Appendixes

Appendix A

Present and Future Value Tables

Many problems in personal finance involve decisions about money values at varying points in time. These values can be directly and fairly compared only when they are adjusted to a common point in time. Chapter 1 introduced the basic time value concepts. This appendix offers more details about the time value of money. In addition, it provides tables listing the future and present value of $1 with which to make calculations.

Four assumptions must be made to eliminate unnecessary complications:

1. Each planning period is one year long.
2. Only annual interest rates are considered.
3. Interest rates are the same during each of the annual periods.
4. Interest is compounded and continues earning a return in subsequent periods.

Tables of present and future values can be constructed to make these adjustments. **Future values** are derived from the principles of compounding the dollar values ahead in time. **Present values** are derived by discounting (which is the inverse of compounding) the dollar values and transferring them to an earlier point in time.

It is usually unnecessary to precisely identify whether the interest is paid/received at the *beginning* of a period or at the *end* of a period, or to know whether interest compounds daily or quarterly instead of annually. (These calculations require even more tables.) The following present and future value tables assume that money is accumulated, received, paid, compounded, or whatever at the *end* of a period. The tables can be used to compute the mathematics of personal finance with high certainty and to confirm (or reject as inaccurate) what people tell you about financial matters.

The most significant task is to find the correct table. Accordingly, each table is clearly described here, and illustrations of its use appear on the facing page where possible. In addition, the appropriate mathematical equation is shown and can be easily solved using a calculator.

Illustrations Using Table A.1: Future Value of a Single Amount ($1)

To use Table A.1 on page A-5, locate the future value factor for the time period and the interest rate.

1. You invest $500 at a 15 percent rate of return for 12 years. How much will you have at the end of that 12-year period?

   The future value factor is 5.350; hence, the solution is $500 × 5.350, or $2675.

2. Property values in your neighborhood are increasing at a rate of 5 percent per year. If your home is presently worth $90,000, what will its worth be in 7 years?

   The future value factor is 1.407; hence, the solution is $90,000 × 1.407, or $126,630.

3. You need to amass $40,000 in the next 10 years to make a balloon payment on your home mortgage. You have $17,000 available to invest. What annual interest rate must be earned to realize the $40,000?

   $40,000 ÷ $17,000 = 2.353. Read down the periods (*n*) column to 10 years and across to 2.367 (close enough), which is found under the 9 percent column. Hence, the $17,000 invested at 9 percent for 10 years will grow to a future value of slightly more than $40,000.

4. An apartment building is currently valued at $160,000, and it has been appreciating at 8 percent per year. If this rate continues, in how many years will it be worth $300,000?

   $300,000 ÷ $160,000 = 1.875. Read down the 8 percent column until you reach 1.851 (close enough to 1.875). This number corresponds to a period of 8 years. Hence, the $160,000 property appreciating at 8 percent annually will grow to a future value of $300,000 in slightly more than 8 years.

5. You have the choice of receiving a down payment from someone who wants to purchase your rental property as $15,000 today or as a personal note for $25,000 payable in 6 years. If you could expect to earn 8 percent on such funds, which is the better choice?

   The future value factor is 1.587; hence, the future value of $15,000 at 8 percent is $15,000 × 1.587, or $23,805. Thus, it would be better to take the note for $25,000.

6. How much will an automobile now priced at $20,000 cost in 4 years, assuming an annual inflation rate of 5 percent?

   Read down the 5 percent column and across the row for 4 years to locate the future value factor of 1.216. Hence, the solution is $20,000 × 1.216, or $24,320.

7. How large a lump-sum investment do you need now to have $20,000 available in 5 years, assuming a 10 percent annual rate of return?

   The $20,000 future value is divided by 1.611 (10 percent at 5 years), resulting in a current lump-sum investment of $12,415.

8. You have $5000 now and need $10,000 in 9 years. What rate of return is needed to reach that goal?

   Divide the future value of $10,000 by the present value of the lump sum of $5000 to obtain a future value factor of 2.0. In the row for 9 years, locate the future value factor of 1.999 (very close to 2.0). Read up the column to find that an 8 percent return on investment is needed.

9. How many years will it take your lump-sum investment of $10,000 to grow to $16,000, given an annual rate of return of 7 percent?

Divide the future value of $16,000 by the present value of the $10,000 lump sum to compute a future value factor of 1.6; look down the 7 percent column to find 1.606 (close enough). Read across the row to find that an investment period of 7 years is needed.

An alternative approach is to use a calculator to determine the future value, *FV,* of a sum of money invested today, assuming that the amount remains in the investment for a specified number of time periods (usually years) and that it earns a certain rate of return each period. The equation is

$$FV = PV\,(1.0 + i)^n \qquad \textbf{(A.1)}$$

where

FV = *Future Value*
PV = *Present Value* of the investment
i = *Interest* rate per period
n = *Number* of periods the PV is invested

Table A.1 Future Value of a Single Amount ($1 at the End of *n* Periods)

