Appendix

Using Your Calculator

In this section we will review how to use your calculator to perform common mathematical operations. This discussion assumes that your calculator uses the algebraic operating system, the system used by most brands.

One very important principle to keep in mind as you use your calculator is that it is not a substitute for your brain. Keep thinking as you do the calculations. Keep asking yourself, "Does the answer make sense?"

Addition, Subtraction, Multiplication, and Division

Performing these operations on a pair of numbers always involves the following steps:

1. Enter the first number, using the numbered keys and the decimal (.) key if needed.
2. Enter the operation to be performed.
3. Enter the second number.
4. Press the "equals" key to display the answer.

For example, the operation

$$15.1 + 0.32$$

is carried out as follows:

Press	***Display***
15.1	15.1
+	15.1
.32	0.32
=	15.42

The answer given by the display is 15.42. If this is the final result of a calculation, you should round it off to the correct number of significant figures (15.4), as discussed in Section 2.5. If this number is to be used in further calculations, use it exactly as it appears on the display. Round off only the final answer in the calculation.

Do the following operations for practice. The detailed procedures are given below.

a. $1.5 + 32.86$ c. 0.33×153

b. $23.5 - 0.41$ d. $\dfrac{9.3}{0.56}$ or $9.3 \div 0.56$

PROCEDURES

a.

Press	***Display***
1.5	1.5
+	1.5
32.86	32.86
=	34.36
Rounded:	34.4

b.

Press	***Display***
23.5	23.5
−	23.5
.41	0.41
=	23.09
Rounded:	23.1

c.

Press	***Display***
.33	0.33
×	0.33
153	153
=	50.49
Rounded:	50.

d.

Press	***Display***
9.3	9.3
÷	9.3
.56	0.56
=	16.607143
Rounded:	17

Squares, Square Roots, Reciprocals, and Logs

Now we will consider four additional operations that we often need to solve chemistry problems.

The *squaring* of a number is done with a key labeled X^2. The *square root* key is usually labeled $\sqrt{X}$. To take the *reciprocal* of a number, you need the 1/X key. The *logarithm* of a number is determined by using a key labeled log or logX.

To perform these operations, take the following steps:

1. Enter the number.
2. Press the appropriate function key.
3. The answer is displayed automatically.

For example, let's calculate the square root of 235.

Press	***Display***
235	235
$\sqrt{X}$	15.32971
Rounded:	15.3

We can obtain the log of 23 as follows:

Press	***Display***
23	23
log	1.3617278
Rounded:	1.36

Often a key on a calculator serves two functions. In this case, the first function is listed on the key and the second is shown on the calculator just above the key. For example, on some calculators the top row of keys appears as follows:

	1/X	X^2		
2nd	R/S	$\sqrt{X}$	off	on/C

To make the calculator square a number, we must use 2nd and then $\sqrt{X}$; pressing 2nd tells the calculator we want the function that is listed *above* the key. Thus we can obtain the square of 11.56 on this calculator as follows:

Press	***Display***
11.56	11.56
2nd then $\sqrt{X}$	133.6336
Rounded:	133.6

We obtain the reciprocal of 384 (1/384) on this calculator as follows:

Press	***Display***
384	384
2nd then R/S	0.0026042
Rounded:	0.00260

Your calculator may be different. See the user's manual if you are having trouble with these operations.

Chain Calculations

In solving problems you often have to perform a series of calculations—a calculation chain. This is generally quite easy if you key in the chain as you read the numbers and operations in order. For example, to perform the calculation

$$\frac{14.68 + 1.58 - 0.87}{0.0850}$$

you should use the appropriate keys as you read it to yourself:

> 14.68 plus 1.58 equals; minus .87 equals;
> divided by 0.0850 equals

The details follow.

