Chapter 1: Review of Real Numbers

Section 1.1

Concept Review 1.1

1. Sometimes true
The only exception to this statement is for zero.
$|0| = 0,$ which is not a positive number.

3. Sometimes true
If x is a positive integer, then $-x$ is a negative integer.
If x is a negative integer, then $-x$ is a positive integer.

5. Never true
$-4 < -2$ but $(-4)^2$ is not less than $(-2)^2$.

7. Always true

Objective 1 Exercises

1. –14: c, e
9: a, b, c, d
0: b, c
53: a, b, c, d
7.8: none
–626: c, e

3. $-\frac{15}{2}$: b, d
0: a, b, d
–3: a, b, d
π: c, d
$2.\overline{33}$: b, d
4.232232223: c, d
$\frac{\sqrt{5}}{4}$: c, d
$\sqrt{7}$: c, d

9. –27

11. $-\frac{3}{4}$

13. 0

15. $\sqrt{33}$

17. 91

19. Replace x with each element in the set and determine whether the inequality is true.
$x < 5$
$-3 < 5$ True
$0 < 5$ True
$7 < 5$ False
The inequality is true for –3 and 0.

21. Replace y with each element in the set and determine whether the inequality is true.
$y > -4$
$-6 > -4$ False
$-4 > -4$ False
$7 > -4$ True
The inequality is true for 7.

23. Replace w with each element in the set and determine whether the inequality is true.
$w \le -1$
$-2 \le -1$ True
$-1 \le -1$ True
$0 \le -1$ False
$1 \le -1$ False
The inequality is true for –2 and –1.

25. Replace b with each element in the set and evaluate the expression.
$-b$
$-(-9) = 9$
$-(0) = 0$
$-(9) = -9$

27. Replace c with each element in the set and evaluate the expression.
$|c|$
$|-4| = 4$
$|0| = 0$
$|4| = 4$

29. Replace m with each element in the set and evaluate the expression.
$-|m|$
$-|-6| = -6$
$-|-2| = -2$
$-|0| = 0$
$-|1| = -1$
$-|4| = -4$

Objective 2 Exercises

33. {–2, –1, 0, 1, 2, 3, 4}

35. {2, 4, 6, 8, 10, 12}

37. {3, 6, 9, 12, 15, 18, 21, 24, 27, 30}

39. {–35, –30, –25, –20, –15, –10, –5}

41. $\{x \mid x > 4,\ x \text{ is an integer}\}$

43. $\{x \mid x \ge -2\}$

45. $\{x \mid 0 < x < 1\}$

47. $\{x \mid 1 \le x \le 4\}$

49. $A \cup B = \{1, 2, 4, 6, 9\}$

51. $A \cup B = \{2, 3, 5, 8, 9, 10\}$

53. $A \cup B = \{-4, -2, 0, 2, 4, 8\}$

55. $A \cup B = \{1, 2, 3, 4, 5\}$

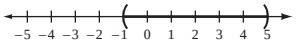
57. $A \cap B = \{6\}$

59. $A \cap B = \{5, 10, 20\}$

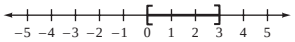
61. $A \cap B = \varnothing$

63. $A \cap B = \{4, 6\}$

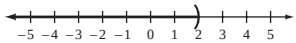
65. $\{x|-1 < x < 5\}$



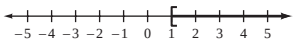
67. $\{x|0 \le x \le 3\}$



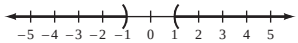
69. $\{x|x < 2\}$



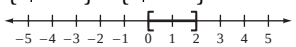
71. $\{x|x \ge 1\}$



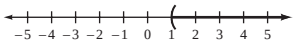
73. $\{x|x > 1\} \cup \{x|x < -1\}$



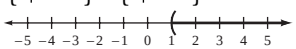
75. $\{x|x \le 2\} \cap \{x|x \ge 0\}$



77. $\{x|x > 1\} \cap \{x|x \ge -2\}$



79. $\{x|x > 2\} \cup \{x|x > 1\}$



81. $\{x|0 < x < 8\}$

83. $\{x|-5 \le x \le 7\}$

85. $\{x|-3 \le x < 6\}$

87. $\{x|x \le 4\}$

89. $\{x|x > 5\}$

91. $(-2, 4)$

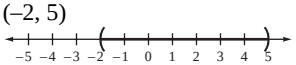
93. $[-1, 5]$

95. $(-\infty, 1)$

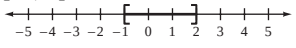
97. $[-2, 6)$

99. $(-\infty, \infty)$

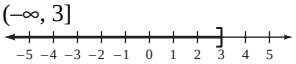
101. $(-2, 5)$



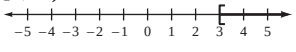
103. $[-1, 2]$



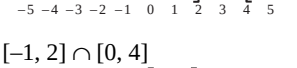
105. $(-\infty, 3]$



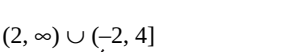
107. $[3, \infty)$



109. $(-\infty, 2] \cup [4, \infty)$



111. $[-1, 2] \cap [0, 4]$



113. $(2, \infty) \cup (-2, 4]$

Applying Concepts 1.1

115. $A \cup B$ is
$\{x|-1 \le x \le 1\} \cup \{x|0 \le x \le 1\} = \{x|-1 \le x \le 1\} = A$

117. $B \cap B$ is set B.

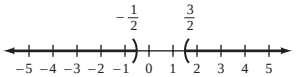
119. $A \cap R$ is $\{x|-1 \le x \le 1\}$, which is set A.

121. $B \cup R$ is the set of real numbers, R.

123. $R \cup R$ is the set R.

