the form

   n(T N) = n(T) + n(N) - n(T ∩ N)

   With n(T N) = 90, n(T) = 75, and n(N) = 55, we get the equation 90 = 75 + 55 - n(T ∩ N), that is, 90 = 130 - n(T ∩ N). Therefore, n(T ∩ N) = 40, which says that **40** families subscribe to both.

7. To do this problem, first draw a Venn diagram. Let C be the set of students taking Chemistry, E be the set taking English, and M the set taking Mathematics. Draw the diagram with three circles as shown. Then start at the end of the given listing to fill in the proper numbers in the various regions

Because 25 are taking all three courses, write **25** in the region common to all three circles. Since 35 are taking Math and Chem, and we have accounted for 25 of these, write **10** in the remainder of the region common to M and C. Likewise, 35 are taking English and Chem, and we have accounted for 25 of these, write **10** in the remainder of the region common to E and C. Similarly, because 30 are taking English and Math, and we have accounted for 25 of these, write **5** in the remainder of the region common to E and M. Then, because 50 are taking Chem, and we have accounted for 10 + 25 + 10 = 45 of these, write **5** in the region of C that is outside of E and M. There are 40 in Math, and we have accounted for 5 + 10 + 25 = 40, so write **0** in the region of M that is outside of E and C**.** Since there are 60 taking English, and we have accounted for 5 + 10 + 25 = 40 of these, write **20** in the region of E that is outside of C and M. Up to this point, we have accounted for 20 + 5 + 5 + 10 + 10 + 25 = 75 students. Because 100 students were surveyed, write **25** in the region outside of the three circles. This completes the diagram and we can now read off the answers.



(a) **No** students were taking Math and neither Chem nor English. (See the region of M that is outside of C and E.)

(b) There were **10** students taking both Math and Chem, but not English. (See the region that is inside both M and C, but is outside of E.)

(c) There were **10** students taking both English and Chem but not

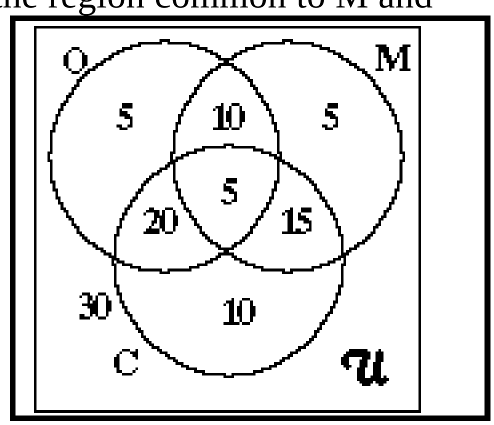
Math. (See the region inside both E and C, but outside of M.)

9. You can get the answers directly from the table.
   (a) Add the numbers in Row P to get 1 + 14 + 7 = **22**.
   (b) Add the numbers in the SK column to get **36**.
   (c) This is the number in Row J, Column SK, **6**.

11. (a) This is the item in the Materials Row of Column 1-2, 120, meaning **$120,000.**
   (b) This would be obtained by adding the items in Column 1-2, which gives a sum of 510, so the answer is **$510,000.**
   (c) This is obtained by adding the items in the first row of the table. The sum is 1305, so the answer is **$1,305,000.**

13. Let Ti be the set of persons who read the Times, and Tr be the set who read the Tribune. Then we can use Equation (1) in the form
   n(Ti Tr) = n(Ti) + n(Tr) - n(Ti ∩ Tr). This gives
   n(Ti Tr) = 130 + 120 - 50 = 200.
   Thus, **200** people were surveyed.

15. Let O stand for onions, M for mustard, and C for catsup. Draw a Venn diagram as shown. The numbers are obtained as follows: Start at the end of the list. Since 5 had all three condiments, write **5** in the region common to the three circles. Because 25 had onions and catsup, the region common to O and C must have a total of 25. We have already put 5 into this region, so **20** goes into the remainder of the region. Next, we see that 20 had mustard and catsup, so the region common to M and C must have a total of 20. Since 5 have already been put in this region, **15** must be put in the remainder of the region. Since 15 had onion and mustard, the region common to O and M must have a total of 15. Because 5 have already been put in this region, **10** must be in the remainder of the region. Now we see that 50 had catsup, so circle C must contain a total of 50. The diagram shows that we already have put 5 + 15 + 20 = 40 in C, so **10** must be put in the remainder of the region. Next, we see that 35 had mustard. Thus, circle M must have a total of 35. Since we have entered 5 + 15 + 10 = 30 in this circle, **5** must go into the remainder of this circle. Since 40 had onions, and the diagram shows that 5 + 10 + 20 = 35 have already



been put in circle O, an additional **5** must go in this circle. Now, the diagram shows that 70 persons have been put into the three circles. Since 100 were surveyed, we must put **30** into the rectangle outside of the three circles. This completes the diagram and we can read off the answers to the questions.

