36. If we set the base of a triangle on the \( x \) axis, we can specify the coordinates of any triangle as follows: \((0, 0), (r, m)\) and \((e, 0)\). However, if we have an isosceles triangle, there is a relationship between some of these coordinates that enables us to be more specific, just as we were in Investigation 8.18 with a parallelogram. If \( BA \) and \( BC \) are the congruent sides, there is a relationship between \( e \) and \( r \) that enables us to specify \( e \) in terms of \( r \). What is the relationship?

37. a. Prove that if we connect the midpoints of the sides of two isosceles triangles, the length of the line segment connecting those two points is one-half the length of the base.

b. Is this true just for isosceles triangles or for all triangles? Support your conclusion.

38. a. Draw a right triangle, and find the midpoint of the hypotenuse; then connect the midpoint and the other vertex of the triangle. What do you think is the relationship between the length of that segment you just constructed and the length of the hypotenuse. Use coordinate geometry to find the answer.

b. You may recall from high school algebra that two lines are perpendicular if the product of their slopes is equal to \(-1\). Use this knowledge to show that the diagonals of a square are perpendicular.

### Section 8.3 THREE-DIMENSIONAL FIGURES

This is the dimension in which we live. Almost everything we interact with is three-dimensional: people and pets, buildings, our rooms, our cars, what we see when we look around. Look at the six pictures on the next page. What do you see? Think, jot some notes, and then read on. . . . Four of the pictures are of objects made by humans, and two are natural—one organic and one inorganic. The snail (in its genetic makeup) and the crystal (in its molecular makeup) have the “blueprints” for the ultimate shape of the shell and of the crystal. Both the pyramid and the Parthenon are over 2000 years old, and yet the geometry that was used to make them is relevant to building today. The soccer ball and the design on the Shrine of Shah Nimatuollähī represent the solution to two different questions about connections between two dimensions and three dimensions. In the former case, someone discovered that a specific combination of two polygons (a regular pentagon and a regular hexagon) will produce a nearly perfect ball (sphere). In the latter case, the designers had to figure out how to take a two-dimensional tessellation design and “fold” it around the roof of the shrine so that it would “work” in three dimensions.

Just as we discovered patterns and relationships among many two-dimensional objects, there are many patterns and relationships among three-dimensional figures, also called space figures. With respect to practical matters, understanding the geometry of human-made objects helps us to make them work better and, in the case of objects such as bridges, overpasses, and airplanes, to make them work more safely. Geometry also helps us to understand natural phenomena better—for example, why certain animals have the shapes they have. An understanding of shapes has many applications in science. For example, many carcinogens are virtually identical in size and shape to other compounds, and thus they fool the body into thinking they are not harmful. The silicon chip has the same structure as the diamond, except that there are silicon atoms instead of carbon atoms at these positions.\(^5\) With respect to aesthetics, geometry helps us to understand why some shapes are so appealing to people and to understand patterns within those shapes (see Figure 8.109).

Figure 8.109

(a) Pyramids at Giza

(b) the Parthenon

(c) Nautilus shell

(d) Pyrite crystals

(e) Soccer ball

(f) Shrine of Shah Nimatuolláhi
In this section, we will begin simple and build up. Let’s do a “What do you see?” investigation here, as we did in Section 8.2. As your ability to “see” three-dimensional objects improves—that is, your ability to see all the various attributes of a solid object, relationships between those attributes, and/or relationships between that object and other similarly shaped objects—your appreciation of the three-dimensional property grows too.

**INVESTIGATION 8.20**

What Do You See?

Examine a cube carefully (see Figure 8.110). What do you see? Write down all the attributes you can think of before reading on. Next, look at the “box” (see Figure 8.111). Which of the attributes of a cube does the box possess? What different attributes does it have? . . .

**DISCUSSION**

A cube has 6 faces, though you might have called them sides. All of the faces are squares, which also means right angles, parallel sides, and so on.

All of the faces are congruent.

We have names for opposite sides: front-back, side-side, top-bottom. There are 12 edges on the cube. There are 8 vertices.