(Used to Compute the Compounded Future Value of a Known Lump Sum)

n	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%
1	1.0100	1.0200	1.0300	1.0400	1.0500	1.0600	1.0700	1.0800	1.0900	1.1000	1.1100	1.1200	1.1300	1.1400	1.1500	1.1600	1.1700	1.1800	1.1900	1.2000
2	1.0201	1.0404	1.0609	1.0816	1.1025	1.1236	1.1449	1.1664	1.1881	1.2100	1.2321	1.2544	1.2769	1.2996	1.3225	1.3456	1.3689	1.3924	1.4161	1.4400
3	1.0303	1.0612	1.0927	1.1249	1.1576	1.1910	1.2250	1.2597	1.2950	1.3310	1.3676	1.4049	1.4429	1.4815	1.5209	1.5609	1.6016	1.6430	1.6852	1.7280
4	1.0406	1.0824	1.1255	1.1699	1.2155	1.2625	1.3108	1.3605	1.4116	1.4641	1.5181	1.5735	1.6305	1.6890	1.7490	1.8106	1.8739	1.9388	2.0053	2.0736
5	1.0510	1.1041	1.1593	1.2167	1.2763	1.3382	1.4026	1.4693	1.5386	1.6105	1.6851	1.7623	1.8424	1.9254	2.0114	2.1003	2.1924	2.2878	2.3864	2.4883
6	1.0615	1.1262	1.1941	1.2653	1.3401	1.4185	1.5007	1.5869	1.6771	1.7716	1.8704	1.9738	2.0820	2.1950	2.3131	2.4364	2.5652	2.6996	2.8398	2.9860
7	1.0721	1.1487	1.2299	1.3159	1.4071	1.5036	1.6058	1.7138	1.8280	1.9487	2.0762	2.2107	2.3526	2.5023	2.6600	2.8262	3.0012	3.1855	3.3793	3.5832
8	1.0829	1.1717	1.2668	1.3686	1.4775	1.5938	1.7182	1.8509	1.9926	2.1436	2.3045	2.4760	2.6584	2.8526	3.0590	3.2784	3.5115	3.7589	4.0214	4.2998
9	1.0937	1.1951	1.3048	1.4233	1.5513	1.6895	1.8385	1.9990	2.1719	2.3579	2.5580	2.7731	3.0040	3.2519	3.5179	3.8030	4.1084	4.4355	4.7854	5.1598
10	1.1046	1.2190	1.3439	1.4802	1.6289	1.7908	1.9672	2.1589	2.3674	2.5937	2.8394	3.1058	3.3946	3.7072	4.0456	4.4114	4.8068	5.2338	5.6947	6.1917
11	1.1157	1.2434	1.3842	1.5395	1.7103	1.8983	2.1049	2.3316	2.5804	2.8531	3.1518	3.4785	3.8359	4.2262	4.6524	5.1173	5.6240	6.1759	6.7767	7.4301
12	1.1268	1.2682	1.4258	1.6010	1.7959	2.0122	2.2522	2.5182	2.8127	3.1384	3.4985	3.8960	4.3345	4.8179	5.3503	5.9360	6.5801	7.2876	8.0642	8.9161
13	1.1381	1.2936	1.4685	1.6651	1.8856	2.1329	2.4098	2.7196	3.0658	3.4523	3.8833	4.3635	4.8980	5.4924	6.1528	6.8858	7.6987	8.5994	9.5964	10.6993
14	1.1495	1.3195	1.5126	1.7317	1.9799	2.2609	2.5785	2.9372	3.3417	3.7975	4.3104	4.8871	5.5348	6.2613	7.0757	7.9875	9.0075	10.1472	11.4198	12.8392
15	1.1610	1.3459	1.5580	1.8009	2.0789	2.3966	2.7590	3.1722	3.6425	4.1772	4.7846	5.4736	6.2543	7.1379	8.1371	9.2655	10.5387	11.9737	13.5895	15.4070
16	1.1726	1.3728	1.6047	1.8730	2.1829	2.5404	2.9522	3.4259	3.9703	4.5950	5.3109	6.1304	7.0673	8.1372	9.3576	10.7480	12.3303	14.1290	16.1715	18.4884
17	1.1843	1.4002	1.6528	1.9479	2.2920	2.6928	3.1588	3.7000	4.3276	5.0545	5.8951	6.8660	7.9861	9.2765	10.7613	12.4677	14.4265	16.6722	19.2441	22.1861
18	1.1961	1.4282	1.7024	2.0258	2.4066	2.8543	3.3799	3.9960	4.7171	5.5599	6.5436	7.6900	9.0243	10.5752	12.3755	14.4625	16.8790	19.6733	22.9005	26.6233
19	1.2081	1.4568	1.7535	2.1068	2.5270	3.0256	3.6165	4.3157	5.1417	6.1159	7.2633	8.6128	10.1974	12.0557	14.2318	16.7765	19.7484	23.2144	27.2516	31.9480
20	1.2202	1.4859	1.8061	2.1911	2.6533	3.2071	3.8697	4.6610	5.6044	6.7275	8.0623	9.6463	11.5231	13.7435	16.3665	19.4608	23.1056	27.3930	32.4294	38.3376
21	1.2324	1.5157	1.8603	2.2788	2.7860	3.3996	4.1406	5.0338	6.1088	7.4002	8.9492	10.8038	13.0211	15.6676	18.8215	22.5745	27.0336	32.3238	38.5910	46.0051
22	1.2447	1.5460	1.9161	2.3699	2.9253	3.6035	4.4304	5.4365	6.6586	8.1403	9.9336	12.1003	14.7138	17.8610	21.6447	26.1864	31.6293	38.1421	45.9233	55.2061
23	1.2572	1.5769	1.9736	2.4647	3.0715	3.8197	4.7405	5.8715	7.2579	8.9543	11.0263	13.5523	16.6266	20.3616	24.8915	30.3762	37.0062	45.0076	54.6487	66.2474
24	1.2697	1.6084	2.0328	2.5633	3.2251	4.0489	5.0724	6.3412	7.9111	9.8497	12.2392	15.1786	18.7881	23.2122	28.6252	35.2364	43.2973	53.1090	65.0320	79.4968
25	1.2824	1.6406	2.0938	2.6658	3.3864	4.2919	5.4274	6.8485	8.6231	10.8347	13.5855	17.0001	21.2305	26.4619	32.9190	40.8742	50.6578	62.6686	77.3881	95.3962
26	1.2953	1.6734	2.1566	2.7725	3.5557	4.5494	5.8074	7.3964	9.3992	11.9182	15.0799	19.0401	23.9905	30.1666	37.8568	47.4141	59.2697	73.9490	92.0918	114.4755
27	1.3082	1.7069	2.2213	2.8834	3.7335	4.8223	6.2139	7.9881	10.2451	13.1100	16.7386	21.3249	27.1093	34.3899	43.5353	55.0004	69.3455	87.2598	109.5893	137.3706
28	1.3213	1.7410	2.2879	2.9987	3.9201	5.1117	6.6488	8.6271	11.1671	14.4210	18.5799	23.8839	30.6335	39.2045	50.0656	63.8004	81.1342	102.9666	130.4112	164.8447
29	1.3345	1.7758	2.3566	3.1187	4.1161	5.4184	7.1143	9.3173	12.1722	15.8631	20.6237	26.7499	34.6158	44.6931	57.5755	74.0085	94.9271	121.5005	155.1893	197.8136
30	1.3478	1.8114	2.4273	3.2434	4.3219	5.7435	7.6123	10.0627	13.2677	17.4494	22.8923	29.9599	39.1159	50.9502	66.2118	85.8499	111.0647	143.3706	184.6753	237.3763
40	1.4889	2.2080	3.2620	4.8010	7.0400	10.2857	14.9745	21.7245	31.4094	45.2593	65.0009	93.0510	132.7816	188.8835	267.8635	378.7212	533.8687	750.3783	1051.668	1469.772
50	1.6446	2.6916	4.3839	7.1067	11.4674	18.4202	29.4570	46.9016	74.3575	117.3909	184.5648	289.0022	450.7359	700.2330	1083.657	1670.704	2566.215	3927.357	5988.914	9100.438