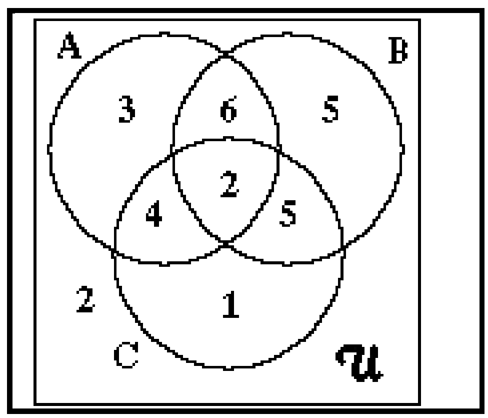
(a) **5** (See the region inside O and outside M and C.)
(b) **30** (See the region in the rectangle outside of the three circles.)
(c) 5 + 5 + 10 = **20** (See the non overlapping regions of the three circles.)

17. Let G be the set of people who liked ground coffee, and I be the set of people who liked instant coffee. Then we can use Equation (1) in the form n(G I) = n(G) + n(I) - n(G ∩ I) to find how many of those surveyed liked coffee (either ground or instant or both). We were given n(G) = 200, n(I) = 270, and n(G ∩ I) = 70, so that

n(G I) = 200 + 270 - 70 = 400.

Since 50 people did not like coffee, 450 people were surveyed, and the company had to pay out **$450.**

19. The Venn diagram for the data reported in Problem 18 is shown at the right. The sum of the numbers in the diagram is **28**, which is the correct number of persons interviewed to give this data.



21. (a) n(A) is the sum of the numbers in A, 70 + 50 = **120.**
(b) n(C) is the sum of the numbers in C, 30 + 50 = **80.**
(c) n(A ∩ C) is the number in the overlapping portion of the two circles, **50.**

23. (a) n(A') is the number of students in **U** and not in A, which is

200 - (70 + 50) = **80.**

(b) n(C') is the number of students in **U** and not in C**,** which is

200 - (50 + 30) = **120.**

(c) n(A' ∩ C') is the number of students outside of both A and C**,** that is, 200 - (70 + 50 + 30) = **50.**

25. (a) This is the sum of the numbers in E and T that are not in the region common to these two circles, 35 + 20 + 12 + 6 = **73.**
(b) This is the sum of the numbers in E and T that are not in M,

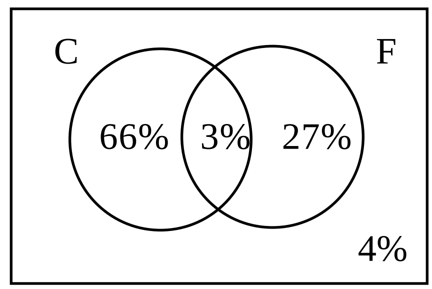
35 + 8 + 12 = **55.**

(c) This is the sum of all the numbers in the diagram except the number in the region common to all three circles. Thus, we get 35 + 8 + 12 + 20 + 6 + 10 = **91.**

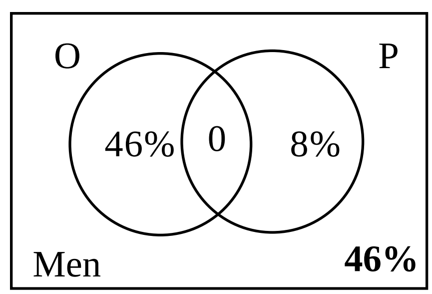
(d) This is the sum of the numbers in all the regions common to at least two of the circles, 8 + 6 + 20 + 4 = **38.**

(e) Since the sum of all the numbers in the diagram is 95 and 100 persons were interviewed, **5** persons must have had none of these three types of investment.