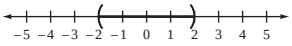
125. $B \cap C$ is $\{x|0 \le x \le 1\} \cap \{x|-1 \le x \le 0\}$, which contains only the number 0.

127.

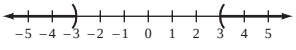


129.

131.



133.



135. $A \cup B = \{x|x > 0,\ x \text{ is an integer}\}$

137. $A \cap B = \{x|x \ge 15,\ x \text{ is an odd integer}\}$

139. The answer is b and c. For example:

a. $\dfrac{5-4}{3-2} \le 0$

$1 \le 0$

False

b. $\dfrac{2-3}{5-4} \le 0$

$-1 \le 0$

True

c. $\dfrac{5-4}{2-3} \le 0$

$-1 \le 0$

True

d. $\dfrac{4-5}{2-3} \le 0$

$1 \le 0$

False

Section 1.2

Concept Review 1.2

1. Sometimes true
(–2) + 4 = 2, a positive number
(–8) + 4 = –4, a negative number

3. Never true
$\dfrac{1}{2}+\dfrac{2}{3}=\dfrac{7}{6}$
$\dfrac{1+2}{2+3}=\dfrac{3}{5}$

5. Always true

7. Never true
The Order of Operations says to work inside parentheses before doing exponents.

9. Always true

Objective 1 Exercises

5. –18 + (–12) = –30

7. 5 – 22 = 5 + (–22) = –17

9. 3 · 4(–8) = 12 · (–8) = –96

11. 18 ÷ (–3) = –6

13. –60 ÷ (–12) = 5

15. –20(35)(–16) = –700(–16) = 11,200

17. (–271)(–365) = 98,915

19. $|12(-8)| = |-96| = 96$

21. $|15-(-8)| = |15+8| = |23| = 23$

23. $|-56 \div 8| = |-7| = 7$

25. $|-153 \div (-9)| = |17| = 17$

27. $-|-8|+|-4| = -8+4 = -4$

29.
$$\begin{aligned} -30+(-16)-14-2 &= -30+(-16)+(-14)+(-2) \\ &= -46+(-14)+(-2) \\ &= -60+(-2) \\ &= -62 \end{aligned}$$

31.
$$\begin{aligned} -2+(-19)-16+12 &= -2+(-19)+(-16)+12 \\ &= -21+(-16)+12 \\ &= -37+12 \\ &= -25 \end{aligned}$$

33.
$$\begin{aligned} 13-|6-12| &= 13-|6+(-12)| \\ &= 13-|-6| \\ &= 13-6 \\ &= 13+(-6) \\ &= 7 \end{aligned}$$

35.
$$\begin{aligned} 738-46+(-105) &= 738+(-46)+(-105) \\ &= 692+(-105) \\ &= 587 \end{aligned}$$

37. –442 ÷ (–17) = 26

39. –4897 ÷ 59 = –83

Objective 2 Exercises

43. $\dfrac{7}{12}+\dfrac{5}{16}=\dfrac{28}{48}+\dfrac{15}{48}=\dfrac{28+15}{48}=\dfrac{43}{48}$

45. $-\dfrac{5}{9}-\dfrac{14}{15}=-\dfrac{25}{45}-\dfrac{42}{45}=\dfrac{-25-42}{45}=-\dfrac{67}{45}$

47.
$$\begin{aligned} -\frac{1}{3}+\frac{5}{9}-\frac{7}{12} &= -\frac{12}{36}+\frac{20}{36}-\frac{21}{36} \\ &= \frac{-12+20-21}{36} \\ &= -\frac{13}{36} \end{aligned}$$

49. $\dfrac{2}{3}-\dfrac{5}{12}+\dfrac{5}{24}=\dfrac{16}{24}-\dfrac{10}{24}+\dfrac{5}{24}=\dfrac{16-10+5}{24}=\dfrac{11}{24}$

51. $\dfrac{5}{8}-\dfrac{7}{12}+\dfrac{1}{2}=\dfrac{15}{24}-\dfrac{14}{24}+\dfrac{12}{24}=\dfrac{15-14+12}{24}=\dfrac{13}{24}$

53. $\left(\dfrac{6}{35}\right)\left(-\dfrac{5}{16}\right)=-\dfrac{6\cdot 5}{35\cdot 16}=-\dfrac{\overset{1}{\cancel{2}}\cdot 3\cdot \overset{1}{\cancel{5}}}{\underset{1}{\cancel{5}}\cdot 7\cdot \underset{1}{\cancel{2}}\cdot 2\cdot 2\cdot 2}=-\dfrac{3}{56}$

55. $-\dfrac{8}{15}\div\dfrac{4}{5}=-\dfrac{8}{15}\cdot\dfrac{5}{4}=-\dfrac{8\cdot 5}{15\cdot 4}=-\dfrac{\overset{1}{\cancel{2}}\cdot\overset{1}{\cancel{2}}\cdot 2\cdot\overset{1}{\cancel{5}}}{3\cdot\underset{1}{\cancel{5}}\cdot\underset{1}{\cancel{2}}\cdot\underset{1}{\cancel{2}}}=-\dfrac{2}{3}$

57.
$$\begin{aligned} -\frac{11}{24}\div\frac{7}{12} &= -\frac{11}{24}\cdot\frac{12}{7} \\ &= -\frac{11\cdot 12}{24\cdot 7} \\ &= -\frac{11\cdot\overset{1}{\cancel{2}}\cdot\overset{1}{\cancel{2}}\cdot\overset{1}{\cancel{3}}}{\underset{1}{\cancel{2}}\cdot\underset{1}{\cancel{2}}\cdot 2\cdot\underset{1}{\cancel{3}}\cdot 7} \\ &= -\frac{11}{14} \end{aligned}$$