Illustrations Using Table A.2: Present Value of a Single Amount ($1)

To use this table, locate the present value factor for the time period and the interest rate.

1. You want to begin a college fund for your newborn child; you hope to accumulate $30,000 by 18 years from now. If a current investment opportunity yields 7 percent, how much must you invest in a lump sum to realize the $30,000 when needed?

   The present value factor is 0.296; hence, the solution is $30,000 × 0.296, or $8880.

2. You hope to retire in 25 years and want to deposit a single lump sum that will grow to $250,000 at that time. If you can now invest at 8 percent, how much must you invest to realize the $250,000 when needed?

   The present value factor is 0.146; hence, the solution is $250,000 × 0.146, or $36,500. The present value of $250,000 received 25 years from now is $36,500 if the interest rate is 8 percent.

3. You have the choice of receiving a down payment from someone who wants to purchase your rental property as $15,000 today or as a personal note for $25,000 payable in 6 years. If you could expect to earn 8 percent on such funds, which is the better choice?

   The present value factor is 0.630; hence, the solution is $25,000 × 0.630, or $15,750. Thus, the present value of $25,000 received in 6 years is greater than $15,000 received now, and the personal note is the better choice.

4. You own a $1000 bond paying 8 percent annually until its maturity in 5 years. You need to sell the bond now, even though the market rate of interest on similar bonds has increased to 10 percent. What discounted market price for the bond will allow the new buyer to earn a yield of 10 percent?

   First, compute the present value of the future interest payments of $80 per year for 5 years at 10 percent (using Table A.4): $80 × 3.791, or $303.28. Second, compute the present value of the future principal repayment of $1000 after 5 years at 10 percent: $1000 × 0.621, or $621.00. Hence, the market price is the sum of the two present values ($303.28 + $621.00), or $924.28.

An alternative approach is to use a calculator to determine the present value, *PV*, of a single payment received some time in the future. The equation, which is a rearrangement of the future value Equation (A.1), is

$$PV = \frac{FV}{(1.0 + i)^n} \qquad \textbf{(A.2)}$$

where

PV = *Present Value* of the investment
FV = *Future Value*
i = *Interest* rate per period
n = *Number* of periods the PV is invested

Table A.2 Present Value of a Single Amount ($1)

(Used to Compute the Discounted Present Value of Some Known Future Single Lump Sum)