Press	***Display***
14.68	14.68
+	14.68
1.58	1.58
=	16.26
−	16.26
.87	0.87
=	15.39
÷	15.39
.0850	0.0850
=	181.05882
Rounded:	181

Note that you must press [=] after every operation to keep the calculation "up to date."

For more practice, consider the calculation

$$(0.360)(298) + \frac{(14.8)(16.0)}{1.50}$$

Here you are adding two numbers, but each must be obtained by the indicated calculations. One procedure is to calculate each number first and then add them. The first term is

$$(0.360)(298) = 107.28$$

The second term,

$$\frac{(14.8)(16.0)}{1.50}$$

can be computed easily by reading it to yourself. It "reads"

> 14.8 times 16.0 equals: divided by 1.50 equals:

and is summarized as follows:

Press	***Display***
14.8	14.8
×	14.8
16.0	16.0
=	236.8
÷	236.8
1.50	1.50
=	157.86667

Now we can keep this last number on the calculator and add it to 107.28 from the first calculation.

Press	***Display***
+	157.86667
107.28	107.28
=	265.14667
Rounded:	265

To summarize,

$$(0.360)(298) + \frac{(14.8)(16.0)}{1.50}$$

becomes

$$107.28 + 157.86667$$

and the sum is 265.14667 or, rounded to the correct number of significant figures, 265. There are other ways to do this calculation, but this is the safest way (assuming you are careful).

A common type of chain calculation involves a number of terms multiplied together in the numerator and the denominator, as in

$$\frac{(323)(.0821)(1.46)}{(4.05)(76)}$$

There are many possible sequences by which this calculation can be carried out, but the following seems the most natural.

> 323 times .0821 equals; times 1.46 equals;
> divided by 4.05 equals; divided by 76 equals

This sequence is summarized as follows:

Press	***Display***
323	323
×	323
.0821	0.0821
=	26.5183
×	26.5183
1.46	1.46
=	38.716718
÷	38.716718
4.05	4.05
=	9.5596835
÷	9.5596835
76	76
=	0.1257853

The answer is 0.1257853, which, when rounded to the correct number of significant figures, is 0.13. Note that when two or more numbers are multiplied in the denominator, you must divide by *each* one.

Here are some additional chain calculations (with solutions) to give you more practice.

a. $15 - (0.750)(243)$

b. $\dfrac{(13.1)(43.5)}{(1.8)(63)}$

c. $\dfrac{(85.8)(0.142)}{(16.46)(18.0)} + \dfrac{(131)(0.0156)}{10.17}$

d. $(18.1)(0.051) - \dfrac{(325)(1.87)}{(14.0)(3.81)} + \dfrac{1.56 - 0.43}{1.33}$

SOLUTIONS

a. $15 - 182 = -167$

b. 5.0

c. $0.0411 + 0.201 = 0.242$

d. $0.92 - 11.4 + 0.850 = -9.6$

In performing chain calculations, take the following steps in the order listed.

1. Perform any additions and subtractions that appear inside parentheses.
2. Complete the multiplications and divisions of individual terms.
3. Add and subtract individual terms as required.

Basic Algebra

In solving chemistry problems you will use, over and over again, relatively few mathematical procedures. In this section we review the few algebraic manipulations that you will need.

Solving an Equation

In the course of solving a chemistry problem, we often construct an algebraic equation that includes the unknown quantity (the thing we want to calculate). An example is

$$(1.5)V = (0.23)(0.08206)(298)$$

We need to "solve this equation for V." That is, we need to isolate V on one side of the equals sign with all the numbers on the other side. How can we do this? The key idea in solving an algebraic equation is that *doing the same thing on both sides of the equals sign* does not change the equality. That is, it is always "legal" to do the same thing to both sides of the equation. Here we want to solve for V, so we must get the number 1.5 on the other side of the equals sign. We can do this by dividing *both sides* by 1.5.