27. Draw a diagram with two circles labeled **C** for cake and **F** for frosting. Since 3% eat them together, place 3% on the region common to C and F. The circle labeled C must contain 69% of the people, so enter 66%(69% - 3%) in the other region of C. The set F has 30% of the people, so we enter 27%(30% - 3%) on the other region of F. We now have accounted for 66% + 3% + 27% = 96% of the people, so 4% must be outside both circles and 27% + 4%= 31% are outside C. Thus, **31%** do not eat cake.



29. Draw a diagram with two circles labeled **O** for optimist and **P** for pessimist. The intersection of O and P is empty, so we enter a 0. According to the data, 46% of the men are optimist (enter 46%) and 8% are optimist (enter 8%). We now have accounted for 46% + 8% = 54% of the men, so there must be **46%** of the men that are neither optimist nor pessimist, that is, outside both circles



31. **False.** A counterexample is A = {1, 2} and B = {m, n}.

33. **False.** A counterexample is A = {1, 2} and B = {1, 2, 3}

35. See diagram in the answer section of the textbook. Thus, with the added information, the statistics in the cartoon are possible.

37. If there are 4 elements and each element can be either included or not included (two choices for each element), then there is a total of $2 \times 2 \times 2 \times 2 = \mathbf{2^4} = 16$ different subsets.

EXERCISE 1.6

STUDY TIPS How do you know if a set is **infinite**? The only elementary way is to show that the set can be put into one-to-one correspondence with one of its proper subsets. **Exercise 1** will show a one-to-one correspondence between the set of natural numbers and one of its proper subsets, the set of odd numbers. Thus, the set of natural numbers is **infinite.** Mathematicians have even tried to do arithmetic with infinite numbers. To see how different this is examine questions 24-27 and the Using Your Knowledge.

1. One such correspondence is

$$\begin{array}{ccccc} 1 & 2 & 3 \ldots & n \ldots \\ \updownarrow & \updownarrow & \updownarrow & \updownarrow \\ 1 & 3 & 5 \ldots & 2n - 1 \ldots \end{array}$$

   This shows that the sets N and O are equivalent.

3. One such correspondence is

$$\begin{array}{cccc} 2 & 4 & 6 \ldots & 2n \ldots \\ \updownarrow & \updownarrow & \updownarrow & \updownarrow \\ 102 & 104 & 106 \ldots & 100 + 2n \ldots \end{array}$$

   Thus, the two sets E and G are equivalent.

5. One such correspondence is

$$\begin{array}{cccc} 202 & 204 & 206 \ldots & 200 + 2n \ldots \\ \updownarrow & \updownarrow & \updownarrow & \updownarrow \\ 302 & 304 & 306 \ldots & 300 + 2n \ldots \end{array}$$

   Thus, the two sets G and T are equivalent.

7. We can set up the correspondence

$$\begin{array}{cccc} 2 & 4 & 8 & 12 \\ \updownarrow & \updownarrow & \updownarrow & \updownarrow \\ 6 & 12 & 24 & 36 \end{array}$$

   Thus, the sets P and Q are equivalent.

9. The correspondence

$$\begin{array}{cccc} -1 & -2 & -3 \ldots & -n \ldots \\ \updownarrow & \updownarrow & \updownarrow & \updownarrow \\ 1 & 2 & 3 \ldots & n \ldots \end{array}$$

   shows that I^- and N are equivalent.

11. n(A) = **26**

13. n(C) = **50**

15. The set E can be put into one-to-one correspondence with the set N of counting numbers. So n(E) = $\aleph_0$.

17. The set {100, 200, 300, . . . } can be put into one-to-one correspondence with a subset of itself, {200, 300, 400, . . .}. This shows that the set is **infinite.**

19. The set $\{\frac{1}{3}, \frac{2}{3}, \frac{3}{3}, \ldots\}$ can be put into one-to-one correspondence with
a subset of itself, $\{\frac{2}{3}, \frac{3}{3}, \frac{4}{3}, \ldots\}$. Thus, the set is **infinite**.

21. Sets B and D are equal and **equivalent**.

23. Set A is neither equivalent nor equal to any of the other sets.

25. Fill in the blank with $\aleph_0$. This is justified by considering the combination of two sets such as {1, 3, 5, . . . } and {2, 4, 6, . . .} into the set N = {1, 2, 3, 4, 5, 6, . . .} . Since all three of these sets have $\aleph_0$ as their cardinality, it follows that $\aleph_0 + \aleph_0 = \aleph_0$.