n	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%
1	0.9901	0.9804	0.9709	0.9615	0.9524	0.9434	0.9346	0.9259	0.9174	0.9091	0.9009	0.8929	0.8850	0.8772	0.8696	0.8621	0.8547	0.8475	0.8403	0.8333
2	0.9803	0.9612	0.9426	0.9246	0.9070	0.8900	0.8734	0.8573	0.8417	0.8264	0.8116	0.7972	0.7831	0.7695	0.7561	0.7432	0.7305	0.7182	0.7062	0.6944
3	0.9706	0.9423	0.9151	0.8890	0.8638	0.8396	0.8163	0.7938	0.7722	0.7513	0.7312	0.7118	0.6931	0.6750	0.6575	0.6407	0.6244	0.6086	0.5934	0.5787
4	0.9610	0.9238	0.8885	0.8548	0.8227	0.7921	0.7629	0.7350	0.7084	0.6830	0.6587	0.6355	0.6133	0.5921	0.5718	0.5523	0.5337	0.5158	0.4987	0.4823
5	0.9515	0.9057	0.8626	0.8219	0.7835	0.7473	0.7130	0.6806	0.6499	0.6209	0.5935	0.5674	0.5428	0.5194	0.4972	0.4761	0.4561	0.4371	0.4190	0.4019
6	0.9420	0.8880	0.8375	0.7903	0.7462	0.7050	0.6663	0.6302	0.5963	0.5645	0.5346	0.5066	0.4803	0.4556	0.4323	0.4104	0.3898	0.3704	0.3521	0.3349
7	0.9327	0.8706	0.8131	0.7599	0.7107	0.6651	0.6227	0.5835	0.5470	0.5132	0.4817	0.4523	0.4251	0.3996	0.3759	0.3538	0.3332	0.3139	0.2959	0.2791
8	0.9235	0.8535	0.7894	0.7307	0.6768	0.6274	0.5820	0.5403	0.5019	0.4665	0.4339	0.4039	0.3762	0.3506	0.3269	0.3050	0.2848	0.2660	0.2487	0.2326
9	0.9143	0.8368	0.7664	0.7026	0.6446	0.5919	0.5439	0.5002	0.4604	0.4241	0.3909	0.3606	0.3329	0.3075	0.2843	0.2630	0.2434	0.2255	0.2090	0.1938
10	0.9053	0.8203	0.7441	0.6756	0.6139	0.5584	0.5083	0.4632	0.4224	0.3855	0.3522	0.3220	0.2946	0.2697	0.2472	0.2267	0.2080	0.1911	0.1756	0.1615
11	0.8963	0.8043	0.7224	0.6496	0.5847	0.5268	0.4751	0.4289	0.3875	0.3505	0.3173	0.2875	0.2607	0.2366	0.2149	0.1954	0.1778	0.1619	0.1476	0.1346
12	0.8874	0.7885	0.7014	0.6246	0.5568	0.4970	0.4440	0.3971	0.3555	0.3186	0.2858	0.2567	0.2307	0.2076	0.1869	0.1685	0.1520	0.1372	0.1240	0.1122
13	0.8787	0.7730	0.6810	0.6006	0.5303	0.4688	0.4150	0.3677	0.3262	0.2897	0.2575	0.2292	0.2042	0.1821	0.1625	0.1452	0.1299	0.1163	0.1042	0.0935
14	0.8700	0.7579	0.6611	0.5775	0.5051	0.4423	0.3878	0.3405	0.2992	0.2633	0.2320	0.2046	0.1807	0.1597	0.1413	0.1252	0.1110	0.0985	0.0876	0.0779
15	0.8613	0.7430	0.6419	0.5553	0.4810	0.4173	0.3624	0.3152	0.2745	0.2394	0.2090	0.1827	0.1599	0.1401	0.1229	0.1079	0.0949	0.0835	0.0736	0.0649
16	0.8528	0.7284	0.6232	0.5339	0.4581	0.3936	0.3387	0.2919	0.2519	0.2176	0.1883	0.1631	0.1415	0.1229	0.1069	0.0930	0.0811	0.0708	0.0618	0.0541
17	0.8444	0.7142	0.6050	0.5134	0.4363	0.3714	0.3166	0.2703	0.2311	0.1978	0.1696	0.1456	0.1252	0.1078	0.0929	0.0802	0.0693	0.0600	0.0520	0.0451
18	0.8360	0.7002	0.5874	0.4936	0.4155	0.3503	0.2959	0.2502	0.2120	0.1799	0.1528	0.1300	0.1108	0.0946	0.0808	0.0691	0.0592	0.0508	0.0437	0.0376
19	0.8277	0.6864	0.5703	0.4746	0.3957	0.3305	0.2765	0.2317	0.1945	0.1635	0.1377	0.1161	0.0981	0.0829	0.0703	0.0596	0.0506	0.0431	0.0367	0.0313
20	0.8195	0.6730	0.5537	0.4564	0.3769	0.3118	0.2584	0.2145	0.1784	0.1486	0.1240	0.1037	0.0868	0.0728	0.0611	0.0514	0.0433	0.0365	0.0308	0.0261
21	0.8114	0.6598	0.5375	0.4388	0.3589	0.2942	0.2415	0.1987	0.1637	0.1351	0.1117	0.0926	0.0768	0.0638	0.0531	0.0443	0.0370	0.0309	0.0259	0.0217
22	0.8034	0.6468	0.5219	0.4220	0.3418	0.2775	0.2257	0.1839	0.1502	0.1228	0.1007	0.0826	0.0680	0.0560	0.0462	0.0382	0.0316	0.0262	0.0218	0.0181
23	0.7954	0.6342	0.5067	0.4057	0.3256	0.2618	0.2109	0.1703	0.1378	0.1117	0.0907	0.0738	0.0601	0.0491	0.0402	0.0329	0.0270	0.0222	0.0183	0.0151
24	0.7876	0.6217	0.4919	0.3901	0.3101	0.2470	0.1971	0.1577	0.1264	0.1015	0.0817	0.0659	0.0532	0.0431	0.0349	0.0284	0.0231	0.0188	0.0154	0.0126
25	0.7798	0.6095	0.4776	0.3751	0.2953	0.2330	0.1842	0.1460	0.1160	0.0923	0.0736	0.0588	0.0471	0.0378	0.0304	0.0245	0.0197	0.0160	0.0129	0.0105
26	0.7720	0.5976	0.4637	0.3607	0.2812	0.2198	0.1722	0.1352	0.1064	0.0839	0.0663	0.0525	0.0417	0.0331	0.0264	0.0211	0.0169	0.0135	0.0109	0.0087
27	0.7644	0.5859	0.4502	0.3468	0.2678	0.2074	0.1609	0.1252	0.0976	0.0763	0.0597	0.0469	0.0369	0.0291	0.0230	0.0182	0.0144	0.0115	0.0091	0.0073
28	0.7568	0.5744	0.4371	0.3335	0.2551	0.1956	0.1504	0.1159	0.0895	0.0693	0.0538	0.0419	0.0326	0.0255	0.0200	0.0157	0.0123	0.0097	0.0077	0.0061
29	0.7493	0.5631	0.4243	0.3207	0.2429	0.1846	0.1406	0.1073	0.0822	0.0630	0.0485	0.0374	0.0289	0.0224	0.0174	0.0135	0.0105	0.0082	0.0064	0.0051
30	0.7419	0.5521	0.4120	0.3083	0.2314	0.1741	0.1314	0.0994	0.0754	0.0573	0.0437	0.0334	0.0256	0.0196	0.0151	0.0116	0.0090	0.0070	0.0054	0.0042
40	0.6717	0.4529	0.3066	0.2083	0.1420	0.0972	0.0668	0.0460	0.0318	0.0221	0.0154	0.0107	0.0075	0.0053	0.0037	0.0026	0.0019	0.0013	0.0010	0.0007
50	0.6080	0.3715	0.2281	0.1407	0.0872	0.0543	0.0339	0.0213	0.0134	0.0085	0.0054	0.0035	0.0022	0.0014	0.0009	0.0006	0.0004	0.0003	0.0002	0.0001