$$\frac{(1.5)V}{1.5} = \frac{(0.23)(0.08206)(298)}{1.5}$$

Now the 1.5 in the denominator on the left cancels the 1.5 in the numerator:

$$\frac{(\cancel{1.5})V}{\cancel{1.5}} = \frac{(0.23)(0.08206)(298)}{1.5}$$

to give

$$V = \frac{(0.23)(0.08206)(298)}{1.5}$$

Using the procedures in "Using Your Calculator" for chain calculations, we can now obtain the value for V with a calculator.

$$V = 3.7$$

Sometimes it is necessary to solve an equation that consists of symbols. For example, consider the equation

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

Let's assume we want to solve for T_2. That is, we want to isolate T_2 on one side of the equation. There are several possible ways to proceed, keeping in mind that we always do the same thing on both sides of the equals sign. First we multiply both sides by T_2.

$$T_2 \times \frac{P_1V_1}{T_1} = \frac{P_2V_2}{\cancel{T_2}} \times \cancel{T_2}$$

This cancels T_2 on the right. Next we multiply both sides by T_1.

$$T_2 \times \frac{P_1V_1}{\cancel{T_1}} \times \cancel{T_1} = P_2V_2T_1$$

This cancels T_1 on the left. Now we divide both sides by P_1V_1.

$$T_2 \times \frac{\cancel{P_1V_1}}{\cancel{P_1V_1}} = \frac{P_2V_2T_1}{P_1V_1}$$

This yields the desired equation,

$$T_2 = \frac{P_2V_2T_1}{P_1V_1}$$

For practice, solve each of the following equations for the variable indicated.

a. $PV = k$; solve for P

b. $1.5x + 6 = 3$; solve for x

c. $PV = nRT$; solve for n

d. $\dfrac{P_1V_1}{T_1} = \dfrac{P_2V_2}{T_2}$; solve for V_2

e. $\dfrac{°F - 32}{°C} = \dfrac{9}{5}$; solve for °C

f. $\dfrac{°F - 32}{°C} = \dfrac{9}{5}$; solve for °F

SOLUTIONS

a. $\frac{PV}{V} = \frac{k}{V}$

$P = \frac{k}{V}$

b. $1.5x + 6 - 6 = 3 - 6$

$1.5x = -3$

$\frac{1.5x}{1.5} = \frac{-3}{1.5}$

$x = -\frac{3}{1.5} = -2$

c. $\frac{PV}{RT} = \frac{nRT}{RT}$

$\frac{PV}{RT} = n$

d. $\frac{P_1V_1}{T_1} \times T_2 = \frac{P_2V_2}{T_2} \times T_2$

$\frac{P_1V_1T_2}{T_1P_2} = \frac{P_2V_2}{P_2}$

$\frac{P_1V_1T_2}{T_1P_2} = V_2$

e. $\frac{°F - 32}{°C} \times °C = \frac{9}{5}°C$

$\frac{5}{9}(°F - 32) = \frac{5}{9} \times \frac{9}{5}°C$

$\frac{5}{9}(°F - 32) = °C$

f. $\frac{°F - 32}{°C} \times °C = \frac{9}{5}°C$

$°F - 32 + 32 = \frac{9}{5}°C + 32$

$°F = \frac{9}{5}°C + 32$

Scientific (Exponential) Notation

The numbers we must work with in scientific measurements are often very large or very small; thus it is convenient to express them using powers of 10. For example, the number 1,300,000 can be expressed as 1.3×10^6, which means multiply 1.3 by 10 six times, or

$$1.3 \times 10^6 = 1.3 \times \underbrace{10 \times 10 \times 10 \times 10 \times 10 \times 10}_{10^6 = 1 \text{ million}}$$

A number written in scientific notation always has the form:

A number (between 1 and 10) times
the appropriate power of 10

To represent a large number such as 20,500 in scientific notation, we must move the decimal point in such a way as to achieve a number between 1 and 10 and then multiply the result by a power of 10 to compensate for moving the decimal point. In this case, we must move the decimal point four places to the left.

