

## Chapter 7

# Quantum Theory of the Atom

### Concept Check 7.1

Laser light of a specific frequency falls on a crystal that converts this light into one with double the original frequency. How is the wavelength of this frequency-doubled light related to the wavelength of the original laser light? Suppose the original laser light was red. What region of the spectrum would the frequency-doubled light be in? (If this is the visible region, what color is the light?)

#### Solution

The frequency and wavelength of light are inversely related. Therefore, if the frequency is doubled, the wavelength is halved. Red light has a wavelength around 700 nm, so doubling its frequency halves its wavelength to about 350 nm, which is in the ultraviolet, just beyond the visible spectrum.

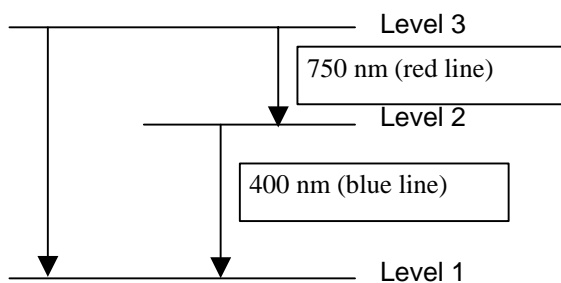
### Concept Check 7.2

An atom has a line spectrum consisting of a red line and a blue line. Assume that each line corresponds to a transition between two adjacent energy levels. Sketch an energy-level diagram with three energy levels that might explain this line spectrum, indicating the transitions on this diagram. Consider the transition from the highest energy level on this diagram to the lowest energy level. How would you describe the color or region of the spectrum corresponding to this transition?

#### Solution

Since the transitions are between adjacent levels, the energy-level diagram must look something like the following diagram, with the red transition between two close levels and

the blue transition between two levels more widely spaced. (The three levels could be spaced so that the red and blue transitions are interchanged, with the blue transition above the red one.)



The transition from the top level to the lowest level would correspond to a transition that is greater in energy change than either of the other two transitions. Thus, the three transitions, from lowest to highest energy change, are in the order: red, blue, and the transition from the highest to lowest level. This last transition would have the highest frequency and therefore the shortest wavelength. It would lie just beyond the blue portion of the visible spectrum, in the ultraviolet region.

### Concept Check 7.3

A proton is approximately 2000 times heavier than an electron. How would the speeds of these particles compare if their corresponding wavelengths were to be about equal?

#### Solution

The de Broglie relation says that the wavelength of a particle is inversely proportional to both mass and speed. So, to maintain the wavelength constant while the mass increases would mean that the speed would have to decrease. In going from a particle with the mass of an electron to that of a proton, the speed would have to decrease by a factor of about 2000 in order to maintain the same wavelength. The proton would have to have a speed approximately 2000 times slower than an electron of the same wavelength.

### Conceptual Problem 7.19

Consider two beams of the same yellow light. Imagine that one beam has its wavelength doubled; the other has its frequency doubled. Which of these two beams is then in the ultraviolet region?

**Solution**

Wavelength and frequency are inversely related. Moreover, ultraviolet light is at higher frequency than yellow light. Doubling the frequency of a beam of light would give that beam a higher frequency than yellow light, whereas doubling the wavelength would give that beam a lower frequency than yellow light. Consequently, the beam with frequency doubled must be the one in the ultraviolet region.

Here is another way to look at the problem. Energy is directly related to the frequency and inversely related to the wavelength. Thus, the beam whose frequency is doubled will increase in energy, whereas the beam whose wavelength is doubled will decrease in energy. Since yellow light is in the visible region of the spectrum, which is lower in energy than the ultraviolet region (Figure 7.5), the beam whose frequency is doubled will be higher in energy and thus in the UV region of the spectrum.

**Conceptual Problem 7.20**

Some infrared radiation has a wavelength that is 1000 times larger than that of a certain visible light. This visible light has a frequency that is 1000 times smaller than that of some X radiation. How many times more energy is there in a photon of this X radiation than there is in a photon of the infrared radiation?

**Solution**

Frequency is inversely related to the wavelength. Thus, the infrared radiation with a wavelength that is one thousand times larger than the visible light would have a frequency one thousand times smaller than the visible light. But, the visible light has a frequency one thousand times smaller than that of the X radiation. This makes the frequency of the X radiation one million times larger than the frequency of the infrared radiation. Therefore, since energy is directly related to the frequency, the energy of the X radiation would be one million times as large as the energy of the infrared radiation.