A box also has 6 faces. Some of the faces are squares and some are rectangles. As you discovered in the last section, our definitions enable us to say that all of the faces are rectangles. Not all of the faces are congruent, but some are. The opposite sides are congruent.

The box, like the cube, has 12 edges. The box, like the cube, has 8 vertices.
This investigation serves as what is called an advance organizer of this section. That is, it got you to grapple with many of the important ideas that we will examine in more detail. Our first order of business toward that end is to learn some of the language we will use with three-dimensional objects. Rather than just presenting you with the new terms, we will do another investigation in which you will be asked to think about what language and concepts from our work with polygons will make sense with our work with polyhedra, and where we need new terms or where the addition of one dimension “changes” things. For example, the addition of one side resulted in a way of naming quadrilaterals different from the way we named triangles.

INVESTIGATION

8.21 Connecting Polygons to Polyhedra

Just as we examined families (subsets) of triangles and quadrilaterals, we will now investigate families of three-dimensional geometric figures (see Figure 8.112). If you did Exploration 8.14, you grappled with describing and classifying three-dimensional figures.

Let us explore the connection between polygons and **polyhedra**, which will be loosely defined (for now) as three-dimensional figures made up of polygons.

The second column of Table 8.5 describes several attributes of polygons, which we investigated in Section 8.2. Which of these attributes do you think hold for polyhedra or can be modified to describe different kinds of polyhedra? Fill in as much of the third column as you can. The questions below are given to help you focus on the connections between how we see and define two- and three-dimensional figures. After you have completed as much of the third column as possible, compare your hypotheses with those of another student. Then read on...

**FIGURE 8.112**

- If a polygon is defined as a simple closed curve, is there an analogous definition for a polyhedron?
- All polygons have vertices, line segments, and angles. Do these terms work for describing component parts of polyhedra? Do we also need new terms?
- Can we classify polyhedra by the number of sides?
- If there are regular polyhedra, how might they be defined?
- If there are convex polyhedra, how might they be defined?

**CLASSROOM CONNECTION**

In the beginning, children view solids (three-dimensional shapes—3D) as entities instead of seeing the parts of the solid as a collection of related shapes.
In Section 8.2, we began with simple closed curves that partitioned a plane (two dimensions) into three disjoint sets: the curve, inside, and outside.

Though we will not rigorously define simple closed surfaces, we can say that they partition space (three dimensions) into three disjoint sets: the surface itself, inside, and outside (see Figure 8.113).

We will use the term **space figure** to describe any three-dimensional object.

We will use the term **polyhedron** (the plural is **polyhedra**) to describe those simple closed surfaces that are composed of polygonal regions.

We will use the term **solid** to describe the union of any space figure and its interior.

**Component parts** Just as the component parts of polygons have special names, so do those of polyhedra.

Each of the separate polygonal regions of a polyhedron is called a **face**; for example, square \( ABEF \) is a face of the cube in Figure 8.114.

The sides of each of the faces are called **edges**; for example, \( AB \) is an edge of the cube in Figure 8.114.

The **vertices** of the polyhedron are simply the vertices of the polygonal regions that form the polyhedron; for example, \( E \) and \( F \) are vertices of the cube in Figure 8.114.

**Convex and concave** Just as polygons can be convex or concave, so can polyhedra. Before reading the definition of a convex polyhedron, think back to the definition of a convex polygon and see whether you can modify that definition for three-dimensional objects. Then read on...
A polyhedron is convex if and only if any line segment connecting two points of the polyhedron is either on the surface or in the interior of the polyhedron (see Figure 8.115).

INVESTIGATION 8.22

Features of Three-Dimensional Objects

Look at the picture of a box and a ramp (see Figure 8.116). In what ways are they “the same”? That is, what characteristics do they have in common that not all three-dimensional objects have? In what ways are they different? Do this before reading on...

DISCUSSION

Some of the things they have in common:

- All the faces (sides) are polygons.
- In both cases, at least some of the sides are quadrilaterals.
- At least one pair of sides are congruent and parallel to each other. In the ramp, the two triangles on the side are parallel and congruent.