2 0 5 0 0
4 3 2 1

to give a number between 1 and 10:

2.05

where we retain only the significant figures (the number 20,500 has three significant figures). To compensate for moving the decimal point four places to the left, we must multiply by 10^4. Thus

$$20{,}500 = 2.05 \times 10^4$$

As another example, the number 1985 can be expressed as 1.985×10^3. To end up with the number 1.985, which is between 1 and 10, we had to move the decimal point three places to the left. To compensate for that, we must multiply by 10^3. Some other examples are given in the accompanying list.

Number	***Exponential Notation***
5.6	5.6×10^0 or 5.6×1
39	3.9×10^1
943	9.43×10^2
1126	1.126×10^3

So far, we have considered numbers greater than 1. How do we represent a number such as 0.0034 in exponential notation? First, to achieve a number between 1 and 10, we start with 0.0034 and move the decimal point three places to the right.

0 . 0 0 3 4
1 2 3

This yields 3.4. Then, to compensate for moving the decimal point to the right, we must multiply by a power of 10 with a negative exponent—in this case, 10^{-3}. Thus

$$0.0034 = 3.4 \times 10^{-3}$$

In a similar way, the number 0.00000014 can be written as 1.4×10^{-7}, because going from 0.00000014 to 1.4 requires that we move the decimal point seven places to the right.

Mathematical Operations with Exponentials

We next consider how various mathematical operations are performed using exponential numbers. First we cover the various rules for these operations; then we consider how to perform them on your calculator.

Multiplication and Division

When two numbers expressed in exponential notation are multiplied, the initial numbers are multiplied and the exponents of 10 are *added.*

$$(M \times 10^m)(N \times 10^n) = (MN) \times 10^{m+n}$$

For example (to two significant figures, as required),

$$(3.2 \times 10^4)(2.8 \times 10^3) = 9.0 \times 10^7$$

When the numbers are multiplied, if a result greater than 10 is obtained for the initial number, the decimal point is moved one place to the left and the exponent of 10 is increased by 1.

$$\begin{aligned}(5.8 \times 10^2)(4.3 \times 10^8) &= 24.9 \times 10^{10}\\ &= 2.49 \times 10^{11}\\ &= 2.5 \times 10^{11} \text{ (two significant figures)}\end{aligned}$$

Division of two numbers expressed in exponential notation involves normal division of the initial numbers and *subtraction* of the exponent of the divisor from that of the dividend. For example,

$$\frac{4.8 \times 10^8}{\underbrace{2.1 \times 10^3}_{\text{Divisor}}} = \frac{4.8}{2.1} \times 10^{(8-3)} = 2.3 \times 10^5$$

If the initial number resulting from the division is less than 1, the decimal point is moved one place to the right and the exponent of 10 is decreased by 1. For example,

$$\begin{aligned}\frac{6.4 \times 10^3}{8.3 \times 10^5} = \frac{6.4}{8.3} \times 10^{(3-5)} &= 0.77 \times 10^{-2}\\ &= 7.7 \times 10^{-3}\end{aligned}$$

Addition and Subtraction

In order for us to add or subtract numbers expressed in exponential notation, *the exponents of the numbers must be the same*. For example, to add 1.31×10^5 and 4.2×10^4, we must rewrite one number so that the exponents of both are the same. The number 1.31×10^5 can be written 13.1×10^4: decreasing the exponent by 1 compensates for moving the decimal point one place to the right. Now we can add the numbers.

$$\begin{array}{r} 13.1 \times 10^4 \\ +\ \ 4.2 \times 10^4 \\ \hline 17.3 \times 10^4 \end{array}$$

In correct exponential notation, the result is expressed as 1.73×10^5.