27. Fill in the blank with $\aleph_0$. This is justified by considering the multiplication table for the set N of counting numbers. The set of products in this table can be put into one-to-one correspondence with the set N itself. This shows that $\aleph_0 \cdot \aleph_0 = \aleph_0$.

29. (a) The next two points are $\mathbf{\frac{7}{9}}$ and $\mathbf{\frac{8}{9}}$.

 (b) $\frac{1}{3} + \frac{2}{9} + \frac{4}{27} + \frac{8}{81} + \cdots$ The sum gets closer and closer to 1.

31. To room **223**.

33. Rooms 1, 3, 5, . . ., 2n + 1, . . .

35. They would move to room **666.**

PRACTICE TEST l

STUDY TIPS There are two additional practice tests, with answers, at the end of this manual Find out if your actual test will be multiple choice or fill in the blank, and how long you will have to take it. Then take the corresponding practice test using the time limit set by your instructor. Find your weaknesses and remedy them before your actual test! We will have more test taking tips later.

1. Look at the difference between successive terms as shown

	1	2	7	19	41	76	**127**
Differences	1	5	12	22	35		
Differences	4	7	10	13			
Difference	3	3	3				

The third differences are constant, so the next number can be constructed by addition. Add the last diagonal from bottom to top. We obtain the next number in the pattern, 3 + 13 + 35 + 76 = 127. Now, we can use the 127 to continue the last three rows as shown. The next term now is 3 + 16 + 51 + 127 = 197

	1	2	7	19	41	76	127	**197**
Differences	1	5	12	22	35	51		
Differences	4	7	10	13	16			
Difference	3	3	3	3				

If you do this one more time, you will find the next term to be **289** .

Once you get four terms you can show that all the following terms can be obtained from the formula: $a_{n+1} = 3 + 3a_n - 3a_{n-1} + a_{n-2}$

2. The counting numbers between 2 and 10 are 3, 4, 5, 6, 7, 8, and 9, so the set is {3, 4, 5, 6, 7, 8, 9}.

3. (a) This is the set of vowels in the English alphabet. In set-builder notation, it is {x | x is a vowel in the English alphabet}.
   (b) This is the set of even counting numbers less than 10. In set builder notation, it is {x | x is an even counting number less than 10}.

4. The proper subsets are , {$}, {¢ }, {%}, {$, ¢ }, {$, %}, {¢ ,

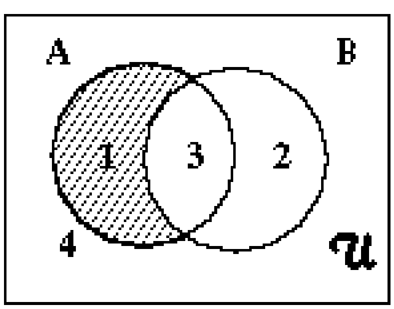
%}.

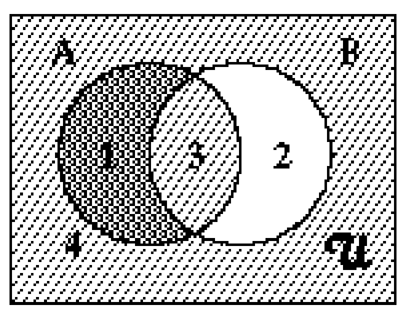
5. (a) Both blanks take the symbol , because A B is the set of all elements that are in A or B.
   (b) The first blank takes the symbol . The second blank takes the symbol , because A ∩ B' is the set of elements that are in A and not in B.
   (c) A' is the complement of A, that is, the set of elements in **U**, but not in A. Thus, the first blank takes the symbol , and the second blank takes the symbol .
   (d) Recall that A - B is the set of elements in A with the elements that are also in B removed, that is, it is the set of elements that are in A and not in B. Thus, both blanks take the symbol .