Some of the differences between them:

- In the box, there are an even number of faces, and opposite faces are congruent. In the ramp, only the triangle faces are opposite. The other three faces are noncongruent rectangles.
- The numbers of faces, edges, and vertices are different.
  - The box: 6, 12, 8
  - The ramp: 5, 9, 6
- However, the relationship between the numbers of faces, edges, and vertices is the same.

See Exploration 8.15 (Relationships Among Polyhedra) for more on this.
Families of Polyhedra

Now let us investigate some of the families of polyhedra. Take a few minutes to examine the figures in Figure 8.117. How are these figures alike? How are they different? Write your thoughts in your notebook before reading on...

All of these figures have at least two sides that are parallel; some students would say that the top and bottom sides are parallel. And the faces are all polygons. All of these figures are called prisms. We use the word **prism** to describe all polyhedra that have two parallel **bases** that are congruent polygons. It is a convention to call the other faces of prisms **lateral faces**.

What one shape can be used to describe the lateral faces of all prisms? In other words, all lateral faces of all prisms are __________. Think and read on...

In all prisms, the lateral faces are parallelograms. In some cases, all of the lateral faces are rectangles. How would you describe the differences between those prisms whose lateral faces are non-rectangular parallelograms and those whose lateral faces are rectangles?

In the latter case, the plane of the base and the plane of the lateral faces are perpendicular (see Figure 8.118). We could also say that the **dihedral angle** formed by either base and any lateral face is a right angle. (A **dihedral angle** is simply a three-dimensional angle—that is, an angle whose vertex is a line and whose sides are planes.)

Thus we can define a **right prism** as a prism in which the lateral faces are rectangles. Alternatively, we could define a right prism as a prism in which the angle formed by either base and any lateral face is a right dihedral angle.

A prism that is not a right prism is an **oblique prism** (see Figure 8.119).

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**Language**

As you can see from this presentation of prisms, the box and ramp in Investigation 8.22 are both prisms. That is, they both have one pair of congruent bases that are parallel to each other.

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Long before they study formal geometry, many children know the names for two special kinds of prisms.

Although this is not a term mathematicians use, what we call a **box** is actually a prism in which all six faces are rectangles. If all six faces are squares, we call the figure a **cube** (see Figure 8.120).
Pyramids

Let us consider now another family of polyhedra. You may recognize the polyhedra in Figure 8.121 as pyramids. How might we define that term? Make your own definition and then read on. . . .

We use the word pyramid to describe those polyhedra whose base is a polygon and whose faces are triangles that have a common vertex. That common vertex is called the apex of the pyramid.

An alternative way to think of a pyramid is to start with any polygon and a point above the plane of the polygon. Now connect that point to each vertex of the polygon.

Most of the pyramids you have seen in pictures (or in person, if you are lucky) have square bases. However, the base can be any polygon. A pyramid is named according to its base: triangular pyramid, square pyramid, and so on.

As you have seen from our work with two-dimensional objects, the question of examining how objects are alike and how they differ is an important part of the learning process. Remember that it begins in preschool, where the teacher might give the children an assortment of buttons and have the children put them in piles so that each button belongs in one pile. The next investigation involves looking at similarities and differences.

INVESTIGATION

Prisms and Pyramids

Look at the set of prisms in Figure 8.117 and the set of pyramids in Figure 8.121. Note that these are just some examples of prisms and pyramids. What attributes do all prisms and pyramids have in common? Write your thoughts before reading on. . . .

DISCUSSION

In all prisms and all pyramids:

There are bases, although prisms have two and pyramids have one.
There are faces, edges, and vertices.
The bases and faces are polygons.
Regular Polyhedra

In Section 8.2, we discussed regular polygons. How might we define a regular polyhedron? Try to do so before reading on. . . .

One of the ways we classified polygons was by the number of sides: triangles, quadrilaterals, pentagons, hexagons, and so on. We could speak of a regular hexagon and a nonregular hexagon. However, that doesn’t work with polyhedra. Do you see why?

We define a regular polyhedron as a convex polyhedron in which the faces are congruent regular polygons and in which the numbers of edges that meet at each vertex are the same.

Stop! Does that definition make sense? The concept of a regular polyhedron is one of the more abstract in the book. What does it mean to say that “the numbers of edges that meet at each vertex are the same”? If you are still not sure, read on, but then go back and check to see whether this definition jibes with that of regular polyhedra.