59. $\left(-\frac{5}{12}\right)\left(\frac{4}{35}\right)\left(\frac{7}{8}\right) = -\frac{5\cdot 4\cdot 7}{12\cdot 35\cdot 8}$

$$= -\frac{\overset{1}{\cancel{5}}\cdot\overset{1}{\cancel{2}}\cdot\overset{1}{\cancel{2}}\cdot\overset{1}{\cancel{7}}}{\underset{1}{\cancel{2}}\cdot\underset{1}{\cancel{2}}\cdot 3\cdot\underset{1}{\cancel{5}}\cdot\underset{1}{\cancel{7}}\cdot 2\cdot 2\cdot 2}$$

$$= -\frac{1}{24}$$

61.
$$\begin{array}{r} -14.270 \\ \underline{+\ 1.296} \\ -12.974 \end{array}$$

$-14.27 + 1.296 = -12.974$

63.
$$\begin{array}{r} -7.840 \\ \underline{+\ 1.832} \\ -6.008 \end{array}$$

$1.832 - 7.84 = -6.008$

65. $(0.03)(10.5)(6.1) = (0.315)(6.1) = 1.9215$

67. $0.9 \to 9$
$5.418 \to 54.18$

$$\begin{array}{r} 6.02 \\ 9\overline{)\ 54.18} \\ \underline{-54} \\ 01 \\ \underline{-\ 0} \\ 18 \\ \underline{-18} \\ 0 \end{array}$$

$5.418 \div (-0.9) = -6.02$

69. $0.065 \to 65$
$0.4355 \to 435.5$

$$\begin{array}{r} 6.7 \\ 65\overline{)\ 435.5} \\ \underline{-390} \\ 455 \\ \underline{-455} \\ 0 \end{array}$$

$-0.4355 \div 0.065 = -6.7$

71. $38.241 \div [-(-6.027)] - 7.453$
$= 38.241 \div 6.027 + (-7.453)$
$\approx 6.345 + (-7.453)$
≈ -1.11

73. $-287.3069 \div 0.1415 \approx -2030.44$

Objective 3 Exercises

75. $5^3 = 5\cdot 5\cdot 5 = 125$

77. $-2^3 = -(2\cdot 2\cdot 2) = -8$

79. $(-5)^3 = (-5)(-5)(-5) = -125$

81. $2^2\cdot 3^4 = (2)(2)\cdot(3)(3)(3)(3) = 4\cdot 81 = 324$

83. $-2^2\cdot 3^2 = -(2)(2)\cdot(3)(3) = -4\cdot 9 = -36$

85. $(-2)^3\cdot(-3)^2 = (-2)(-2)(-2)\cdot(-3)(-3)$
$= -8\cdot 9$
$= -72$

87. $4\cdot 2^3\cdot 3^3 = 4\cdot(2)(2)(2)\cdot(3)(3)(3)$
$= 4\cdot 8\cdot 27$
$= 32\cdot 27$
$= 864$

89. $2^2\cdot(-10)(-2)^2 = (2)(2)\cdot(-10)\cdot(-2)(-2)$
$= 4\cdot(-10)(4)$
$= -40(4)$
$= -160$

91. $(-3)^3\cdot 15\cdot(-2)^4 = (-27)\cdot 15\cdot(16)$
$= -405\cdot(16)$
$= -6480$

93. $2^5\cdot(-3)^4\cdot 4^5 = 32\cdot(81)\cdot 1024$
$= 2592\cdot 1024$
$= 2{,}654{,}208$

Objective 4 Exercises

97. $5 - 3(8\div 4)^2 = 5 - 3(2)^2 = 5 - 3(4) = 5 - 12 = -7$

99. $16 - \frac{2^2-5}{3^2+2} = 16 - \frac{4-5}{9+2}$
$= 16 - \frac{-1}{11}$
$= 16 + \frac{1}{11}$
$= \frac{177}{11}$

101. $\frac{3+\frac{2}{3}}{\frac{11}{16}} = \frac{\frac{11}{3}}{\frac{11}{16}} = \frac{11}{3}\cdot\frac{16}{11} = \frac{16}{3}$

103. $5[(2-4)\cdot 3 - 2] = 5[(-2)\cdot 3 - 2]$
$= 5[-6-2]$
$= 5[-8]$
$= -40$

105. $16-4\left(\frac{8-2}{3-6}\right)\div\frac{1}{2}=16-4\left(\frac{6}{-3}\right)\div\frac{1}{2}$
$=16-4(-2)\div\frac{1}{2}$
$=16-(-8)\div\frac{1}{2}$
$=16-(-8)\cdot 2$
$=16-(-16)$
$=16+16$
$=32$

107. $6[3-(-4+2)\div 2]=6[3-(-2)\div 2]$
$=6[3-(-1)]$
$=6[3+1]$
$=6[4]$
$=24$

109. $\frac{1}{2}-\left(\frac{2}{3}\div\frac{5}{9}\right)+\frac{5}{6}=\frac{1}{2}-\left(\frac{2}{3}\cdot\frac{9}{5}\right)+\frac{5}{6}$
$=\frac{1}{2}-\frac{6}{5}+\frac{5}{6}$
$=\frac{15}{30}-\frac{36}{30}+\frac{25}{30}$
$=\frac{15-36+25}{30}$
$=\frac{4}{30}$
$=\frac{2}{15}$

111. $\frac{1}{2}-\frac{\frac{17}{25}}{4-\frac{3}{5}}+\frac{1}{5}=\frac{1}{2}-\frac{\frac{17}{25}}{\frac{17}{5}}+\frac{1}{5}$
$=\frac{1}{2}-\left(\frac{17}{25}\cdot\frac{5}{17}\right)+\frac{1}{5}$
$=\frac{1}{2}-\frac{1}{5}+\frac{1}{5}$
$=\frac{1}{2}$