**Conceptual Problem 7.21**

One photon of green light has less than twice the energy of two photons of red light. Consider two hypothetical experiments. In one experiment, potassium metal is exposed to one photon of green light; in another experiment, potassium metal is exposed to two photons of red light. In one of these experiments, no electrons are ejected by the photoelectric effect (no matter how many times this experiment is repeated), whereas in the other experiment at least one electron is ejected. What is the maximum number of electrons that could be ejected during this experiment, one or two?

**Solution**

That one color of light does not result in an ejection of electrons implies that that color has too little energy per photon. Of the two colors, red and green, red light has less energy per photon. So you expect that the experiment with red light results in no ejection of photons, whereas the experiment with green light must be the one that ejects electrons. (Two red photons have more than enough energy to eject an electron, but this energy needs to be concentrated in only one photon to be effective.) In the photoelectric effect, one photon of light ejects at most one electron. Therefore, in the experiment with green light, one electron is ejected.

**Conceptual Problem 7.22**

An atom in its ground state absorbs a photon (photon 1), then quickly emits another photon (photon 2). One of these photons corresponds to ultraviolet radiation, whereas the other one corresponds to red light. Explain what is happening. Which electromagnetic radiation, ultraviolet or red light, is associated with the emitted photon (photon 2)?

**Solution**

Ultraviolet radiation is higher in energy than red light (Figure 7.5). Since an atom that started in the ground state cannot emit more energy than it absorbed, the absorbed photon must be higher in energy than the emitted photon. This makes the emitted photon (photon 2) the red light.

**Conceptual Problem 7.23**

Three emission lines involving three energy levels in the magnesium atom occur at wavelengths  $x$ ,  $1.5x$ , and  $3.0x$  nm. Which wavelength corresponds to the transition from the highest to the lowest of these three energy levels?

**Solution**

Energy is inversely proportional to the wavelength of the radiation. The transition from the highest to the lowest energy levels would involve the greatest energy change and thus the shortest wavelength,  $x$  nm.

**Conceptual Problem 7.24**

An atom emits yellow light when an electron makes the transition from the  $n = 5$  to the  $n = 1$  level. In separate experiments, suppose you bombarded the  $n = 1$  level of this atom with red light, yellow light (obtained from the previous emission), and blue light. In which experiment or experiments would the electron be promoted to the  $n = 5$  level?

### Solution

In a transition from the  $n = 1$  to the  $n = 5$  energy level, an atom will absorb a photon with the same energy as the photon that was emitted in the transition from the  $n = 5$  to the  $n = 1$  energy level. Since yellow light was emitted, the experiment using yellow light will promote the electron to the  $n = 5$  level.

### Conceptual Problem 7.25

Which of the following particles has the longest wavelength?

- an electron traveling at  $x$  m/s
- a proton traveling at  $x$  m/s
- a proton traveling at  $2x$  m/s

### Solution

A proton is approximately 2000 times the weight of an electron. Also, from the de Broglie relation,  $\lambda = h/mv$ , you see that the wavelength is inversely proportional to both the mass and the speed of the particle. Considering the protons in parts b. and c., since the mass is the same in both parts, the proton with the smaller speed, part b., will have a longer wavelength. Now, comparing the electron in part a. with the proton in part b., since both have the same speed, the electron in part a. with the smaller mass will have the longer wavelength. Therefore, the electron in part a. will have the longest wavelength.

### Conceptual Problem 7.26

Imagine a world in which the rule for the  $l$  quantum number is that values start with 1 and go up to  $n$ . The rules for the  $n$  and  $m_l$  quantum numbers are unchanged from those of our world. Write the quantum numbers for the first two shells (i.e.,  $n = 1$  and  $n = 2$ ).

### Solution

For the first shell, the quantum numbers would have the following allowable values:

$$n = 1; l = 1; m_l = 0, +1, -1.$$

For the second shell, the quantum numbers would have the following allowable values:

$$n = 2; l = 1; m_l = 0, +1, -1$$

$$l = 2; m_l = 0, +1, -1, +2, -2$$