Illustrations Using Table A.3: Future Value of a Series of Equal Amounts (an Annuity of $1 per Period)

To use this table, locate the future value factor for the time period and the interest rate.

1. You plan to retire after 16 years. To provide for that retirement, you initiate a savings program of $7000 per year in an investment yielding 8 percent. What will the value of the retirement fund be at the beginning of the seventeenth year?

   Your last payment into the fund will occur at the end of the sixteenth year, so scan down the periods (n) column for period 16, and then move across until you reach the column for 8 percent. The future value factor is 30.32. Hence, the solution is $7000 $\times$ 30.32, or $212,240.

2. What will be the value of an investment if you put $2000 into a retirement plan yielding 7 percent annually for 25 years?

   The future value factor is 63.250. Hence, the solution is $2000 $\times$ 63.250, or $126,500.

3. You are trying to decide between putting $3000 or $4000 annually for the next 20 years into an investment yielding 7 percent for retirement purposes. What is the difference in the value of investing the extra $1000 for 20 years?

   The future value factor is 41.0. Hence, the solution is $1000 $\times$ 41.0, or $41,000.

4. You will receive an annuity payment of $1200 at the end of each year for 6 years. What will be the total value of this stream of income invested at 7 percent by the time you receive the last payment?

   The appropriate future value factor for 6 years at 7 percent is 7.153. Hence, the solution is $1200 $\times$ 7.153, or $8584.

5. How many years of investing $1200 annually at 9 percent will it take to reach a goal of $11,000?

   Divide the future value of $11,000 by the lump sum of $1200 to find a future value factor of 9.17. Look down the 9 percent column to find 9.200 (close enough). Read across the row to find that an investment period of 7 years is needed.

6. If you plan to invest $1200 annually for 9 years, what rate of return is needed to reach a goal of $15,000?

   Divide the future value goal of $15,000 by $1200 to derive the future value factor 12.5. Look across the row for 9 years to locate the future value factor of 12.49 (close enough). Read up the column to find that you need an 8 percent return.

An alternative approach is to use a calculator to determine the total future value, FV, of a stream of equal payments (an annuity). The equation is

$$FV = \frac{[(1.0 + i)^n - 1.0] \times A}{i} \qquad \textbf{(A.3)}$$

where

FV = *Future Value* of the investment
i = *Interest* rate per period
n = *Number* of periods the PV is invested
A = *Amount* of the annuity

Table A.3 Future Value of a Series of Equal Amounts (an Annuity of $1 Paid at the End of Each Period)

(Used to Compute the Compounded Future Value of a Stream of Income Payments)