113. $\frac{2}{3}-\left[\frac{3}{8}+\frac{5}{6}\right]\div\frac{3}{5}=\frac{2}{3}-\left[\frac{9}{24}+\frac{20}{24}\right]\div\frac{3}{5}$
$=\frac{2}{3}-\frac{29}{24}\div\frac{3}{5}$
$=\frac{2}{3}-\frac{29}{24}\cdot\frac{5}{3}$
$=\frac{2}{3}-\frac{145}{72}$
$=\frac{48}{72}-\frac{145}{72}$
$=-\frac{97}{72}$

115. $0.4(1.2-2.3)^2+5.8=0.4(-1.1)^2+5.8$
$=0.4(1.21)+5.8$
$=0.484+5.8$
$=6.284$

117. $1.75\div 0.25-(1.25)^2=1.75\div 0.25-1.5625$
$=7-1.5625$
$=5.4375$

119. $25.76\div(6.96-3.27)^2=25.76\div(3.69)^2$
$=25.76\div 13.6161$
≈ 1.891878

Applying Concepts 1.2

121. 0

123. No, the multiplicative inverse of zero is undefined.

125. $7^{18}=1,628,413,597,910,449$
The ones digit is 9.

127. 5^{234} has over 150 digits. The last three are 625.

129. The order of operations is $a^{(b^c)}$.

Section 1.3

Concept Review 1.3

1. Sometimes true
The reciprocal of 1 is 1, a whole number. The reciprocal of 2 is $\frac{1}{2}$, not a whole number.

3. Sometimes true
$2xy$ and $3xy$ are like terms with the same variables. $2xy$ and $2x^2y$ are unlike terms with same variables.

5. Always true

Objective 1 Exercises

1. $3\cdot 4=4\cdot 3$

3. $(3+4)+5=3+(4+5)$

5. $\frac{5}{0}$ is undefined.

7. $3(x+2)=3x+6$

9. $\frac{0}{-6}=0$

11. $\frac{1}{mn}(mn)=1$

13. $2(3x)=(2\cdot 3)\cdot x$

15. A Division Property of Zero

17. The Inverse Property of Multiplication

19. The Addition Property of Zero

21. A Division Property of Zero

23. The Distributive Property

25. The Associative Property of Multiplication

Objective 2 Exercises

29. $ab + dc$

$(2)(3) + (-4)(-1) = 6 + 4 = 10$

31. $4cd \div a^2$

$$\begin{aligned} 4(-1)(-4) \div (2)^2 &= 4(-1)(-4) \div 4 \\ &= (-4)(-4) \div 4 \\ &= 16 \div 4 \\ &= 4 \end{aligned}$$

33. $(b-2a)^2 + c$

$$\begin{aligned} [3-2(2)]^2 + (-1) &= [3-4]^2 + (-1) \\ &= [-1]^2 + (-1) \\ &= 1 + (-1) \\ &= 0 \end{aligned}$$

35. $(bc+a)^2 \div (d-b)$

$$\begin{aligned} [(3)(-1)+2]^2 \div (-4-3) &= [-3+2]^2 \div (-7) \\ &= [-1]^2 \div (-7) \\ &= -\frac{1}{7} \end{aligned}$$

37. $\frac{1}{4}a^4 - \frac{1}{6}bc$

$$\begin{aligned} \frac{1}{4}(2)^4 - \frac{1}{6}(3)(-1) &= \frac{1}{4}(16) - \frac{1}{6}(3)(-1) \\ &= 4 - \frac{1}{6}(3)(-1) \\ &= 4 - \frac{1}{2}(-1) \\ &= 4 - \left(-\frac{1}{2}\right) \\ &= 4 + \frac{1}{2} \\ &= \frac{9}{2} \end{aligned}$$

39. $\frac{3ac}{-4} - c^2$

$$\begin{aligned} \frac{3(2)(-1)}{-4} - (-1)^2 &= \frac{6(-1)}{-4} - (-1)^2 \\ &= \frac{-6}{-4} - (-1)^2 \\ &= \frac{3}{2} - (-1)^2 \\ &= \frac{3}{2} - 1 \\ &= \frac{1}{2} \end{aligned}$$

41. $\frac{3b-5c}{3a-c}$

$$\frac{3(3)-5(-1)}{3(2)-(-1)} = \frac{9-(-5)}{6-(-1)} = \frac{9+5}{6+1} = \frac{14}{7} = 2$$

43. $\frac{a-d}{b+c}$

$$\frac{2-(-4)}{3+(-1)} = \frac{2+4}{3+(-1)} = \frac{6}{2} = 3$$

45. $-a|a+2d|$

$$\begin{aligned} -2|2+2(-4)| &= -2|2+(-8)| \\ &= -2|-6| \\ &= -2(6) \\ &= -12 \end{aligned}$$

47. $\frac{2a-4d}{3b-c}$

$$\begin{aligned} \frac{2(2)-4(-4)}{3(3)-(-1)} &= \frac{4-(-16)}{9-(-1)} \\ &= \frac{4+16}{9+1} \\ &= \frac{20}{10} \\ &= 2 \end{aligned}$$

49. $-3d \div \left|\frac{ab-4c}{2b+c}\right|$

$$\begin{aligned} -3(-4) \div \left|\frac{2(3)-4(-1)}{2(3)+(-1)}\right| &= -3(-4) \div \left|\frac{6-(-4)}{6+(-1)}\right| \\ &= -3(-4) \div \left|\frac{6+4}{6+(-1)}\right| \\ &= -3(-4) \div \left|\frac{10}{5}\right| \\ &= -3(-4) \div |2| \\ &= -3(-4) \div 2 \\ &= 12 \div 2 \\ &= 6 \end{aligned}$$