Which of the prisms and pyramids we have discussed so far do you think might be regular polyhedra? Think before reading on. . . .

A cube is a regular polyhedron. A triangular pyramid composed of equilateral triangles is a regular polyhedron and has a special name, tetrahedron. The origin of the name is Greek: tetra (“four”) and hedron (“face”).

A fact that surprises many people is that there are not a large number of regular polyhedra. In fact, there are only five regular polyhedra: the tetrahedron, the cube, the octahedron (with 8 triangular faces), the dodecahedron (with 12 pentagonal faces), and the icosahedron (with 20 triangular faces) (see Figure 8.122). The solids made from the regular polyhedra are called Platonic solids after the Greek philosopher Plato.

Source: The five regular solids drawn by Johannes Kepler in Harmonices Mundi, Book II, 1619.

FIGURE 8.122
Relationships Among Polyhedra

We examined relationships among quadrilaterals and among polygons in Section 8.2, and we examined relationships among polyhedra in this section, for example, the close connection between prisms and cylinders. One, of many more, relationships we will consider here was discovered by Leonard Euler and bears his name. If you did Exploration 8.15, you discovered this on your own. If not, you will see it here. It is included both because it is a famous formula and because it is one that children can discover, with some guidance, and it is another example of the rich interconnectedness that permeates mathematics which is not well appreciated by most people.

Any polyhedron has a certain number of faces, vertices, and edges. For example, count the number of faces, vertices, and edges of the cube on p. (552), the square pyramid on p. (549), and the truncated pyramid (the polyhedron to the right of the square pyramid) on p. (549). Then read on...
The cube has 6 faces, 8 vertices, and 12 edges.
The pyramid has 5 faces, 5 vertices, and 8 edges.
The truncated pyramid has 6 faces, 8 vertices, and 12 edges.

In chapter 2, you found that in each growing pattern, there is a relationship that enables us to construct a formula to predict the number of dots or squares in the nth figure in a growing pattern. Similarly, there is a relationship among the number of faces (F), vertices (V), and edges (E) in any polyhedron, and knowing this relationship enables us to construct a formula that connects the number of faces, vertices, and edges. Can you guess it from these three examples?

What Euler discovered is that the sum of the number of vertices and faces is always two more than the number of edges, and this is true for all polyhedra. In symbols, we write:

\[ V + F = E + 2 \]

**Connecting Two-Dimensional Representations to Three-Dimensional Objects**

Although we live in a three-dimensional world, much of our interaction with this world is on the two dimensions of books, magazines, and newspapers and on the two dimensions of computer and television screens.

There are many ways in which the two-dimensional and three-dimensional worlds connect. All buildings, from small sheds to large skyscrapers, are designed before they are built. For major projects, scale models are built. To enable the architects and the engineers to communicate, blueprints are designed and studied. So that the electricians, plumbers, and other members of the building team will know where to place the appropriate wires and fixtures, other kinds of drawings are used. Each of these drawings requires someone to think about the object in three dimensions and then represent that information two dimensionally, although computer simulation is changing the nature of these representations.

And this is only one example. Archaeologists work with the three dimensions of the excavation site and the two-dimensional representations of the “dig.” Painters need a thorough understanding of geometry so that their paintings (on a two-dimensional surface) will look like the three-dimensional objects or landscapes that they are representing. In this section, we will examine several ways in which the two-dimensional and three-dimensional worlds connect: cross sections, nets, and simple (isometric) drawings.

First, we will focus on simple buildings, the kind that can be made with cubes. Most elementary classrooms have blocks, and many powerful geometric ideas can be developed by playing with blocks. One of those is for children to build block buildings and then give directions for making the buildings.