6. (a) A', the complement of A, = {King}.
   (b) In this problem, A B = **U**, so the complement of A B is the empty set, .
   (c) A ∩ B is the set of elements that are in both A and B, so the answer is **{Queen}.**
   (d) The complement of A ∩ B is {Ace, King, Jack}. Taking this set away from **U** gives **U** - {A ∩ B}' = **{Queen}.**

7. (a) In Problem 6 (c), we found that A ∩ B = {Queen}. The union of this set with the set **C** gives (A ∩ B) C = **{Ace, Queen, Jack}**.
   (b) In Problem 6(a), we found that A' = {King}, so that A' C = {Ace, King, Jack}, which has only the element King in common with B. Thus, (A' C) ∩ B = **{King}**.

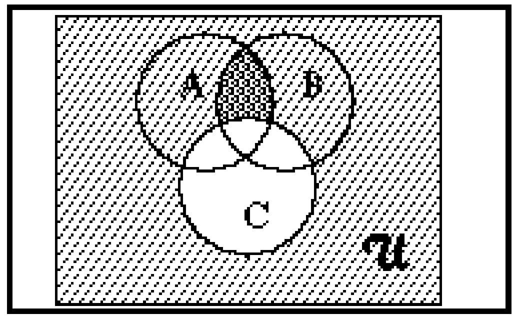
8. The two diagrams for this problem are shown here.





Refer to the diagram on the left. Since A - B is the set A with the elements common to A and B removed, this corresponds to Region 1, that is, circle A with Region 3 removed. Shade Region l. Now, refer to the diagram on the right. A ∩ B' is the set of elements common to A and the complement of B. To show this, shade circle A one way and the region outside of B another way. The cross-hatched region corresponds to A ∩ B'. The diagrams show that **A - B = A ∩ B'.**

9. In the diagram at the right, first shade the region outside the circle C. This region corresponds to C'. Then cross hatch that portion of the shaded region that lies inside both A and B. The cross-hatched region corresponds to the set **A ∩ B ∩ C'.**



10. (a) A B is the set of elements in A or in B, so the corresponding regions are l, 2, 4, 5, 6, 7. C' is the set of elements in **U** but not in C**,** so the corresponding regions are l, 2, 5, 8. The regions common to (A B) and C' are **1, 2, 5.**

    (b) A' is the complement of A, so the corresponding regions are those in **U** that are not in A, Regions 2, 3, 6, 8. Similarly, the regions corresponding to B' are those in **U** that are not in B, Regions 1, 3, 4, 8. The regions common to B' and C, Regions 3, 4, correspond to B' ∩ C**.** Thus, the regions corresponding to A' (B' ∩ C) are those in A' or in (B' ∩ C), Regions **2, 3, 4, 6, 8.**

11. A ∩ B corresponds to the regions inside both circles A and B. Regions 5, 7; (A ∩ B) C corresponds to the regions in (A ∩ B) or in C, Regions 3, 4, 5, 6, 7. Similarly, (A C) corresponds to the regions in A or C, Regions 1, 3, 4, 5, 6, 7; (B C) corresponds to the regions in B or C, Regions 2, 3, 4, 5, 6, 7; (A C) ∩ (B C) corresponds to the regions common to (A C) and (B C), Regions 3, 4, 5, 6, 7. This verifies the equation **(A ∩ B) C = (A C) ∩ (B C).**

12. (A ∩ B)' is the complement of A ∩ B, so the corresponding regions are those in **U** and not in A ∩ B This gives us Regions 1, 2, 3, 4, 6, 8 (all except 5 and 7). A' is the complement of A, so it corresponds to Regions 2, 3, 6, 8; B' is the complement of B, so it corresponds to Regions 1, 3, 4, 8. Since A' B' must correspond to the regions in A' or in B', we get Regions l, 2, 3, 4, 6, 8, as before. This verifies that **(A ∩ B)' = A' B'.**

13. The **(b)** part is correct. (A C) ∩ B.

14. None of these. B ∩ C is represented by regions 6, 7.

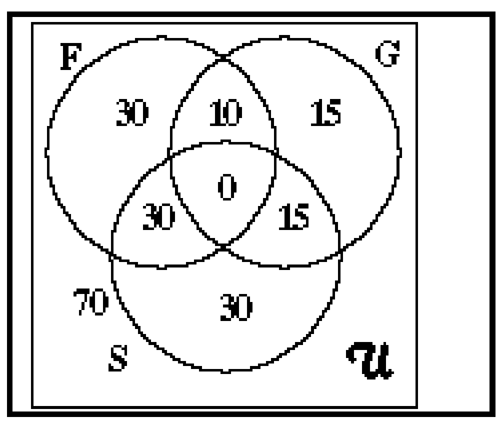
15. (a) A C = {1, 3, 4, 5, 7, 8}, so n(A C) = 6.
    (b) B ∩ C = {4, 8}, so n(B ∩ C) = 2 ≠ 3.