To perform addition or subtraction with numbers expressed in exponential notation, we add or subtract only the initial numbers. The exponent of the result is the same as the exponents of the numbers being added or subtracted. To subtract 1.8×10^2 from 8.99×10^3, we first convert 1.8×10^2 to 0.18×10^3 so that both numbers have the same exponent. Then we subtract.

$$\begin{array}{r} 8.99 \times 10^3 \\ -0.18 \times 10^3 \\ \hline 8.81 \times 10^3 \end{array}$$

Powers and Roots

When a number expressed in exponential notation is taken to some power, the initial number is taken to the appropriate power and the exponent of 10 is *multiplied* by that power.

$$(N \times 10^n)^m = N^m \times 10^{m \times n}$$

For example,

$$\begin{aligned}(7.5 \times 10^2)^2 &= (7.5)^2 \times 10^{2\times2}\\ &= 56. \times 10^4\\ &= 5.6 \times 10^5\end{aligned}$$

When a root is taken of a number expressed in exponential notation, the root of the initial number is taken and the exponent of 10 is divided by the number representing the root. For example, we take the square root of a number as follows:

$$\sqrt{N \times 10^n} = (N \times 10^n)^{1/2} = \sqrt{N} \times 10^{n/2}$$

For example,

$$\begin{aligned}(2.9 \times 10^6)^{1/2} &= \sqrt{2.9} \times 10^{6/2}\\ &= 1.7 \times 10^3\end{aligned}$$

Using a Calculator to Perform Mathematical Operations on Exponentials

In dealing with exponential numbers, you must first learn to enter them into your calculator. First the number is keyed in and then the exponent. There is a special key that must be pressed just before the exponent is entered. This key is often labeled [EE] or [exp]. For example, the number 1.56×10^6 is entered as follows:

Press	***Display***
1.56	1.56
EE or exp	1.56 00
6	1.56 06

To enter a number with a negative exponent, use the change-of-sign key [+/−] after entering the exponent number. For example, the number 7.54×10^{-3} is entered as follows:

Press	***Display***
7.54	7.54
EE or exp	7.54 00
3	7.54 03
+/−	7.54 −03

Once a number with an exponent is entered into your calculator, the mathematical operations are performed exactly the same as with a "regular" number. For example, the numbers 1.0×10^3 and 1.0×10^2 are multiplied as follows:

Press	***Display***
1.0	1.0
EE or exp	1.0 00
3	1.0 03
×	1 03
1.0	1.0
EE or exp	1.0 00
2	1.0 02
=	1 05

The answer is correctly represented as 1.0×10^5.

The numbers 1.50×10^5 and 1.1×10^4 are added as follows:

Press	***Display***	
1.5	1.50	
EE or exp	1.50	00
5	1.50	05
+	1.5	05
1.1	1.1	
EE or exp	1.1	00
4	1.1	04
=	1.61	05

The answer is correctly represented as 1.61×10^5. Note that when exponential numbers are added, the calculator automatically takes into account any difference in exponents.

To take the power, root, or reciprocal of an exponential number, enter the number first, then press the appropriate key or keys. For example, the square root of 5.6×10^3 is obtained as follows:

Press	***Display***	
5.6	5.6	
EE or exp	5.6	00
3	5.6	03
$\sqrt{X}$	7.4833148	01

The answer is correctly represented as 7.5×10^1.

Practice by performing the following operations that involve exponential numbers. The answers follow the exercises.

a. $7.9 \times 10^2 \times 4.3 \times 10^4$

b. $\dfrac{5.4 \times 10^3}{4.6 \times 10^5}$

c. $1.7 \times 10^2 + 1.63 \times 10^3$

d. $4.3 \times 10^{-3} + 1 \times 10^{-4}$

e. $(8.6 \times 10^{-6})^2$

f. $\dfrac{1}{8.3 \times 10^2}$

g. $\log(1.0 \times 10^{-7})$

h. $-\log(1.3 \times 10^{-5})$

i. $\sqrt{6.7 \times 10^9}$

SOLUTIONS

a. 3.4×10^7

b. 1.2×10^{-2}

c. 1.80×10^3

d. 4.4×10^{-3}

e. 7.4×10^{-11}

f. 1.2×10^{-3}

g. -7.00

h. 4.89

i. 8.2×10^4

Graphing Functions

In interpreting the results of a scientific experiment, it is often useful to make a graph. If possible, the function to be graphed should be in a form that gives a straight line. The equation for a straight line (a *linear equation*) can be represented in the general form