51. $2(d-b) \div (3a-c)$

$$\begin{aligned} 2(-4-3) \div [3(2)-(-1)] &= 2(-7) \div [6-(-1)] \\ &= 2(-7) \div [6+1] \\ &= 2(-7) \div 7 \\ &= -14 \div 7 \\ &= -2 \end{aligned}$$

53. $-d^2 - c^3 a$

$$\begin{aligned} -(-4)^2 - (-1)^3(2) &= -16 - (-1)(2) \\ &= -16 + 2 \\ &= -14 \end{aligned}$$

55. $-d^3 + 4ac$

$$\begin{aligned} -(-4)^3 + 4(2)(-1) &= -(-64) + 8(-1) \\ &= 64 - 8 \\ &= 56 \end{aligned}$$

57. $4^{(a)^2}$

$4^{(2^2)} = 4^4 = 256$

59. $V = LWH$

$V = (14)(10)(6)$

$V = 840$

The volume is 840 in^3.

61. $V = \frac{1}{3}s^2 h$

$V = \frac{1}{3}(3^2)5$

$V = 15$

The volume is 15 ft^2.

63. $V = \frac{4}{3}\pi r^3 \qquad r = \frac{1}{2}d = \frac{1}{2}(3) = 1.5$

$V = \frac{4}{3}\pi(1.5)^3$

$V = 4.5\pi$

$V \approx 14.14$

The volume is 4.5π cm^3.

The volume is approximately 14.14 cm^3.

65. SA = $2LW + 2LH + 2WH$

SA = 2(5)(4) + 2(5)(3) l+ 2(4)(3)

SA = 40 + 30 + 24

SA = 94

The surface area is 94 m^2.

67. $SA = s^2 + 4\left(\frac{1}{2}\right)bh$

$SA = 4^2 + 2(4)(5)$

$SA = 16 + 40 = 56$

The surface area is 56 m^2.

69. $SA = 2\pi r^2 + 2\pi rh$

$SA = 2\pi(6^2) + 2\pi(6)(2)$

$SA = 72\pi + 24\pi$

$SA = 96\pi$

$SA \approx 301.59$

The surface area is 96π in^2.

The surface area is approximately 301.59 in^2.

71. $\text{Density of statue} = \frac{\text{weight of statue}}{\text{volume of statue}}$

$\text{Density of statue} = \frac{15}{60.48}$

Density of statue = 0.25 lb / in^3

The statue has a density of 0.25 lb/in^3.

Objective 3 Exercises

73. $3x + 10x = 13x$

75. $-2x + 5x - 7x = 3x - 7x = -4x$

77. $-2a + 7b + 9a = 7a + 7b$

79. $12\left(\frac{1}{12}\right)x = x$

81. $-3(x - 2) = -3x + 6$

83. $(x + 2)5 = 5x + 10$

85. $-(-x - y) = x + y$

87. $3(x - 2y) - 5 = 3x - 6y - 5$

89. $-2a - 3(3a - 7) = -2a - 9a + 21 = -11a + 21$

91. $2x - 3(x - 2y) = 2x - 3x + 6y = -x + 6y$

93. $5[-2 - 6(a - 5)] = 5[-2 - 6a + 30]$
$= 5[28 - 6a]$
$= 140 - 30a$

95. $5[y - 3(y - 2x)] = 5[y - 3y + 6x]$
$= 5[-2y + 6x]$
$= -10y + 30x$

97. $4(-a - 2b) - 2(3a - 5b) = -4a - 8b - 6a + 10b$
$= -10a + 2b$

99. $-7(2a - b) + 2(-3b + a) = -14a + 7b - 6b + 2a$
$= -12a + b$

101. $2x - 4[x - 4(y - 2[5y + 3])]$
$= 2x - 4[x - 4(y - 10y - 6)]$
$= 2x - 4[x - 4(-9y - 6)]$
$= 2x - 4[x + 36y + 24]$
$= 2x - 4x - 144y - 96$
$= -2x - 144y - 96$

103. $3x + 8(x - 4) - 3(2x - y)$
$= 3x + 8x - 32 - 6x + 3y$
$= 5x - 32 + 3y$

105. $\frac{1}{4}[14x - 3(x - 8) - 7x] = \frac{1}{4}[14x - 3x + 24 - 7x]$
$= \frac{1}{4}[4x + 24]$
$= x + 6$

Applying Concepts 1.3

107. $4(3y + 1) = 12y + 4$

The statement is correct; it uses the Distributive Property.

109. $2 + 3x = (2 + 3)x = 5x$

The statement is not correct; it mistakenly uses the Distributive Property. It is in an irreducible statement. That is, the answer is $2 + 3x$.

111. $2(3y) = (2 \cdot 3)(2y) = 12y$

The statement is not correct; it incorrectly uses the Associative Property of Multiplication. The correct answer is $(2 \cdot 3)y = 6y$.

113. $-x^2 + y^2 = y^2 - x^2$

The statement is correct; it uses the Commutative Property of Addition.