n	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0100	2.0200	2.0300	2.0400	2.0500	2.0600	2.0700	2.0800	2.0900	2.1000	2.1100	2.1200	2.1300	2.1400	2.1500	2.1600	2.1700	2.1800	2.1900	2.2000
3	3.0301	3.0604	3.0909	3.1216	3.1525	3.1836	3.2149	3.2464	3.2781	3.3100	3.3421	3.3744	3.4069	3.4396	3.4725	3.5056	3.5389	3.5724	3.6061	3.6400
4	4.0604	4.1216	4.1836	4.2465	4.3101	4.3746	4.4399	4.5061	4.5731	4.6410	4.7097	4.7793	4.8498	4.9211	4.9934	5.0665	5.1405	5.2154	5.2913	5.3680
5	5.1010	5.2040	5.3091	5.4163	5.5256	5.6371	5.7507	5.8666	5.9847	6.1051	6.2278	6.3528	6.4803	6.6101	6.7424	6.8771	7.0144	7.1542	7.2966	7.4416
6	6.1520	6.3081	6.4684	6.6330	6.8019	6.9753	7.1533	7.3359	7.5233	7.7156	7.9129	8.1152	8.3227	8.5355	8.7537	8.9775	9.2068	9.4420	9.6830	9.9299
7	7.2135	7.4343	7.6625	7.8983	8.1420	8.3938	8.6540	8.9228	9.2004	9.4872	9.7833	10.0890	10.4047	10.7305	11.0668	11.4139	11.7720	12.1415	12.5227	12.9159
8	8.2857	8.5830	8.8923	9.2142	9.5491	9.8975	10.2598	10.6366	11.0285	11.4359	11.8594	12.2997	12.7573	13.2328	13.7268	14.2401	14.7733	15.3270	15.9020	16.4991
9	9.3685	9.7546	10.1591	10.5828	11.0266	11.4913	11.9780	12.4876	13.0210	13.5795	14.1640	14.7757	15.4157	16.0853	16.7858	17.5185	18.2847	19.0859	19.9234	20.7989
10	10.4622	10.9497	11.4639	12.0061	12.5779	13.1808	13.8164	14.4866	15.1929	15.9374	16.7220	17.5487	18.4197	19.3373	20.3037	21.3215	22.3931	23.5213	24.7089	25.9587
11	11.5668	12.1687	12.8078	13.4864	14.2068	14.9716	15.7836	16.6455	17.5603	18.5312	19.5614	20.6546	21.8143	23.0445	24.3493	25.7329	27.1999	28.7551	30.4035	32.1504
12	12.6825	13.4121	14.1920	15.0258	15.9171	16.8699	17.8885	18.9771	20.1407	21.3843	22.7132	24.1331	25.6502	27.2707	29.0017	30.8502	32.8239	34.9311	37.1802	39.5805
13	13.8093	14.6803	15.6178	16.6268	17.7130	18.8821	20.1406	21.4953	22.9534	24.5227	26.2116	28.0291	29.9847	32.0887	34.3519	36.7862	39.4040	42.2187	45.2445	48.4966
14	14.9474	15.9739	17.0863	18.2919	19.5986	21.0151	22.5505	24.2149	26.0192	27.9750	30.0949	32.3926	34.8827	37.5811	40.5047	43.6720	47.1027	50.8180	54.8409	59.1959
15	16.0969	17.2934	18.5989	20.0236	21.5786	23.2760	25.1290	27.1521	29.3609	31.7725	34.4054	37.2797	40.4175	43.8424	47.5804	51.6595	56.1101	60.9653	66.2607	72.0351
16	17.2579	18.6393	20.1569	21.8245	23.6575	25.6725	27.8881	30.3243	33.0034	35.9497	39.1899	42.7533	46.6717	50.9804	55.7175	60.9250	66.6488	72.9390	79.8502	87.4421
17	18.4304	20.0121	21.7616	23.6975	25.8404	28.2129	30.8402	33.7502	36.9737	40.5447	44.5008	48.8837	53.7391	59.1176	65.0751	71.6730	78.9791	87.0680	96.0217	105.9306
18	19.6147	21.4123	23.4144	25.6454	28.1324	30.9057	33.9990	37.4502	41.3013	45.5992	50.3959	55.7497	61.7251	68.3941	75.8364	84.1407	93.4056	103.7403	115.2659	128.1167
19	20.8109	22.8406	25.1169	27.6712	30.5390	33.7600	37.3790	41.4463	46.0185	51.1591	56.9395	63.4397	70.7494	78.9692	88.2118	98.6032	110.2846	123.4135	138.1664	154.7400
20	22.0190	24.2974	26.8704	29.7781	33.0660	36.7856	40.9955	45.7620	51.1601	57.2750	64.2028	72.0524	80.9468	91.0249	102.4436	115.3797	130.0329	146.6280	165.4180	186.6880
21	23.2392	25.7833	28.6765	31.9692	35.7193	39.9927	44.8652	50.4229	56.7645	64.0025	72.2651	81.6987	92.4699	104.7684	118.8101	134.8405	153.1385	174.0210	197.8474	225.0256
22	24.4716	27.2990	30.5368	34.2480	38.5052	43.3923	49.0057	55.4568	62.8733	71.4027	81.2143	92.5026	105.4910	120.4360	137.6316	157.4150	180.1721	206.3448	236.4384	271.0307
23	25.7163	28.8450	32.4529	36.6179	41.4305	46.9958	53.4361	60.8933	69.5319	79.5430	91.1479	104.6029	120.2048	138.2970	159.2764	183.6014	211.8013	244.4868	282.3618	326.2368
24	26.9735	30.4219	34.4265	39.0826	44.5020	50.8156	58.1767	66.7648	76.7898	88.4973	102.1741	118.1552	136.8315	158.6586	184.1678	213.9776	248.8075	289.4945	337.0105	392.4842
25	28.2432	32.0303	36.4593	41.6459	47.7271	54.8645	63.2490	73.1059	84.7009	98.3471	114.4133	133.3339	155.6196	181.8708	212.7930	249.2140	292.1048	342.6035	402.0424	471.9811
26	29.5256	33.6709	38.5530	44.3117	51.1135	59.1564	68.6765	79.9544	93.3240	109.1818	127.9988	150.3339	176.8501	208.3327	245.7120	290.0883	342.7626	405.2721	479.4305	567.3773
27	30.8209	35.3443	40.7096	47.0842	54.6691	63.7058	74.4838	87.3508	102.7231	121.0999	143.0786	169.3740	200.8406	238.4993	283.5688	337.5024	402.0323	479.2211	571.5223	681.8527
28	32.1291	37.0512	42.9309	49.9676	58.4026	68.5281	80.6977	95.3388	112.9682	134.2099	159.8173	190.6989	227.9499	272.8892	327.1041	392.5027	471.3778	566.4808	681.1116	819.2233
29	33.4504	38.7922	45.2188	52.9663	62.3227	73.6398	87.3465	103.9659	124.1354	148.6309	178.3972	214.5827	258.5834	312.0937	377.1697	456.3032	552.5120	669.4474	811.5228	984.0679
30	34.7849	40.5681	47.5754	56.0849	66.4389	79.0582	94.4608	113.2832	136.3075	164.4940	199.0209	241.3327	293.1992	356.7868	434.7451	530.3117	647.4390	790.9479	966.7121	1181.882
40	48.8864	60.4020	75.4013	95.0255	120.7998	154.7620	199.6351	259.0565	337.8824	442.5925	581.8260	767.0914	1013.704	1342.025	1779.090	2360.757	3134.522	4163.212	5529.829	7343.856
50	64.4632	84.5794	112.7969	152.6671	209.3480	290.3359	406.5289	573.7701	815.0834	1163.908	1668.771	2400.018	3459.507	4994.522	7217.714	10435.65	15089.50	21813.09	31515.33	45497.17

Illustrations Using Table A.4: Present Value of Series of Equal Amounts (an Annuity of $1 per Period)

To use this table, locate the present value factor for the time period and the interest rate.