**INVESTIGATION 8.24**

Look at the building at the right. Following are the profile views of the building from the front, from the right, from the back, from the left, and from the top (imagine flying over the building as you approach it from the front). Look at those views. Can you see how those views have been made? For example, can you see why the front view consists of three cubes stacked on one another and then a stack of two cubes to the right?
Look again at those views. What do you see? Recall that we found in high school geometry that in order for two triangles to be congruent, all three pairs of corresponding sides were congruent and all three pairs of corresponding angles were congruent. When we examined triangles more closely, we didn’t need to know all six pieces of information in order to say that two triangles were congruent. For example, if all three corresponding sides were congruent, that was sufficient for us to conclude that the two triangles were congruent. The analogous question here: Do we need all five views in order to make the figure? Why or why not? A slightly simpler, but related question: Are some of the views related to each other? Think about these questions before reading on.

**DISCUSSION**

As you may have noticed, in this case, the right and left views are mirror images of each other. Similarly, the front and back views are mirror images of each other. Do you think this is true just in this case, in some cases, or all cases? It turns out that it will be true in all cases. Thus we can cut out two pieces of information. For the sake of convention, we will denote the front and right-side views. What about the top? Is that really necessary? For example, if you were given only the front and right-side views of the building above, could you make the building? Think and read on.

There is another building that has the same front and right-side views as the one pictured above. It is shown below. However, its top view is different. Both the building and its top view are shown below. Thus the front, right, and top views are all necessary. A curious reader might be wondering whether the top, front, and right views will be sufficient in all cases. That is a great question and will be left as an exercise.

Another way to represent our three-dimensional block buildings would be to sketch them as was done above. Before proceeding further, cover the rest of this page and look at the block building shown above. Sketch it on a blank sheet of paper. Then read on.

Having looked at representing a building with three views, let us look at how we can draw the building. Some of you remember being told to make a cube by first drawing two overlapping squares. If you try to use Geoboard Dot Paper, you won’t draw very good models. But it turns out that Isometric Dot Paper will enable us to draw pictures of buildings quite nicely.
INVESTIGATION

8.25

As we noted in Section 8.1, paintings became much more realistic when artists learned perspective during the Renaissance. I'm not sure when isometric drawings were developed, but they offer us a tool to sketch simple polyhedra. First, look at the Isometric Dot Paper in Figure 8.124. What do you see?

The dots are not in a rectangular array. Rather, the dots in each row are staggered so that when you connect dots, you form equilateral triangles. Believe it or not, this equilateral triangular array is an efficient way to draw objects whose angles are right angles. Now go back and try to sketch the block building from the previous investigation on the isometric grid below. Then read on...

DISCUSSION

Some people find this easy to do. I wasn’t one of them! Now that I have done it hundreds of times, it is easy for me, but I can remember just not being able to figure it out and then finally “getting it”—only to discover some months later, when I was faced with the task again, I had forgotten it. Therefore, I understand completely if you are one of the readers who feels baffled at this point. Let us begin at the beginning. Figure 8.125(a) below shows 1 cube. Figure 8.125(b) shows a column of 2 cubes, and Figure 8.125(c), show a column of 2 cubes next to a column of 3 cubes. Finally, the earlier block figure illustrates how to put columns of cubes side by side, especially when there is a blank spot in the building.

Now that we have examined buildings from different views and isometric drawings, let us examine another connection between two- and three-dimensional objects. A cross section of a solid is what the exposed face would look like if we sliced through the solid. Because there are many ways that we might slice through a solid, the shape of the cross section will depend on the nature of the slice. Let’s examine a few.
**INVESTIGATION 8.26**

If we sliced the cube as shown in the Figure 8.126, what would the cross section look like? If we sliced the cube as shown in the Figure 8.127, what would the cross section look like? If we sliced the cube as shown in the Figure 8.128, what would the cross section look like? Could we get other shapes from slicing through a cube?

**DISCUSSION**

In the first case, the cross section is a square. In the second case, the cross section is a rectangle. In the third case, the cross section is a triangle. There are different ways of slicing that will result in different rectangles and triangles.

One last connection between two- and three-dimensional figures that we will explore here is nets. A net is simply a two-dimensional representation of a three-dimensional object, in which:

1. Every face of the object is represented.
2. If you cut out the net and fold along the edges, it will fold up into an actual object.