$$y = mx + b$$

where y is the *dependent variable,* x is the *independent variable,* m is the *slope,* and b is the *intercept* with the y axis.

To illustrate the characteristics of a linear equation, the function $y = 3x + 4$ is plotted in Figure A.1. For this equation $m = 3$ and $b = 4$. Note that the y intercept occurs when $x = 0$. In this case the y intercept is 4, as can be seen from the equation ($b = 4$).

The slope of a straight line is defined as the ratio of the rate of change in y to that in x:

$$m = \text{slope} = \frac{\Delta y}{\Delta x}$$

For the equation $y = 3x + 4$, y changes three times as fast as x (because x has a coefficient of 3). Thus the slope in this case is 3. This can be verified from the graph. For the triangle shown in Figure A.1,

$$\Delta y = 15 - 16 = 36 \qquad \text{and} \qquad \Delta x = 15 - 3 = 12$$

Thus

$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{36}{12} = 3$$

This example illustrates a general method for obtaining the slope of a line from the graph of that line. Simply draw a triangle with one side parallel to the y axis and the other side parallel to the x axis, as shown in Figure A.1. Then determine the lengths of the sides to get Δy and Δx, respectively, and compute the ratio $\Delta y/\Delta x$.

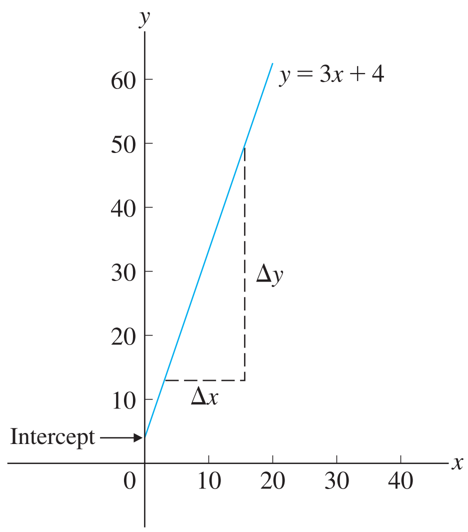

Figure A.1
Graph of the linear equation $y = 3x + 4$.

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SI Units and Conversion Factors

These conversion factors are given with more significant figures than those typically used in the body of the text.

Length

SI Unit: Meter (m)

1 meter	= 1.0936 yards
1 centimeter	= 0.39370 inch
1 inch	= 2.54 centimeters (exactly)
1 kilometer	= 0.62137 mile
1 mile	= 5280. feet
	= 1.6093 kilometers

Volume

SI Unit: Cubic Meter (m^3)

1 liter	= 10^{-3} m^3
	= 1 dm^3
	= 1.0567 quarts
1 gallon	= 4 quarts
	= 8 pints
	= 3.7854 liters
1 quart	= 32 fluid ounces
	= 0.94635 liter

Mass

SI Unit: Kilogram (kg)

1 kilogram	= 1000 grams
	= 2.2046 pounds
1 pound	= 453.59 grams
	= 0.45359 kilogram
	= 16 ounces
1 atomic mass unit	= 1.66057×10^{-27} kilograms

Pressure

SI Unit: Pascal (Pa)

1 atmosphere	= 101.325 kilopascals
	= 760. torr (mm Hg)
	= 14.70 pounds per square inch

Energy

SI Unit: Joule (J)

1 joule	= 0.23901 calorie
1 calorie	= 4.184 joules