1. You are entering into a contract that will provide you with an income of $1000 at the end of the year for the next 10 years. If the annual interest rate is 7 percent, what is the present value of that stream of payments?

   The present value factor is 7.024; hence, the solution is $1000 × 7.024, or $7024.

2. You expect to have $250,000 available in a retirement plan when you retire. If the amount invested yields 8 percent and you hope to live an additional 20 years, how much can you withdraw each year so that the fund will just be liquidated after 20 years?

   The present value factor for 20 years at 8 percent is 9.818. Hence, the solution is $250,000 ÷ 9.818, or $25,463.

3. You have received an inheritance of $60,000 that you invested so that it earns 9 percent. If you withdraw $8000 annually to supplement your income, in how many years will the fund run out?

   Solving for n, $60,000 ÷ $8000 = 7.5. Scan down the 9 percent column until you find a present value factor close to 7.5, which is 7.487. The row indicates 13 years; thus, the fund will be depleted in approximately 13 years with $8000 annual withdrawals.

4. A seller offers to finance the sale of a building to you as an investment. The mortgage loan of $280,000 will be for 20 years and requires an annual mortgage payment of $24,000. Should you finance the purchase through the seller or borrow the funds from a financial institution at a current rate of 10 percent?

   $280,000 ÷ $24,000 = 11.667. Scan down the periods (n) column to 20 years and then read across to locate the figure closest to 11.667, which is 11.470. The column indicates 6 percent; thus, seller financing offers a lower interest rate.

5. You have the opportunity to purchase an office building for $750,000 with an expected life of 20 years. Looking over the financial details, you see that the before-tax net rental income is $90,000. If you want a return of at least 15 percent, how much should you pay for the building?

   The present value factor for 20 years at 15 percent is 6.259, and $90,000 × 6.259 = $563,310. Thus, the price is too high for you to earn a return of 15 percent.

An alternative approach is to use a calculator to determine the present value, *PV,* of a stream of payments. The equation is

$$PV = \frac{[1.0 - 1.0 / (1.0 + i)^n] \times A}{i} \qquad \textbf{(A.4)}$$

where

PV = *Present Value* of the investment
i = *Interest* rate per period
n = *Number* of periods the *PV* is invested
A = *Amount* of the annuity

Table A.4 Present Value of a Series of Equal Amounts (an Annuity of $1 Received at the End of Each Period)

(Used to Compute the Discounted Present Value of a Stream of Income Payments)