115. $3a + 4(b + a)$

a. $3a + (4b + 4a)$ Distributive Property

b. $3a + (4a + 4b)$ Commutative Property of Addition

c. $(3a + 4a) + 4b$ Associative Property of Addition

d. $(3 + 4)a + 4b$ Distributive Property
$7a + 4b$

b. $(5 \cdot 3)a + 5(1)$ Associative Property of Multiplication
$15a + 5(1)$

117. $5(3a + 1)$

a. $5(3a) + 5(1)$ Distributive Property

c. $15a + 5$ Multiplication Property of One

Section 1.4

Concept Review 1.4

1. Never true
The smaller number is represented by $12 - x$.

3. Never true
The sum of twice x and 4 is represented by $2x + 4$.

5. Sometimes true
The square of $-x$ is represented by $(-x)^2$. The only exception is for the number 0.
$-0^2 = (-0)^2 = 0$

Objective 1 Exercises

1. the unknown number: n
The sum of the number and two: $n + 2$
$n - (n + 2) = n - n - 2 = -2$

3. the unknown number: n
one-third of the number: $\frac{1}{3}n$
four-fifths of the number: $\frac{4}{5}n$
$\frac{1}{3}n + \frac{4}{5}n = \frac{5}{15}n + \frac{12}{15}n = \frac{17}{15}n$

5. the unknown number: n
the product of eight and the number: $8n$
$5(8n) = 40n$

7. the unknown number: n
the product of seventeen and the number: $17n$
twice the number: $2n$
$17n - 2n = 15n$

9. the unknown number: n
the square of the number: n^2
the total of twelve and the square of the number: $12 + n^2$
$n^2 - (12 + n^2) = n^2 - 12 - n^2 = -12$

11. the unknown number: n
the sum of five times the number and 12: $5n + 12$
the product of the number and fifteen: $15n$
$15n + (5n + 12) = 15n + 5n + 12 = 20n + 12$

13. Let the smaller number be x.
The larger number is $15 - x$.
The sum of twice the smaller number and two more than the larger number
$2x + (15 - x + 2) = 2x + (17 - x) = x + 17$

15. Let the larger number be x.
Then the smaller number is $34 - x$.
The difference between two more than the smaller number and twice the larger number
$[(34 - x) + 2] - 2x = 34 - x + 2 - 2x = 36 - 3x$

Objective 2 Exercises

17. The population of Milan, Italy: P
The population of San Paolo, Brazil: $4P$

19. Amount earned by Arnold Palmer: A
Amount earned by Dennis Rodman: $\frac{2}{3}A$

21. The measure of angle B: x
The measure of angle A is twice that of angle B: $2x$
The measure of angle C is twice that of angle A: $2(2x) = 4x$

23. The flying time between Los Angeles and New York: t
The flying time between New York and Los Angeles: y
The total round-trip time: $t + y = 12$
The trip from New York to Los Angeles can be expressed as $y = 12 - t$.

Applying Concepts 1.4

25. Three more than twice a number.

27. The product of two and three more than a number.

29. One-half the acceleration due to gravity: $\frac{1}{2}g$
Time squared: t^2
The product: $\frac{1}{2}gt^2$

31. The product of A and v^2: Av^2

Focus on Problem Solving

1. a. *Understand the problem.* We must determine the weight of water in the cup. To do this, we need the volume of the cup and the density (weight per unit volume) of water. The dimensions of the cup are in inches, so the volume will be in cubic inches. Therefore, the density must be found in ounces per cubic inch.

b. *Devise a plan.* Consult a reference book to find the formula for the volume of a cone and the density of water. The formula for the volume of a cone is $V = \frac{1}{3}\pi r^2 h$. The density of water is 62.4 lb/ft^3. The plan is to convert the density to ounces per cubic inch and then use the formula $w = dv$ where w is the weight in ounces, d is the density of water in ounces per cubic inch, and v is the volume in cubic inches.

F **c.** *Carry out the plan.* Find the volume of the cone.

$r = 1.5$, $h = 4$

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(1.5)^2(4) \approx 9.425 \text{ in}^3$$

Convert 62.4 lb/ft^3 to ounces per cubic inch.

$$d = 62.4\frac{\text{lb}}{\text{ft}^3}$$
$$= 62.4\frac{\text{lb}}{\text{ft}^3}\cdot\frac{1 \text{ ft}^3}{1728 \text{ in}^3}\cdot\frac{16 \text{ oz}}{\text{lb}}$$
$$\approx 0.578\frac{\text{oz}}{\text{in}^3}$$

Substitute the values in the formula $w = dv$.

$$w \approx 0.578\frac{\text{oz}}{\text{in}^3}\cdot 9.425 \text{ in}^3 \approx 5.45 \text{ oz}$$

The cup will hold 5.45 oz of water.

d. *Review the solution.* A cup 4 in. tall is a fairly large cup, so it seems reasonable that it would hold about one-third of a pound.

2. a. *Understand the problem.* We are to determine the dimensions of a 12-oz soft drink can. We are to use an approximation of the distance that a hand can reach around 75% of the circumference of the can. We need to know the formula for the volume of a right circular cylinder and the volume in cubic inches of 12 fl oz. We also need to make an approximation of the length of a hand.

b. *Devise a plan.* From a resource book, we find that the volume of a right circular cylinder is $V = \pi r^2 h$. Approximate the length of a hand is 7 in. From this approximation, we can use the formula $C = 2\pi r$ to find the radius of the can. After finding the volume of 12 fl oz and the radius of the can, we find the height of the can.

c. *Carry out the plan.* The length of the hand is 75% of the circumference.

$$7 = 0.75C$$
$$9.33 \approx C$$

Use the formula $C = 2\pi r$ to find the radius.

$$C = 2\pi r$$
$$9.333 = 2\pi r$$
$$1.485 \approx r$$

Use the fact that 128 fl oz = 1 gal and 1 gal = 231 in^3 to find the volume of 12 fl oz.

$$V = 12 \text{ fl oz}$$
$$= 12 \text{ fl oz}\cdot\frac{1 \text{ gal}}{128 \text{ fl oz}}\cdot\frac{231 \text{ in}^3}{1 \text{ gal}}$$
$$\approx 21.656 \text{ in}^3$$

Use the formula for the volume of a right circular cylinder to find the height of the can.

$$V = \pi r^2 h$$
$$21.656 = \pi(1.485)^2 h$$
$$\frac{21.656}{\pi(1.495)^2} = h$$
$$3.13 \approx h$$

The radius of the can is approximately 1.5 in., and the height is approximately 3.1 in.

d. *Review the solution.* The diameter of the can is approximately the same as the height of the can. The diameter seems too large and the height seems too small. The approximation of the distance of the hand reaching around the can may be too large.

