EXPLORATION 2.7 **Pendulum**

The development of more and more accurate ways to keep track of time has a fascinating history. Thousands of years ago, people developed water clocks, sandglasses, and sundials, but each of these had limitations. At some point in time, someone, somewhere realized that there was a functional relationship between the length of a pendulum and the time it took the pendulum to swing back and forth (called the *period* of the pendulum). However, it was not until the seventeenth century that the pendulum clock was perfected. Thus, the antique grandfather clock actually represents a rather great scientific advance, for the pendulum inside the clock enabled people to keep track of time more accurately than they had in the past.

PART 1: Initial explorations

1. What questions — both “mathematical” and “nonmathematical” — do you have about pendula (plural of pendulum)?

2. **a.** What variables do you think will affect the length of time it takes the pendulum to go back and forth once? This length of time is the pendulum’s period.

b. Select one variable that you think might affect the pendulum’s period and briefly describe how you would gather data to determine how that variable affects the period of the pendulum.

3. Describe your exploration:

a. What did you do?

b. What data did you collect?

c. What conclusions can you make? How confident are you about your conclusions?

d. What other observations from the exploration seem worth sharing?

e. If you encountered and overcame difficulties or obstacles, describe one of them and how you overcame it.

4. We will examine the relationship between one variable (length of the pendulum) and the period. However, before we can do that, we will have to determine how long a pendulum is. For example, look at the figure below, which shows a ring tied onto a string.

Is the length of the pendulum the distance from the end of the string to the top of the ring, the distance from the end of the string to the bottom of the ring, or something else?

a. What distance do you choose as the length of the pendulum? Why?

b. Describe how we might find out which of these distances is the actual length of the pendulum.

c. Now do the experiment. First, describe what you did to answer the question. Then state and justify your conclusion.

PART 2: Predicting the period from the length

There are many questions we can answer concerning the relationship between the length and the period of a pendulum. In this part of the exploration, we will gather data on the periods of pendula of various lengths and then use our data to predict the period of a pendulum given its length.

1. **a.** First, take a minute to think about how you might determine the period. Note down your thoughts and discuss your ideas with other members of your group.

b. Determine the periods of a pendulum 40 cm in length and 50 cm in length.

c. Compare data and methods with other groups. Then note down the method the class has decided to use and briefly explain the reasoning behind the choice of this method.

2. **a.** Gather the data below.

Length of string	Period of your pendulum	Other groups' periods	Class period
40 cm			
50 cm			
60 cm			
70 cm			
80 cm			
90 cm			

b. Compare your group's data with other groups' data and add their data to your table.

c. Since there are differences among the groups' numbers, describe your thoughts about how to resolve these differences. That is, what is your proposal to determine the period you will use for each length; that is, the number in the last column? Briefly justify your proposal.

d. After the class discussion, write and briefly explain the class's choice.

e. Note the class period for each length.

3. **a.** Now use these data to predict the period of a 1-meter pendulum. Explain how you arrived at your prediction.

b. Now determine the period of a 1-meter pendulum. What is the class result?

c. How close was your prediction? Why do you think some groups were closer than others?

4. **a.** Use these data to predict the period of a 2-meter pendulum. Explain how you arrived at your prediction.

Most students report that they are less confident of their prediction for a 2-meter pendulum than they were for a 1-meter pendulum. The jump from 1 meter to 2 meters is too big to make an accurate prediction. This example illustrates a difference
between empirical research and theoretical research. The data from 40 centimeters,
50 centimeters, etc., were sufficient for predicting the period of a 1-meter pendulum, but such empirical data are not as useful for the 2-meter question. Several hundred years ago, scientists were able to combine their data and their growing knowledge about physics to discover the following relationship between the length of a pendulum and its period.

The actual formula is

$$P = 2\pi\sqrt{\frac{L}{g}}$$

where P is the period (in seconds), L is the length (in centimeters), and g is the acceleration due to gravity (980 cm/sec^2).

b. Predict the period of a 2-meter pendulum using this formula and then compare your first prediction with the prediction from the formula.

c. Determine the period of a 2-meter pendulum. Note your result, and then determine the class result.

d. How close is the class result to the number predicted by the formula?

In Chapter 10 you will further investigate the topic of measurement. At this point, it is interesting to note that in 1790, Thomas Jefferson made a sweeping series of proposals to standardize the various systems of measurement then in use. Specifically, he proposed to have a pendulum of length x represent 1 second, so that 1 second would be the same throughout the United States.

5. Determine the length of a pendulum that would have a period of 1 second.

6. Show that the physics formula is approximately equivalent to $P = 0.2\sqrt{L}$.

Looking Back on Exploration 2.7

1. Describe something that you learned from this exploration.

2. Describe something that you are still not clear about.

3. Examine the first four NCTM Curriculum Standards: Mathematics as problem Solving, Mathematics as Communication, Mathematics as Reasoning, and Mathematical Connections. Select one of the subheadings from this set that you feel you "own" more as a result of this exploration. Describe what you learned.

4. What did you learn about the use of measurement in experiments?