n	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%
1	0.9901	0.9804	0.9709	0.9615	0.9524	0.9434	0.9346	0.9259	0.9174	0.9091	0.9009	0.8929	0.8850	0.8772	0.8696	0.8621	0.8547	0.8475	0.8403	0.8333
2	1.9704	1.9416	1.9135	1.8861	1.8594	1.8334	1.8080	1.7833	1.7591	1.7355	1.7125	1.6901	1.6681	1.6467	1.6257	1.6052	1.5852	1.5656	1.5465	1.5278
3	2.9410	2.8839	2.8286	2.7751	2.7232	2.6730	2.6243	2.5771	2.5313	2.4869	2.4437	2.4018	2.3612	2.3216	2.2832	2.2459	2.2096	2.1743	2.1399	2.1065
4	3.9020	3.8077	3.7171	3.6299	3.5460	3.4651	3.3872	3.3121	3.2397	3.1699	3.1024	3.0373	2.9745	2.9137	2.8550	2.7982	2.7432	2.6901	2.6386	2.5887
5	4.8534	4.7135	4.5797	4.4518	4.3295	4.2124	4.1002	3.9927	3.8897	3.7908	3.6959	3.6048	3.5172	3.4331	3.3522	3.2743	3.1993	3.1272	3.0576	2.9906
6	5.7955	5.6014	5.4172	5.2421	5.0757	4.9173	4.7665	4.6229	4.4859	4.3553	4.2305	4.1114	3.9975	3.8887	3.7845	3.6847	3.5892	3.4976	3.4098	3.3255
7	6.7282	6.4720	6.2303	6.0021	5.7864	5.5824	5.3893	5.2064	5.0330	4.8684	4.7122	4.5638	4.4226	4.2883	4.1604	4.0386	3.9224	3.8115	3.7057	3.6046
8	7.6517	7.3255	7.0197	6.7327	6.4632	6.2098	5.9713	5.7466	5.5348	5.3349	5.1461	4.9676	4.7988	4.6389	4.4873	4.3436	4.2072	4.0776	3.9544	3.8372
9	8.5660	8.1622	7.7861	7.4353	7.1078	6.8017	6.5152	6.2469	5.9952	5.7590	5.5370	5.3282	5.1317	4.9464	4.7716	4.6065	4.4506	4.3030	4.1633	4.0310
10	9.4713	8.9826	8.5302	8.1109	7.7217	7.3601	7.0236	6.7101	6.4177	6.1446	5.8892	5.6502	5.4262	5.2161	5.0188	4.8332	4.6586	4.4941	4.3389	4.1925
11	10.3676	9.7868	9.2526	8.7605	8.3064	7.8869	7.4987	7.1390	6.8052	6.4951	6.2065	5.9377	5.6869	5.4527	5.2337	5.0286	4.8364	4.6560	4.4865	4.3271
12	11.2551	10.5753	9.9540	9.3851	8.8633	8.3838	7.9427	7.5361	7.1607	6.8137	6.4924	6.1944	5.9176	5.6603	5.4206	5.1971	4.9884	4.7932	4.6105	4.4392
13	12.1337	11.3484	10.6350	9.9856	9.3936	8.8527	8.3577	7.9038	7.4869	7.1034	6.7499	6.4235	6.1218	5.8424	5.5831	5.3423	5.1183	4.9095	4.7147	4.5327
14	13.0037	12.1062	11.2961	10.5631	9.8986	9.2950	8.7455	8.2442	7.7862	7.3667	6.9819	6.6282	6.3025	6.0021	5.7245	5.4675	5.2293	5.0081	4.8023	4.6106
15	13.8651	12.8493	11.9379	11.1184	10.3797	9.7122	9.1079	8.5595	8.0607	7.6061	7.1909	6.8109	6.4624	6.1422	5.8474	5.5755	5.3242	5.0916	4.8759	4.6755
16	14.7179	13.5777	12.5611	11.6523	10.8378	10.1059	9.4466	8.8514	8.3126	7.8237	7.3792	6.9740	6.6039	6.2651	5.9542	5.6685	5.4053	5.1624	4.9377	4.7296
17	15.5623	14.2919	13.1661	12.1657	11.2741	10.4773	9.7632	9.1216	8.5436	8.0216	7.5488	7.1196	6.7291	6.3729	6.0472	5.7487	5.4746	5.2223	4.9897	4.7746
18	16.3983	14.9920	13.7535	12.6593	11.6896	10.8276	10.0591	9.3719	8.7556	8.2014	7.7016	7.2497	6.8399	6.4674	6.1280	5.8178	5.5339	5.2732	5.0333	4.8122
19	17.2260	15.6785	14.3238	13.1339	12.0853	11.1581	10.3356	9.6036	8.9501	8.3649	7.8393	7.3658	6.9380	6.5504	6.1982	5.8775	5.5845	5.3162	5.0700	4.8435
20	18.0456	16.3514	14.8775	13.5903	12.4622	11.4699	10.5940	9.8181	9.1285	8.5136	7.9633	7.4694	7.0248	6.6231	6.2593	5.9288	5.6278	5.3527	5.1009	4.8696
21	18.8570	17.0112	15.4150	14.0292	12.8212	11.7641	10.8355	10.0168	9.2922	8.6487	8.0751	7.5620	7.1016	6.6870	6.3125	5.9731	5.6648	5.3837	5.1268	4.8913
22	19.6604	17.6580	15.9369	14.4511	13.1630	12.0416	11.0612	10.2007	9.4424	8.7715	8.1757	7.6446	7.1695	6.7429	6.3587	6.0113	5.6964	5.4099	5.1486	4.9094
23	20.4558	18.2922	16.4436	14.8568	13.4886	12.3034	11.2722	10.3711	9.5802	8.8832	8.2664	7.7184	7.2297	6.7921	6.3988	6.0442	5.7234	5.4321	5.1668	4.9245
24	21.2434	18.9139	16.9355	15.2470	13.7986	12.5504	11.4693	10.5288	9.7066	8.9847	8.3481	7.7843	7.2829	6.8351	6.4338	6.0726	5.7465	5.4509	5.1822	4.9371
25	22.0232	19.5235	17.4131	15.6221	14.0939	12.7834	11.6536	10.6748	9.8226	9.0770	8.4217	7.8431	7.3300	6.8729	6.4641	6.0971	5.7662	5.4669	5.1951	4.9476
26	22.7952	20.1210	17.8768	15.9828	14.3752	13.0032	11.8258	10.8100	9.9290	9.1609	8.4881	7.8957	7.3717	6.9061	6.4906	6.1182	5.7831	5.4804	5.2060	4.9563
27	23.5596	20.7069	18.3270	16.3296	14.6430	13.2105	11.9867	10.9352	10.0266	9.2372	8.5478	7.9426	7.4086	6.9352	6.5135	6.1364	5.7975	5.4919	5.2151	4.9636
28	24.3164	21.2813	18.7641	16.6631	14.8981	13.4062	12.1371	11.0511	10.1161	9.3066	8.6016	7.9844	7.4412	6.9607	6.5335	6.1520	5.8099	5.5016	5.2228	4.9697
29	25.0658	21.8444	19.1885	16.9837	15.1411	13.5907	12.2777	11.1584	10.1983	9.3696	8.6501	8.0218	7.4701	6.9830	6.5509	6.1656	5.8204	5.5098	5.2292	4.9747
30	25.8077	22.3965	19.6004	17.2920	15.3725	13.7648	12.4090	11.2578	10.2737	9.4269	8.6938	8.0552	7.4957	7.0027	6.5660	6.1772	5.8294	5.5168	5.2347	4.9789
40	32.8347	27.3555	23.1148	19.7928	17.1591	15.0463	13.3317	11.9246	10.7574	9.7791	8.9511	8.2438	7.6344	7.1050	6.6418	6.2335	5.8713	5.5482	5.2582	4.9966
50	39.1961	31.4236	25.7298	21.4822	18.2559	15.7619	13.8007	12.2335	10.9617	9.9148	9.0417	8.3045	7.6752	7.1327	6.6605	6.2463	5.8801	5.5541	5.2623	4.9995