Projects and Group Activities

Water Displacement

1. Volume of the cylinder is $V = \pi r^2 h$, where $r = 2$ and $h = 10$.

$$V = \pi(2)^2(10)$$
$$V = 40\pi$$

The volume of the water displaced is $V = LWH$, where $L = 30$, $W = 20$, and $H = x$.

$$40\pi = (30)(20)x$$
$$\frac{2}{30}\pi = x$$
$$0.21 \approx x$$

The water will rise approximately 0.21 cm.

2. The volume of $\frac{2}{3}$ of the sphere is $V = \frac{2}{3}\left(\frac{4}{3}\pi r^3\right)$, where $r = 6$.

$$V = \frac{8}{9}\pi(6)^3$$
$$V = 192\pi$$

The volume of the water displaced is $V = LWH$, where $L = 20$, $W = 16$ and $H = x$.

$$192\pi = (20)(16)x$$
$$\frac{3}{5}\pi = x$$
$$1.88 \approx x$$

The water will rise approximately 1.88 in.

3. Find the volume of the statue by finding the volume of the water displaced by the statue.
$V = LWH$, where $L = 12$, $W = 12$ and $H = 0.42$.
$V = (12)(12)(0.42) = 60.48$
The volume of the statue is 60.48 cubic inches.
density = weight ÷ volume
density = 15 ÷ 60.48 ≈ 0.25
The density of the statue is approximately 0.25 lb/in^3.

Chapter Review Exercises

1. $\frac{3}{4} : -\frac{3}{4} + \frac{3}{4} = 0$

2. Replace x with the elements in the set and determine whether the inequality is true.
$x > -1$
$-4 > -1$ False
$-2 > -1$ False
$0 > -1$ True
$2 > -1$ True
The inequality is true for 0 and 2.

3. $p \in \{-4, 0, 7\}$
$-|p|$
$-|-4| = -4$
$-|0| = 0$
$-|7| = -7$

4. $\{-2, -1, 0, 1, 2, 3\}$

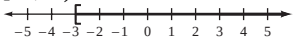
5. $\{x \mid x < -3,\ x \in \text{real numbers}\}$

6. $\{x \mid -2 \le x \le 3\}$

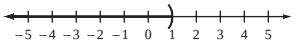
7. $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8\}$

8. $A \cap B = \{2, 3\}$

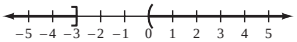
9. $[-3, \infty)$



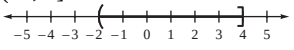
10. $\{x \mid x < 1\}$



11. $\{x \mid x \le -3\} \cup \{x \mid x > 0\}$



12. $(-2, 4]$



13. $-10 - (-3) - 8 = -10 + 3 + (-8) = -7 + (-8) = -15$

14. $-204 \div (-17) = 12$

15. $18 - |-12 + 8| = 18 - |-4| = 18 - 4 = 14$

16. $-2 \cdot (4^2) \cdot (-3)^2 = -2 \cdot 16 \cdot 9 = -32 \cdot 9 = -288$

17.
$$-\frac{3}{8} + \frac{3}{5} - \frac{1}{6} = -\frac{45}{120} + \frac{72}{120} - \frac{20}{120}$$
$$= \frac{-45 + 72 - 20}{120}$$
$$= \frac{7}{120}$$

18.
$$\frac{3}{5}\left(-\frac{10}{21}\right)\left(-\frac{7}{15}\right) = \frac{3 \cdot 10 \cdot 7}{5 \cdot 21 \cdot 15}$$
$$= \frac{\overset{1}{\cancel{3}} \cdot 2 \cdot \overset{1}{\cancel{5}} \cdot \overset{1}{\cancel{7}}}{\underset{1}{\cancel{5}} \cdot \underset{1}{\cancel{3}} \cdot \underset{1}{\cancel{7}} \cdot 3 \cdot 5}$$
$$= \frac{2}{15}$$

19.
$$-\frac{3}{8} \div \frac{3}{5} = -\frac{3}{8} \cdot \frac{5}{3}$$
$$= \frac{\overset{1}{\cancel{3}} \cdot 5}{8 \cdot \underset{1}{\cancel{3}}}$$
$$= -\frac{5}{8}$$

20. $-4.07 + 2.3 - 1.07 = -1.77 - 1.07 = -2.84$

21. $-3.286 \div (-1.06) = 3.1$

22.
$$20 \div \frac{3^2 - 2^2}{3^2 + 2^2} = 20 \div \frac{9 - 4}{9 + 4}$$
$$= 20 \div \frac{5}{13}$$
$$= 20 \cdot \frac{13}{5}$$
$$= 52$$

23.
$$2a^2 - \frac{3b}{a} = 2(-3)^2 - \frac{3(2)}{-3}$$
$$= 2(-3)^2 - \frac{6}{-3}$$
$$= 2(-3)^2 - (-2)$$
$$= 2(9) - (-2)$$
$$= 18 + 2$$
$$= 20$$

24. $(a - 2b^2) \div ab$
$$(4 - 2(-3)^2) \div (4)(-3) = (4 - 2(9)) \div (4)(-3)$$
$$= (4 - 18) \div (4)(-3)$$
$$= -14 \div [(4)(-3)]$$
$$= -14 \div -12$$
$$= \frac{-14}{-12}$$
$$= \frac{7}{6}$$