16. You can use the equation n(A B) = n(A) + n(B) - n(A ∩ B) for both parts of this problem.
    (a) n(A) = 25, n(B) = 35, and n(A ∩ B) = 0. Therefore
    n(A B) = 25 + 35 - 0 = **60**
    (b) n(A) and n(B) are as in Part a, but n(A ∩ B) = 5. Therefore,
    n(A B) = 25 + 35 - 5 = **55.**

17. (a) You can use the same equation as in Problem 16, but with n(A) = 15, n(B) = 25, and n(A B) = 35. With these values, you get 35 = 15 + 25 - n(A ∩ B), or 35 = 40 - n(A ∩ B), which means that n(A ∩ B) = **5**.
    (b) n(A' ∩ B') = 8 means there are 8 elements outside of both A and B. Since n(A B) = 35, n(**U**) = 35 + 8 = **43**.

18. The shaded region is the region inside of B and outside of A, so it may be described as **B - A** or as **B ∩ A'**. (A' ∩ B is also correct.)

19. First make a Venn diagram showing the various sets. Then start at the end of the list and work back through it. Since no students are taking all three of the courses, put a 0 in the region common to the circles F, G, and S. Because 70 are taking no language, write **70** in the rectangle outside of the three circles. As 15 are taking German and Spanish, write **15** in the region common to G and S, but outside of F. 30 are taking French and Spanish, so



write **30** in the region common to F and S, but outside of G. 10 are taking French and German, so write **10** in the region common to F and G but outside of S. 75 are taking Spanish and of these, you have accounted for 0 + 30 + 15 = 45. (See the diagram.) Hence, you must write **30** in the region inside S but outside both F and G. 40 are taking German, and you have accounted for 0 + 10 + 15 = 25 of these, so write **15** in the region inside G, but outside F and S. 70 are taking French, and you have accounted for 0 + 10 + 30 = 40 of these, so write **30** in the region inside F, but outside G and S. This completes the diagram and you can get the answers from it.

(a) To get the number of students taking two languages, add the numbers common to two of the circles: 10 + 15 + 30 = **55.**

(b) You can read this directly as the number **30**, which is inside S and outside the other two circles.

(c) This number is the sum of all the numbers inside S, but outside F: 15 + 30 = **45**.

20. (a) This is the sum of all the numbers in the diagram:
10 + 5 + 15 + 2 + 4 + 3 + 8 = **47**.

(b) This is the sum of the numbers that are in A, but not in B:
10 + 2 = **12.**

(c) This is the sum of the numbers that are in B or C, but not in A:
15 + 3 + 8 = **26**.

(d) This is the sum of the numbers that are in both B and C, but not in A, so the answer is **3**.

(e) This is the sum of all the numbers in the diagram that are not in either B or C, so the answer is **10.**

21. The one-to-one correspondence

$1 \quad 3 \; . \; . \; . \; 2n - 1 \; . \; . \; .$
$\updownarrow \quad \updownarrow \qquad \updownarrow$
$2 \quad 4 \; . \; . \; . \quad 2n \; . \; . \; .$

shows that the two sets have the same cardinal number.

22. The one-to-one correspondence

$1 \quad 2 \; . \; . \; . \qquad n \; . \; . \; .$
$\updownarrow \quad \updownarrow \qquad \updownarrow$
$4 \quad 16 \; . \; . \; . \quad (2n)^2 \; . \; . \; .$

shows that the two sets are equivalent.

23. $n\{1, 4, \ldots, n^2, \ldots, 144\}$ = **12**

24. Since you can set up a one-to-one correspondence between the given set and the set {1, 2, . . ., n, . . .}, the cardinality of the given set is the same as that of the set of counting numbers, $\aleph_0$.

25. The one-to-one correspondence

$$\begin{array}{ccccccc} \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \cdots & \frac{1}{n} & \cdots \\ \updownarrow & \updownarrow & \updownarrow & & \updownarrow & \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{6} & \cdots & \frac{1}{2n-2} & \cdots \end{array}$$

between the given set and a subset of itself shows that the given set is **infinite.**