The figure at the left in Figure 8.129 is a net of a cube, whereas the figure at the right is not. If you fold the first figure up, you will get a cube. Do you see that? If not, try to fold it in your mind. One way is to make use of the properties of a cube. I have labeled the faces: Bo, T, F, Ba, S, and S for bottom, top, front, back, and sides. Does that help? In the second case, if you cut out the figure, it won’t fold up. Two faces will overlap.
INVESTIGATION

8.27

One of the nets that people experience regularly (especially if they recycle) is a flattened cereal box. One net for a standard cereal box is shown in Figure 8.130. What do you notice about this net? This includes, but is not restricted to, the question “What attributes and characteristics do you see?”

FIGURE 8.130

DISCUSSION

One of the key aspects of “really” understanding nets is to see certain attributes of nets and then to connect that information. One important observation is that the net has six faces. If you recall, cubes have six faces. The cereal box is a rectangular prism, and thus it has many of the attributes of a cube. Another observation is there are three pairs of congruent faces.

If we think of a cereal box, we think of front, back, sides, top, and bottom. If we label those faces on our net (see Figure 8.131), this leads to another observation: Two congruent faces are never side-by-side. Do you see why?

A good way to deepen your understanding of the connection between the three-dimensional and two-dimensional worlds is to sketch several other nets for the cereal box. Try this yourself before reading on...

If your understanding of nets is not well connected, this is a very difficult task. If it is connected, the job is much easier. One thing that makes this task easier is to realize that each face of the box is connected to three other faces. Thus we can take our original net and slide the bottom underneath the back, as shown in Figure 8.132(a)—it still folds up. In the original net, the top and bottom were connected to the front. However, on the actual box, they are also connected to the sides. Figure 8.132(b) represents that connection. Finally, we can move the back so that it is connected to the bottom, as shown in Figure 8.132(c). My students have worked on this problem and have found many, many nets!

FIGURE 8.132

Cylinders, Cones, Spheres

The polyhedra we have defined thus far have all been simple, closed surfaces in which all the faces are polygons. There are three other kinds of three-dimensional...
objects that are commonly found and that elementary school children study. Cylinders, cones, and spheres are related to polyhedra we have studied. Before we examine these three, stop for a moment and consider which polyhedra are related to cylinders, which to cones, and which to spheres. Then read on. . .

Think of a prism with more and more sides (see the prism at the left in Figure 8.133). At some point, a prism with a lot of sides begins to look more like a cylinder than like a prism. From one perspective, we can think of a cylinder as a prism in which the bases are circles. Technically, this is not true, because the bases of prisms are polygons, and a circle is not a polygon.

![Figure 8.133](image)

Thus we will describe a cylinder more formally as a simple, closed surface that is bounded by two congruent circles that lie in parallel planes.

Earlier, we talked about right prisms and right pyramids. A cylinder is a right cylinder if and only if the line segments joining two corresponding points on the two bases are perpendicular to the planes of the bases. If a cylinder is not a right cylinder, it is called an oblique cylinder.

Now think of a pyramid with more and more sides (see the pyramid at the left in Figure 8.134). At some point, a pyramid with a lot of sides begins to look more like a cone than like a pyramid. From one perspective, we can think of a cone as a pyramid in which the base is a circle. Technically, this is not true, because the base of a pyramid is a polygon, and a circle is not a polygon.

![Figure 8.134](image)

As we did with cylinders, we can define the term cone by using set language and say that a cone is constructed by starting with a simple, closed surface and a point not on the circle. A cone consists of the union of: the surface, the union of all line segments congruent to and parallel to the given line segment, and the surface formed by connecting the other endpoints of each of the line segments.

![Classroom Connection](image)

When children are asked to describe a cone, they say things like “A triangle with a flat bottom,” “A round triangle,” “A large circle with smaller and smaller circles on top until it reaches a point,” “A cylinder, triangle, and a circle in one.” From Examining Features of Shape: Casebook by Deborah Schifter, Virginia Bastable, and Susan Jo Russell, with Danielle Harrington and Marion Reynolds (Parsippany, NJ: Dale Seymour, 2002), p. 26.
starting with a dodecahedron made of clay and then slicing the various faces at an angle to make more and more faces. Eventually, the figure would begin to resemble a sphere more than a polyhedron.