25. 3

26. y

27. (ab)

28. 4

29. The Inverse Property of Addition

30. The Associative Property of Multiplication

31. $-2(x-3)+4(2-x) = -2x+6+8-4x = -6x+14$

32. $$\begin{aligned} 4y-3[x-2(3-2x)-4y] &= 4y-3[x-6+4x-4y] \\ &= 4y-3[5x-6-4y] \\ &= 4y-15x+18+12y \\ &= 16y-15x+18 \end{aligned}$$

33. The unknown number: x
The sum of the number and four: $x+4$
$4(x+4) = 4x+16$

34. The unknown number: x
The difference between the number and two: $x-2$
Twice the difference between the number and two: $2(x-2)$
$2(x-2)+8 = 2x-4+8 = 2x+4$

35. Let x be the smaller of the numbers. Then the larger number is $40-x$
The sum of twice x and five more than $40-x$.
$2x+(40-x+5) = x+45$

36. Let x be the larger number.
Then the smaller number is $9-x$.
The difference between three more than twice $(9-x)$ and one more than x.
$$\begin{aligned} [2(9-x)+3]-(x+1) &= 18-2x+3-x-1 \\ &= -3x+20 \end{aligned}$$

37. The width of the rectangle: W
The length is 3 feet less than $3W$.
The length is $3W-3$.

38. Let the first integer be n.
The second integer is five more than four times n.
$4n+5$ is the magnitude of the second integer.

Chapter Test

1. 12

2. Replace x with each element in the set and determine whether the inequality is true.
$-1 > x$
$-1 > -5$ True
$-1 > 3$ False
$-1 > 7$ False
The inequality is true for -5.

3. $$\begin{aligned} 2-(-12)+3-5 &= 2+12+3+(-5) \\ &= 14+3+(-5) \\ &= 17+(-5) \\ &= 12 \end{aligned}$$

4. $(-2)(-3)(-5) = (6)(-5) = -30$

5. $-180 \div 12 = -15$

6. $\left|-3-(-5)\right| = \left|-3+5\right| = \left|2\right| = 2$

7. $-5^2 \cdot 4 = -25 \cdot 4 = -100$

8. $(-2)^3(-3)^2 = (-8)(9) = -72$

9. $$\begin{aligned} \frac{2}{3}-\frac{5}{12}+\frac{4}{9} &= \frac{24}{36}-\frac{15}{36}+\frac{16}{36} \\ &= \frac{24-15+16}{36} \\ &= \frac{25}{36} \end{aligned}$$

10. $$\left(-\frac{2}{3}\right)\left(\frac{9}{15}\right)\left(\frac{10}{27}\right) = -\frac{2 \cdot \cancel{3}^{1} \cdot \cancel{3}^{1} \cdot 2 \cdot \cancel{5}^{1}}{\cancel{3} \cdot \cancel{3} \cdot \cancel{5} \cdot 3 \cdot 3 \cdot 3} = -\frac{4}{27}$$

11. $4.27 - 6.98 + 1.3 = -2.71 + 1.3 = -1.41$

12. $-15.092 \div 3.08 = -4.9$

13. $$\begin{aligned} 12-4\left(\frac{5^2-1}{3}\right)\div 16 &= 12-4\left(\frac{25-1}{3}\right)\div 16 \\ &= 12-4\left(\frac{24}{3}\right)\div 16 \\ &= 12-4(8)\div 16 \\ &= 12-32\div 16 \\ &= 12-2 \\ &= 10 \end{aligned}$$

14. $$\begin{aligned} 8-4(2-3)^2 \div 2 &= 8-4(-1)^2 \div 2 \\ &= 8-4(1)\div 2 \\ &= 8-4\div 2 \\ &= 8-2 \\ &= 6 \end{aligned}$$

15. $$\begin{aligned} (a-b)^2 \div (2b+1) &= (2-(-3))^2 \div (2(-3)+1) \\ &= (5)^2 \div (-6+1) \\ &= (5)^2 \div (-5) \\ &= 25 \div (-5) \\ &= -5 \end{aligned}$$

16. $$\begin{aligned} \frac{b^2-c^2}{a-2c} &= \frac{(3)^2-(-1)^2}{2-2(-1)} \\ &= \frac{9-1}{2-(-2)} \\ &= \frac{8}{4} \\ &= 2 \end{aligned}$$

17. 4

18. The Distributive Property

19. $3x-2(x-y)-3(y-4x)$
$= 3x-2x+2y-3y+12x$
$= 13x-y$

20. $2x-4[2-3(x+4y)-2]$
$= 2x-4[2-3x-12y-2]$
$= 2x-4[-3x-12y]$
$= 2x+12x+48y$
$= 14x+48y$

21. the unknown number: n
three less than the number: $n - 3$
the product of three less than the number and nine: $(n - 3)(9)$
$13 - (n - 3)(9) = 13 - 9n + 27 = 40 - 9n$

22. The unknown number: n
The total of twelve times the number and twenty-seven: $12n + 27$
$\frac{1}{3}(12n + 27) = 4n + 9$

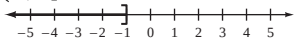
23. $A \cup B = \{1, 2, 3, 4, 5, 7\}$

24. $A \cup B = \{-2, -1, 0, 1, 2, 3\}$

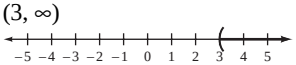
25. $A \cap B = \{5, 7\}$

26. $A \cap B = \{-1, 0, 1\}$

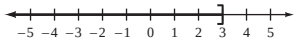
27. $(\infty, 1]$



28. $(3, \infty)$



29. $\{x|x \le 3\} \cup \{x|x < -2\}$



30. $\{x|x < 3\} \cap \{x|x > -2\}$