A sphere is also conceptually related to a circle. Can you apply the earlier definition of a circle to define the term sphere? Try to do so before reading on. . . .

A sphere is the set of points in space equidistant from a given point, which is called the center. How would you define the radius and diameter of a sphere? Try to do so before reading on. . . .

Any line segment joining the center of the sphere to a point on the surface is called a radius. Any line segment whose endpoints lie on the surface of the sphere and that contains the center is called a diameter.

**Exercises 8.3**

1. Given the tetrahedron at the right, name the following:
   a. A face
   b. A vertex
   c. An edge

2. Identify the numbers of vertices, edges, and faces of the following figures.

3. Consider a prism whose base is a regular n-gon—that is, a regular polygon with n sides. How many vertices would such a prism have? How many faces? How many edges? You may want to start with a triangular prism, square prism, pentagonal prism, and so on, and look for patterns.

4. Consider a pyramid whose base is a regular n-gon—that is, a regular polygon with n sides. How many vertices would such a pyramid have? How many faces? How many edges?
   a. Describe the relationship between the numbers of vertices of an n-gon prism and of an n-gon pyramid.
   b. Describe the relationship between the numbers of faces of an n-gon prism and of an n-gon pyramid.
   c. Describe the relationship between the numbers of edges of an n-gon prism and of an n-gon pyramid.

5. a. What attributes do all cylinders and all prisms have in common that not all polyhedra have?
   b. What attributes do all prisms have that only prisms have?
   c. What attributes do all cylinders and cones have that not all three-dimensional figures have?

6. Name the figures below.

7. Which of the polyhedra below are convex?

8. Draw a nonconvex rectangular or pentagonal prism.

9. We defined a regular polyhedron as a convex polyhedron in which the faces are congruent regular polygons and in which the numbers of edges that meet at each vertex are the same. Carlos says that instead of saying that
the numbers of edges that meet at each vertex are the same, we could have said that the numbers of faces that meet at each vertex are the same. What do you think? Support your choice.

10. Can you make a pyramid in which the triangular faces are not all congruent?

11. Write a definition of diagonal for polyhedra.

12. There is a relationship between the number of diagonals and the base of a prism. That is, triangular prisms have a certain number of diagonals, square prisms have a certain number of diagonals, and so on. Determine this relationship so that you can answer the following question: How many diagonals does a prism have whose base is a regular polygon with \(n\) sides? [Your instructor may or may not give you hints for this problem. If not, I suggest looking at the 4 Steps for Problem Solving on the inside front cover of the Explorations volume.]

13. At the center of every tissue of toilet paper is a cardboard cylinder. Find and examine one of these cylinders. You can see a curved line running along the face of the cylinder.

   a. If you cut the cylinder along this line, what would the unfolded shape look like? Predict the shape and explain your reasoning.

   b. Why do you think these cylinders are manufactured this way instead of having a vertical cut?

14. Is there one geometric shape that describes all the sides of (right) pyramids? If there is, name it and justify your answer. If there is more than one shape, describe the shapes and justify your response.

15. a. Write directions for making each of the following block buildings any way you want.

   b. Write directions for the same block buildings, using a different method.

16. Sketch the figures below on Isometric Dot Paper.

17. Sketch the front, side, and top views of the buildings below.

18. Below are three nets for a cereal box. They count as one family because all it takes is a simple transformation—in this case a translation (slide)—to change one into another. Draw three more nets for a cereal box that are all in different families. Each net needs to have 6 whole faces; that is, do not cut one face into two or more pieces—otherwise, we have an almost infinite number of possibilities.

19. Using Polyomino Grid Paper, make as many nets as you can for the triangular prism shown below. There are 5 faces in the figure—the 2 triangular bases and 3 sides. Do not cut any of the faces—this would create many, many possible nets. That is, each net will consist of 5 distinct faces, joined together. The three lateral faces are all \(1 \times 3\) rectangles, and the two bases are equilateral triangles. There are fewer than 10 possible nets.

20. How many different heptominoes (made from 7 squares) are there that have 5 squares in a column and 1 square attached to opposite sides of the column? One is sketched below. Templates are given to make it easier to